Engineering Weekly Exam 03 [Set 02] EV - Answer Sheet

Complex Numbers + Polynomials and Polynomial Equations + Newtonian Mechanics + Quantitative Chemistry

Total Marks: 600 **Time:** 3 Hours

Physics: 200 Mathematics: 200 Chemistry: 200

Physics 200 marks

1. **Since both blocks move with the same acceleration, apply Newton's second law to the whole system, subtracting the total friction.**

$f = (4+6)(0.20)(9.8) = 19.6\,\text{N}$

$a = \frac{40-19.6}{4+6} = 2.04\,\text{ms}^{-2}$

For the $6\,\text{kg}$ **block,** $T - f_2 = 6a \Rightarrow T = 6(2.04) + 6(0.20)(9.8) = 24.0\,\text{N}$

The tension accelerates the $6\,\text{kg}$ **block to the right; subtracting its own friction and the tension from the force applied to the** $4\,\text{kg}$ **block gives the same acceleration.**

2. **First check the limit of static friction.**

$N = 2g\cos 30° = 16.97\,\text{N}$, $f_{s,\max} = 0.40N = 6.79\,\text{N}$

The friction needed on the incline to stay at rest is $3g - 2g\sin 30° = 19.6\,\text{N}$**, which exceeds** $f_{s,\max}$**; so the** $3\,\text{kg}$ **mass descends.**

$f_k = 0.25(16.97) = 4.24\,\text{N}$

$a = \frac{3g-2g\sin 30°-f_k}{3+2} = 3.07\,\text{ms}^{-2}$

$T - 2g\sin 30° - f_k = 2a \Rightarrow T = 20.2\,\text{N}$

3. **The centrifugal pseudo-force is** $\vec{F}_{cf} = -m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = m\Omega^2\vec{r}$

$\vec{F}_{cf} = 0.5(4)^2(2\hat{i}) = 16\hat{i}\,\text{N}$

The Coriolis pseudo-force is $\vec{F}_{cor} = -2m\vec{\Omega}\times\vec{v}_{rel}$

$\vec{F}_{cor} = -2(0.5)(4\hat{k}\times 3\hat{j}) = 12\hat{i}\,\text{N}$

The Euler pseudo-force is $\vec{F}_E = -m\left(\frac{d\vec{\Omega}}{dt}\right)\times\vec{r}$

$\vec{F}_E = -0.5(1.5\hat{k}\times 2\hat{i}) = -1.5\hat{j}\,\text{N}$

Total pseudo-force $\vec{F}_{pseudo} = 28\hat{i} - 1.5\hat{j}\,\text{N}$

The centrifugal force points away from the axis, the Coriolis force is perpendicular to the relative velocity, and the Euler force arises from the angular acceleration.

4. **In the ground frame, the truck's velocity adds to the horizontal component, while the vertical component is unchanged.**

$u_x = 15 + 20\cos 30° = 32.32\,\text{ms}^{-1}$, $u_y = 20\sin 30° = 10\,\text{ms}^{-1}$

$10 + 10t - 4.9t^2 = 0 \Rightarrow t = 2.78\,\text{s}$

$x = u_x t = 32.32(2.78) = 89.8\,\text{m}$

5. **Energy conservation relates the speeds at the bottom and the top.**

$v_{top}^2 = v_{bottom}^2 - 4gR = 20^2 - 4(9.8)(5) = 204$

$v_{top} = 14.28\,\text{ms}^{-1}$

$N_{bottom} - mg = \frac{mv_{bottom}^2}{R} \Rightarrow N_{bottom} = mg + 1000\left(\frac{400}{5}\right) = 89.8\,\text{kN}$

$N_{top} + mg = \frac{mv_{top}^2}{R} \Rightarrow N_{top} = 1000\left(\frac{204}{5} - 9.8\right) = 31.0\,\text{kN}$

At both points, the normal force points toward the centre of the circle; at the bottom it is upward, and at the top it is downward.

6. $\vec{J} = m(\vec{v}_f - \vec{v}_i) = 0.20[(-10-15)\hat{i} + (6-6)\hat{j}] = -5\hat{i}\,\text{Ns}$

$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t} = \frac{-5\hat{i}}{0.020} = -250\hat{i}\,\text{N}$

The wall changes only the perpendicular component; the tangential component's impulse and average force are zero.

$|\vec{J}| = 5\,\text{Ns}$, $|\vec{F}_{avg}| = 250\,\text{N}$

7. **With no external horizontal force, the displacement of the centre of mass is zero. Let the boat's displacement be x, taken in the opposite direction.**

$240x + 80(x + 2.5) = 0 \Rightarrow x = -\frac{80(2.5)}{240+80} = -0.625\ \text{m}$

The person's displacement relative to the ground $= 2.5 - 0.625 = 1.875\ \text{m}$

The boat's length does not affect the result; only the relative displacement the person walked matters.

8. $J = \int_0^3 (6 - 2t)\,dt = \left[6t - t^2\right]_0^3 = 9\ \text{Ns}$

$\Delta v = \frac{J}{m} = \frac{9}{2} = 4.5\ \text{ms}^{-1}$

$v(3) = 1 + 4.5 = 5.5\ \text{ms}^{-1}$ **to the right**

Although the force is zero at $t = 3\ \text{s}$**, the total impulse is not zero; it is the area under the force-time curve over the whole interval.**

9. $\omega(t) = \omega_0 + \int_0^t 4t\,dt = 2 + 2t^2$

$\omega(3) = 2 + 2(9) = 20\ \text{rads}^{-1}$

$\theta = \int_0^3 (2 + 2t^2)\,dt = 2(3) + \frac{2}{3}(27) = 24\ \text{rad}$

$\alpha(3) = 4(3) = 12\ \text{rads}^{-2}$

$a_t = \alpha R = 12(0.30) = 3.6\ \text{ms}^{-2}$

$a_c = \omega^2 R = 20^2(0.30) = 120\ \text{ms}^{-2}$

10. **The distance of each mass from the axis is** $r = \frac{0.40}{\sqrt{2}}\ \text{m}$

$I = 4mr^2 = 4(1)\left(\frac{0.40^2}{2}\right) = 0.32\ \text{kg m}^2$

$\alpha = \frac{\tau}{I} = \frac{2}{0.32} = 6.25\ \text{rads}^{-2}$

$\omega = \alpha t = 6.25(5) = 31.25\ \text{rads}^{-1}$

$K = \frac{1}{2}I\omega^2 = 156.25\ \text{J}$

11. **The moment of inertia of a solid cylinder is** $I = \frac{1}{2}mR^2$**; for pure rolling,** $v = R\omega$**.**

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{3}{4}mv^2$

$v = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4(9.8)(1.4)}{3}} = 4.28\ \text{ms}^{-1}$

$\omega = \frac{v}{R} = \frac{4.28}{0.20} = 21.4\ \text{rads}^{-1}$

$a = \frac{g \sin 37^\circ}{1+\frac{1}{2}} = 3.92\ \text{ms}^{-2}$

$f = mg \sin 37^\circ - ma = 7.84\ \text{N}$

The friction acts up the incline and accelerates the cylinder's rotation; static friction is needed to maintain pure rolling.

12. **The wall's reaction** R **on the ladder is horizontal, and the floor's normal reaction is** $N = mg$**. Take torques about the bottom end.**

$R(L \sin 60^\circ) = mg\left(\frac{L}{2}\right)\cos 60^\circ$

$R = \frac{mg \cos 60^\circ}{2 \sin 60^\circ} = \frac{mg}{2\sqrt{3}}$

The floor's friction $f = R$ **and** $f \le \mu_s mg$

$\mu_{s,\min} = \frac{R}{mg} = \frac{1}{2\sqrt{3}} = 0.289$

13. **Find the mass striking the plate per second from the cross-sectional area of the flow.**

$A = \pi(0.0075)^2 = 1.767 \times 10^{-4}\ \text{m}^2$

$\dot{m} = \varrho A v = 1000(1.767 \times 10^{-4})(30) = 5.30\ \text{kgs}^{-1}$

$F = \dot{m}(v_{in} - v_{out}) = 5.30[30 - (-30)] = 318\ \text{N}$

Since the jet's momentum reverses completely, the rate of change of momentum is $\dot{m}$ **multiplied by** $60\ \text{ms}^{-1}$**.**

14. $m(t) = 10 + 0.50t$**; since horizontal momentum is conserved,** $mv = m_0 v_0 = 20\ \text{kg ms}^{-1}$

$v(4) = \frac{20}{10+0.50(4)} = 1.667\ \text{ms}^{-1}$

$m\frac{dv}{dt} = -v\frac{dm}{dt} \Rightarrow a = -\frac{(0.50)(1.667)}{12} = -0.0694\ \text{ms}^{-2}$

Horizontal momentum is conserved as the stationary sand joins the cart, but the increasing total mass reduces the velocity.

15. **For equal masses and a stationary target, apply momentum and kinetic energy conservation together.**

$u^2 = v_1^2 + v_2^2$

From the y-component of momentum, $0 = v_1 \sin 30° - v_2 \sin \phi$

From the x-component and the energy equation, $v_1 = u \cos 30° = 8.66\,\text{ms}^{-1}$**,** $v_2 = u \sin 30° = 5.00\,\text{ms}^{-1}$

$\phi = 60°$**; the two final velocities are perpendicular to each other.**

16. **Let** A **be the wedge's acceleration; work out the normal reaction on the block in the wedge's frame.**

$N = m(g \cos 30° + A \sin 30°)$

For equilibrium along the incline, $|m(g \sin 30° - A \cos 30°)| \leq \mu_s N$

At the lower limit (A_{min}**, friction at its maximum acting up the slope),** $m(g \sin 30° - A \cos 30°) = \mu_s m(g \cos 30° + A \sin 30°)$

$A_{min} = \frac{g(\sin 30° - \mu_s \cos 30°)}{\cos 30° + \mu_s \sin 30°} = 3.32\,\text{ms}^{-2}$

At the upper limit (A_{max}**, friction at its maximum acting down the slope),** $m(A \cos 30° - g \sin 30°) = \mu_s m(g \cos 30° + A \sin 30°)$

$A_{max} = \frac{g(\sin 30° + \mu_s \cos 30°)}{\cos 30° - \mu_s \sin 30°} = 8.62\,\text{ms}^{-2}$

Since the wedge and block move together with the same acceleration, $P = (M + m)A$**, where** $M + m = 8\,\text{kg}$**.**

$P_{min} = 8(3.32) = 26.5\,\text{N}$**,** $P_{max} = 8(8.62) = 68.9\,\text{N}$

Within this range, friction can act in whichever direction is needed; outside it, the block slips relative to the wedge.

17. $v = \frac{dx}{dt} = 6t^2 - 6t \Rightarrow v(2) = 12\,\text{ms}^{-1}$

$a = \frac{dv}{dt} = 12t - 6 \Rightarrow a(2) = 18\,\text{ms}^{-2}$

$F = ma = 2(18) = 36\,\text{N}$

$v(0) = 0$**; from the work-energy theorem,** $W_{net} = \Delta K = \frac{1}{2}(2)(12^2 - 0) = 144\,\text{J}$

The work-energy theorem gives the work of the net force without needing a displacement-based integration.

18. $I = mr^2 = 2(0.5)^2 = 0.50\,\text{kg m}^2$**,** $\tau = F r = 4(0.5) = 2\,\text{Nm}$

$\alpha = \frac{\tau}{I} = \frac{2}{0.50} = 4\,\text{rads}^{-2}$

$\omega = 6 + 4(3) = 18\,\text{rads}^{-1}$

$\Delta L = I(\omega - \omega_0) = 0.50(12) = 6\,\text{kg m}^2\text{s}^{-1}$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 18 + 18 = 36\,\text{rad}$

$W = \tau\theta = 72\,\text{J}$

19. $x_{cm} = \frac{2(0)+3(4)+5(0)}{10} = 1.2\,\text{m}$

$y_{cm} = \frac{2(0)+3(0)+5(3)}{10} = 1.5\,\text{m}$

$\vec{F}_{ext} = 3\hat{i} + (-\hat{i} + 2\hat{j}) + 4\hat{j} = 2\hat{i} + 6\hat{j}\,\text{N}$

$\vec{a}_{cm} = \frac{\vec{F}_{ext}}{M} = \frac{2\hat{i}+6\hat{j}}{10} = 0.2\hat{i} + 0.6\hat{j}\,\text{ms}^{-2}$

The internal forces cancel each other out; the acceleration of the centre of mass depends only on the total external force.

20. **Initial momentum** $= (100 + 50)(2) = 300\,\text{kg ms}^{-1}$

$v_{person} - v_{cart} = 4$**,** $50v_{person} + 100v_{cart} = 300$

Solving the two equations, $v_{cart} = 0.667\,\text{ms}^{-1}$**,** $v_{person} = 4.667\,\text{ms}^{-1}$

$K_i = \frac{1}{2}(150)(2)^2 = 300\,\text{J}$

$K_f = \frac{1}{2}(100)(0.667)^2 + \frac{1}{2}(50)(4.667)^2 = 566.7\,\text{J}$

Increase in kinetic energy $= 566.7 - 300 = 266.7\,\text{J}$

Although horizontal momentum is conserved, the person's muscular energy input increases the total kinetic energy.

Mathematics — 200 marks

21. **Let** $z^6 = -729$

We know, $-1 = \cos\pi + i\sin\pi = e^{i\pi}$

In general form, $-1 = e^{i(2k\pi+\pi)}$ **where** $k \in \mathbb{Z}$

$\therefore z^6 = 729e^{i(2k\pi+\pi)} = 3^6 e^{i(2k\pi+\pi)}$

Raising both sides to the power $\frac{1}{6}$, by De Moivre's Theorem:

$z = (3^6)^{\frac{1}{6}} e^{i\frac{(2k+1)\pi}{6}}$

$\Rightarrow z = 3\left[\cos\left(\frac{2k+1}{6}\pi\right) + i\sin\left(\frac{2k+1}{6}\pi\right)\right]$

To get the 6 roots, putting $k = 0, 1, 2, 3, 4, 5$:

$k = 0 \Rightarrow z_1 = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 3\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$k = 1 \Rightarrow z_2 = 3\left(\cos\frac{3\pi}{6} + i\sin\frac{3\pi}{6}\right) = 3(0 + i) = 3i$

$k = 2 \Rightarrow z_3 = 3\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 3\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$k = 3 \Rightarrow z_4 = 3\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = 3\left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$

$k = 4 \Rightarrow z_5 = 3\left(\cos\frac{9\pi}{6} + i\sin\frac{9\pi}{6}\right) = 3(0 - i) = -3i$

$k = 5 \Rightarrow z_6 = 3\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = 3\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} - \frac{3}{2}i$

22. **Let $z = x + iy$**

Given equation: $|x + iy - 5| - |x + iy + 5| = 8$

$\Rightarrow \sqrt{(x-5)^2 + y^2} - \sqrt{(x+5)^2 + y^2} = 8$

$\Rightarrow \sqrt{(x-5)^2 + y^2} = 8 + \sqrt{(x+5)^2 + y^2}$

$\Rightarrow (x-5)^2 + y^2 = 64 + 16\sqrt{(x+5)^2 + y^2} + (x+5)^2 + y^2$

$\Rightarrow x^2 - 10x + 25 = 64 + 16\sqrt{(x+5)^2 + y^2} + x^2 + 10x + 25$

$\Rightarrow -20x - 64 = 16\sqrt{(x+5)^2 + y^2}$

$\Rightarrow -5x - 16 = 4\sqrt{(x+5)^2 + y^2}$

$\Rightarrow 25x^2 + 160x + 256 = 16(x^2 + 10x + 25 + y^2)$

$\Rightarrow 25x^2 + 160x + 256 = 16x^2 + 160x + 400 + 16y^2$

$\Rightarrow 9x^2 - 16y^2 = 144$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$

This represents the standard equation of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (Proved)

Here $a^2 = 16$ and $b^2 = 9$

Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$.

23. **Given,** $(1 + x)^n = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + \ldots$

Putting $x = i$ in this equation:

$(1 + i)^n = c_0 + c_1i + c_2i^2 + c_3i^3 + c_4i^4 + \ldots$

We know, $i^2 = -1, i^3 = -i, i^4 = 1$**, etc.**

$\therefore (1 + i)^n = c_0 + ic_1 - c_2 - ic_3 + c_4 + \ldots$

$\Rightarrow (1 + i)^n = (c_0 - c_2 + c_4 - \ldots) + i(c_1 - c_3 + c_5 - \ldots)$ **.....(i)**

Modulus $r = \sqrt{1^2 + 1^2} = \sqrt{2}$ **and argument** $\theta = \tan^{-1}(1) = \frac{\pi}{4}$

$\therefore 1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Applying De Moivre's Theorem:

$(1 + i)^n = (\sqrt{2})^n\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right) = 2^{\frac{n}{2}}\cos\frac{n\pi}{4} + i2^{\frac{n}{2}}\sin\frac{n\pi}{4}$ **.....(ii)**

Comparing equations (i) and (ii) and equating real and imaginary parts:

Real part: $c_0 - c_2 + c_4 - \cdots = 2^{\frac{n}{2}}\cos\left(\frac{n\pi}{4}\right)$ **(Proved)**

Imaginary part: $c_1 - c_3 + c_5 - \cdots = 2^{\frac{n}{2}}\sin\left(\frac{n\pi}{4}\right)$ **(Proved)**

24. **Given equation:** $|z - (4 + 3i)| = 2$

This represents a circle on the Argand plane with centre $C(4, 3)$ and radius $R = 2$.

The distance from any point z on the circle to the origin $O(0, 0)$ is $|z|$.

By geometric property, if d is the distance from the centre of the circle to the origin,

Maximum distance $= d + R$

Minimum distance $= |d - R|$

Distance from the centre $C(4, 3)$ to the origin $O(0, 0)$,

$d = OC = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$\therefore$ **Maximum distance from the origin to a point on the circle** $= 5 + 2 = 7$ **units**

and minimum distance $= 5 - 2 = 3$ **units.**

25. **Given,** $x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow \frac{x^2+1}{x} = 2\cos\theta$

$\Rightarrow x^2 - 2x\cos\theta + 1 = 0$

This is a quadratic equation in x. Solving it:

$x = \frac{-(-2\cos\theta) \pm \sqrt{(-2\cos\theta)^2 - 4(1)(1)}}{2(1)}$

$= \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \cos\theta \pm \sqrt{-(1-\cos^2\theta)} = \cos\theta \pm \sqrt{-\sin^2\theta} = \cos\theta \pm i\sin\theta$

Let, $x = \cos\theta + i\sin\theta$

(If we take the negative sign, the proof works out the same way).

Applying De Moivre's Theorem:

$x^n = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ **.....(i)**

and $\frac{1}{x^n} = x^{-n} = (\cos\theta + i\sin\theta)^{-n} = \cos(-n\theta) + i\sin(-n\theta)$

$\Rightarrow \frac{1}{x^n} = \cos n\theta - i\sin n\theta$ **.....(ii)**

Adding (i) and (ii):

$x^n + \frac{1}{x^n} = (\cos n\theta + i\sin n\theta) + (\cos n\theta - i\sin n\theta)$

$= 2\cos n\theta$ **(Proved)**

26. **Let** $w = \frac{z_1+z_2}{z_1-z_2}$

A complex number w is purely imaginary if and only if $\bar{w} = -w$.

Given, $|z_1| = 1 \Rightarrow |z_1|^2 = 1 \Rightarrow z_1\bar{z_1} = 1 \Rightarrow \bar{z_1} = \frac{1}{z_1}$

Similarly, $|z_2| = 1 \Rightarrow \bar{z_2} = \frac{1}{z_2}$

Now we find the conjugate of w:

$\bar{w} = \overline{\left(\frac{z_1+z_2}{z_1-z_2}\right)} = \frac{\bar{z_1}+\bar{z_2}}{z_1-z_2}$

$\Rightarrow \bar{w} = \frac{\frac{1}{z_1}+\frac{1}{z_2}}{\frac{1}{z_1}-\frac{1}{z_2}} = \frac{\frac{z_2+z_1}{z_1z_2}}{\frac{z_2-z_1}{z_1z_2}} = \frac{z_1+z_2}{-(z_1-z_2)}$

$\Rightarrow \bar{w} = -\left(\frac{z_1+z_2}{z_1-z_2}\right)$

$\Rightarrow \bar{w} = -w$

Since $\bar{w} = -w$**, therefore** $\frac{z_1+z_2}{z_1-z_2}$ **is a purely imaginary number. (Proved)**

27. **Let** $z = x + iy$

Given equation: $\left|\frac{x+iy-3}{x+iy+3}\right| = 2$

$\Rightarrow |(x-3)+iy| = 2|(x+3)+iy|$

$\Rightarrow (x-3)^2 + y^2 = 4[(x+3)^2 + y^2]$

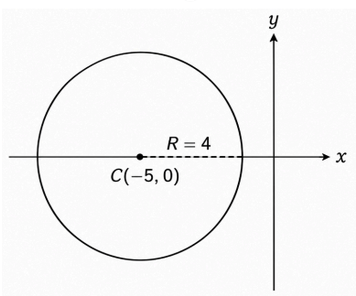
$\Rightarrow x^2 - 6x + 9 + y^2 = 4(x^2 + 6x + 9 + y^2)$

$\Rightarrow x^2 - 6x + 9 + y^2 = 4x^2 + 24x + 36 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 + 30x + 27 = 0$

$\Rightarrow x^2 + y^2 + 10x + 9 = 0$

This is of the general form of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$**. (Proved)**



Centre $(-g, -f) = (-5, 0)$

Radius $R = \sqrt{g^2 + f^2 - c} = \sqrt{(-5)^2 + 0^2 - 9} = \sqrt{25-9} = \sqrt{16} = 4$ **units.**

28. **Let the complex number be** $z = r(\cos\theta + i\sin\theta)$
Given, $\theta = \frac{\pi}{4}$
$\therefore z = r\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \frac{r}{\sqrt{2}} + i\frac{r}{\sqrt{2}}$
Given, $|z - 6| = 3\sqrt{2}$
$\Rightarrow \left|\left(\frac{r}{\sqrt{2}} - 6\right) + i\frac{r}{\sqrt{2}}\right| = 3\sqrt{2}$
$\Rightarrow \left(\frac{r}{\sqrt{2}} - 6\right)^2 + \left(\frac{r}{\sqrt{2}}\right)^2 = 18$
$\Rightarrow \frac{r^2}{2} - 6\sqrt{2}r + 36 + \frac{r^2}{2} = 18$
$\Rightarrow r^2 - 6\sqrt{2}r + 18 = 0$
$\Rightarrow (r - 3\sqrt{2})^2 = 0$
$\Rightarrow r = 3\sqrt{2}$
Now putting the value of r **into** z**:**
$z = \frac{3\sqrt{2}}{\sqrt{2}} + i\frac{3\sqrt{2}}{\sqrt{2}} = 3 + 3i$**.**

29. **Let** $y = \frac{x^2-2x+4}{x^2+2x+4}$
$\Rightarrow y(x^2 + 2x + 4) = x^2 - 2x + 4$
$\Rightarrow yx^2 + 2yx + 4y - x^2 + 2x - 4 = 0$
$\Rightarrow (y - 1)x^2 + 2(y + 1)x + 4(y - 1) = 0$ **.....(i)**
Since x **is real, considering equation (i) as a quadratic in** x**, its discriminant** $D \geq 0$**.**
$D = b^2 - 4ac \geq 0$
$\Rightarrow \{2(y + 1)\}^2 - 4(y - 1)\{4(y - 1)\} \geq 0$
$\Rightarrow 4(y + 1)^2 - 16(y - 1)^2 \geq 0$
$\Rightarrow (y + 1)^2 - 4(y - 1)^2 \geq 0$
$\Rightarrow (y + 1)^2 - \{2(y - 1)\}^2 \geq 0$
$\Rightarrow \{(y + 1) - (2y - 2)\}\{(y + 1) + (2y - 2)\} \geq 0$
$\Rightarrow (3 - y)(3y - 1) \geq 0$
$\Rightarrow (y - 3)(3y - 1) \leq 0$
By the rule of inequalities, the solution is: $\frac{1}{3} \leq y \leq 3$
$\therefore$ **The value of** $f(x)$ **lies between** $\frac{1}{3}$ **and** 3**. (Shown)**

30. **The given points are:** $A(z), B(iz)$ **and** $C(z + iz)$**.**
Now, we find the lengths of the sides of the triangle:
$AB = |z - iz| = |z(1 - i)| = |z| \cdot \sqrt{1^2 + (-1)^2} = \sqrt{2}|z|$
$BC = |iz - (z + iz)| = |-z| = |z|$
$CA = |(z + iz) - z| = |iz| = |i| \cdot |z| = 1 \cdot |z| = |z|$
Here, $BC = CA = |z|$
So, the triangle is isosceles.
Also, considering the squares of the sides:
$BC^2 + CA^2 = |z|^2 + |z|^2 = 2|z|^2$
$AB^2 = (\sqrt{2}|z|)^2 = 2|z|^2$
That is, $BC^2 + CA^2 = AB^2$**, which supports the Pythagorean theorem.**
So, ABC **is a right triangle with** $\angle C = 90°$**.**
$\therefore ABC$ **is an isosceles right triangle. (Proved)**
Area of a right triangle $= \frac{1}{2} \times$ base $\times$ height
$\Rightarrow \Delta = \frac{1}{2} \times BC \times CA = \frac{1}{2} \times |z| \times |z| = \frac{1}{2}|z|^2$
According to the question, the area of the triangle is 18 **square units.**
$\therefore \frac{1}{2}|z|^2 = 18$
$\Rightarrow |z|^2 = 36$
$\Rightarrow |z| = 6$ **(since the modulus is always non-negative)**
Given, the argument of z**,** $\theta = \frac{\pi}{6}$
Using polar form we can write:
$z = |z|(\cos\theta + i\sin\theta)$
$\Rightarrow z = 6\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
$\Rightarrow z = 6\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$
$\Rightarrow z = 3\sqrt{3} + 3i$**.**

31. **Given equation:** $x^3 + 2x^2 - 3x + 4 = 0$

Since the roots of the equation are α, β, γ, the sum of the roots:

$\sum \alpha = \alpha + \beta + \gamma = -2$

Expressing the roots of the required equation using the variable y,

$y = \frac{\alpha}{\beta+\gamma}$

We know, $\beta + \gamma = -2 - \alpha$

$\therefore y = \frac{\alpha}{-2-\alpha}$

Now we express α in terms of y:

$y(-2 - \alpha) = \alpha$

$\Rightarrow -2y - \alpha y = \alpha$

$\Rightarrow \alpha + \alpha y = -2y$

$\Rightarrow \alpha(1 + y) = -2y$

$\Rightarrow \alpha = \frac{-2y}{y+1}$

Since α is a root of the original equation, it satisfies $x^3 + 2x^2 - 3x + 4 = 0$.

That is, putting $x = \frac{-2y}{y+1}$ into the equation:

$\left(\frac{-2y}{y+1}\right)^3 + 2\left(\frac{-2y}{y+1}\right)^2 - 3\left(\frac{-2y}{y+1}\right) + 4 = 0$

Multiplying both sides by $(y + 1)^3$:

$-8y^3 + 2(4y^2)(y + 1) - 3(-2y)(y + 1)^2 + 4(y + 1)^3 = 0$

$\Rightarrow -8y^3 + 8y^2(y + 1) + 6y(y + 1)^2 + 4(y + 1)^3 = 0$

$\Rightarrow -8y^3 + 8y^3 + 8y^2 + 6y(y^2 + 2y + 1) + 4(y^3 + 3y^2 + 3y + 1) = 0$

$\Rightarrow 8y^2 + 6y^3 + 12y^2 + 6y + 4y^3 + 12y^2 + 12y + 4 = 0$

$\Rightarrow (6 + 4)y^3 + (8 + 12 + 12)y^2 + (6 + 12)y + 4 = 0$

$\Rightarrow 10y^3 + 32y^2 + 18y + 4 = 0$

$\therefore 5x^3 + 16x^2 + 9x + 2 = 0$**.**

32. **Let the common root of the two equations be α**

Then,

$a\alpha^2 + b\alpha + c = 0$ **.....(i)**

$b\alpha^2 + c\alpha + a = 0$ **.....(ii)**

Solving by the cross-multiplication method:

$\frac{\alpha^2}{ab-c^2} = \frac{\alpha}{bc-a^2} = \frac{1}{ca-b^2}$

From this we get:

$\alpha = \frac{bc-a^2}{ca-b^2}$ **and** $\alpha^2 = \frac{ab-c^2}{ca-b^2}$

Squaring the first relation and equating it with the second:

$\left(\frac{bc-a^2}{ca-b^2}\right)^2 = \frac{ab-c^2}{ca-b^2}$

$\Rightarrow (bc - a^2)^2 = (ab - c^2)(ca - b^2)$

$\Rightarrow b^2c^2 - 2a^2bc + a^4 = a^2bc - ab^3 - ac^3 + b^2c^2$

$\Rightarrow a^4 + ab^3 + ac^3 - 3a^2bc = 0$

Assuming $a \neq 0$ and dividing by a:

$a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

Either $a + b + c = 0$

Or, $a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow \frac{1}{2}\{(a - b)^2 + (b - c)^2 + (c - a)^2\} = 0$

which is possible only when $a = b = c$**. (Proved)**

33. **Given equation** $x^3 + 2x^2 + 3x + 4 = 0$

From the relations between the roots of the equation:

$\sum \alpha = \alpha + \beta + \gamma = -2$

$\sum \alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = 3$

$\alpha\beta\gamma = -4$

We know, $\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta$

$= (-2)^2 - 2(3) = 4 - 6 = -2$

By the algebraic identity, we know,

$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$

$\Rightarrow \sum \alpha^3 - 3\alpha\beta\gamma = (\sum \alpha)(\sum \alpha^2 - \sum \alpha\beta)$

Substituting values:

$\sum \alpha^3 - 3(-4) = (-2)(-2 - 3)$

$\Rightarrow \sum \alpha^3 + 12 = (-2)(-5)$

$\Rightarrow \sum \alpha^3 + 12 = 10$

$\Rightarrow \sum \alpha^3 = 10 - 12 = -2$

$\therefore \sum \alpha^3 = -2$**.**

34. **Since the roots are in the ratio** $m : n$**, let the roots of the equation be** $m\alpha$ **and** $n\alpha$

Sum of roots:

$m\alpha + n\alpha = -\frac{b}{a}$

$\Rightarrow \alpha(m + n) = -\frac{b}{a}$

$\Rightarrow \alpha = \frac{-b}{a(m+n)}$ **.....(i)**

Product of roots:

$(m\alpha)(n\alpha) = \frac{c}{a}$

$\Rightarrow mn\alpha^2 = \frac{c}{a}$ **.....(ii)**

Putting the value of α **from (i) into (ii):**

$mn\left(\frac{-b}{a(m+n)}\right)^2 = \frac{c}{a}$

$\Rightarrow \frac{mnb^2}{a^2(m+n)^2} = \frac{c}{a}$

$\therefore mnb^2 = ac(m + n)^2$ **(Proved)**

35. **Since the three roots of the cubic equation are in arithmetic progression (A.P), let the three roots be:** $a - d, a, a + d$

Sum of the roots:

$(a - d) + a + (a + d) = -\frac{-12}{1}$

$\Rightarrow 3a = 12$

$\Rightarrow a = 4$

Since $a = 4$ **is a root of the equation, it satisfies it.**

Putting 4 **in place of** x**:**

$(4)^3 - 12(4)^2 + 39(4) + k = 0$

$\Rightarrow 64 - 12(16) + 156 + k = 0$

$\Rightarrow 64 - 192 + 156 + k = 0$

$\Rightarrow 28 + k = 0 \Rightarrow k = -28$

Now, from the sum of products of roots taken two at a time:

$(a - d)a + a(a + d) + (a - d)(a + d) = \frac{39}{1}$

$\Rightarrow a^2 - ad + a^2 + ad + a^2 - d^2 = 39$

$\Rightarrow 3a^2 - d^2 = 39$

Putting $a = 4$**:**

$3(4)^2 - d^2 = 39$

$\Rightarrow 3(16) - d^2 = 39$

$\Rightarrow 48 - d^2 = 39$

$\Rightarrow d^2 = 48 - 39 = 9$

$\Rightarrow d = \pm 3$

Whether d **is positive or negative, the roots are the same.**

For $d = 3$**, the roots are:**

$4 - 3, 4, 4 + 3$

That is, $1, 4, 7$

$\therefore$ **The value of** k **is** -28 **and the required roots are:** $1, 4, 7$**.**

36. **We know that, in an equation with real coefficients, complex roots always occur in conjugate pairs.**

Since one root of the equation is $2 - i\sqrt{3}$**, another root must be** $2 + i\sqrt{3}$**.**

Let the third root of the cubic equation be α**.**

According to the question, sum of roots $= 5$

$\therefore (2 - i\sqrt{3}) + (2 + i\sqrt{3}) + \alpha = 5$

$\Rightarrow 4 + \alpha = 5$

$\Rightarrow \alpha = 1$

So the three roots of the equation are: $1, 2 - i\sqrt{3}$ **and** $2 + i\sqrt{3}$**.**

Quadratic factor formed by the complex conjugate roots:

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$\Rightarrow x^2 - (4)x + (2^2 + (\sqrt{3})^2) = 0$

$\Rightarrow x^2 - 4x + 7 = 0$

The factor for the root $\alpha = 1$ **is** $(x - 1)$**.**

$\therefore$ **The complete cubic equation will be:**

$(x - 1)(x^2 - 4x + 7) = 0$

$\Rightarrow x^3 - 4x^2 + 7x - x^2 + 4x - 7 = 0$

$\Rightarrow x^3 - 5x^2 + 11x - 7 = 0$**.**

37. **Given, the roots of** $x^2 - 3x + 5 = 0$ **are** α, β

$\therefore \alpha + \beta = 3$ **and** $\alpha\beta = 5$

Let the first root of the new equation be r_1**:**

$r_1 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 3^2 - 2(5) = 9 - 10 = -1$

Let the second root of the new equation be r_2**:**

$r_2 = \alpha^{-2} + \beta^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$= \frac{r_1}{(\alpha\beta)^2} = \frac{-1}{5^2} = -\frac{1}{25}$

Sum of roots of the new equation:

$S = r_1 + r_2 = -1 - \frac{1}{25} = -\frac{26}{25}$

Product of roots of the new equation:

$P = r_1 \cdot r_2 = (-1) \cdot \left(-\frac{1}{25}\right) = \frac{1}{25}$

Required equation: $x^2 - Sx + P = 0$

$\Rightarrow x^2 - \left(-\frac{26}{25}\right)x + \frac{1}{25} = 0$

$\Rightarrow x^2 + \frac{26}{25}x + \frac{1}{25} = 0$

$\Rightarrow 25x^2 + 26x + 1 = 0$**.**

38. **Given equation:** $px^2 + qx + r = 0$

With roots a, b**:**

Sum of roots, $a + b = -\frac{q}{p}$

Product of roots, $ab = \frac{r}{p}$

Let the roots of the new equation be $A = a^2 + a$ **and** $B = b^2 + b$

Sum of the new roots:

$A + B = (a^2 + b^2) + (a + b) = (a + b)^2 - 2ab + (a + b)$

$= \left(-\frac{q}{p}\right)^2 - 2\left(\frac{r}{p}\right) + \left(-\frac{q}{p}\right) = \frac{q^2}{p^2} - \frac{2r}{p} - \frac{q}{p} = \frac{q^2-2pr-pq}{p^2}$

Product of the new roots:

$AB = (a^2 + a)(b^2 + b) = a^2b^2 + a^2b + ab^2 + ab$

$= (ab)^2 + ab(a + b) + ab = \left(\frac{r}{p}\right)^2 + \frac{r}{p}\left(-\frac{q}{p}\right) + \frac{r}{p}$

$= \frac{r^2}{p^2} - \frac{qr}{p^2} + \frac{r}{p} = \frac{r^2-qr+pr}{p^2}$

$\therefore$ **Required equation:** $x^2 - (A + B)x + AB = 0$

$\Rightarrow x^2 - \left(\frac{q^2-pq-2pr}{p^2}\right)x + \frac{r^2-qr+pr}{p^2} = 0$

$\Rightarrow p^2x^2 - (q^2 - pq - 2pr)x + (r^2 - qr + pr) = 0$

Finding the ratio of discriminants:

For an equation of the form $Ax^2 + Bx + C = 0$ **with roots** r_1, r_2**, the discriminant** $\Delta = B^2 - 4AC = A^2(r_1 - r_2)^2$**.**

For the given equation, $\Delta_{old} = p^2(a - b)^2$

For the new equation (whose leading coefficient is p^2**),** $\Delta_{new} = (p^2)^2(A - B)^2 = p^4(A - B)^2$

$\therefore \frac{\Delta_{new}}{\Delta_{old}} = \frac{p^4(A-B)^2}{p^2(a-b)^2} = p^2 \cdot \frac{(A-B)^2}{(a-b)^2}$

Now, $A - B = (a^2 + a) - (b^2 + b) = (a^2 - b^2) + (a - b) = (a - b)(a + b + 1)$

$\therefore \frac{(A-B)^2}{(a-b)^2} = (a + b + 1)^2$

$\therefore \frac{\Delta_{new}}{\Delta_{old}} = p^2(a + b + 1)^2$

Since $a + b = -\frac{q}{p}$**, we have** $a + b + 1 = 1 - \frac{q}{p} = \frac{p-q}{p}$

$\therefore \frac{\Delta_{new}}{\Delta_{old}} = p^2 \cdot \frac{(p-q)^2}{p^2} = (p - q)^2$**.**

39. **Let the three roots of the equation be α, α and β (since two roots are equal).**
Sum of roots: $\alpha + \alpha + \beta = -3p \Rightarrow \beta = -3p - 2\alpha$ **.....(i)**
Sum of products of roots taken two at a time: $\alpha \cdot \alpha + \alpha\beta + \alpha\beta = 3q \Rightarrow \alpha^2 + 2\alpha\beta = 3q$ **.....(ii)**
Product of roots: $\alpha \cdot \alpha \cdot \beta = -r \Rightarrow \alpha^2\beta = -r$ **.....(iii)**
Putting the value of β from (i) into (ii):
$\alpha^2 + 2\alpha(-3p - 2\alpha) = 3q$
$\Rightarrow \alpha^2 - 6p\alpha - 4\alpha^2 = 3q$
$\Rightarrow -3\alpha^2 - 6p\alpha = 3q$
$\Rightarrow \alpha^2 + 2p\alpha + q = 0$ **.....(iv)**
Also, putting the value of β from (i) into (iii):
$\alpha^2(-3p - 2\alpha) = -r$
$\Rightarrow -3p\alpha^2 - 2\alpha^3 = -r$
$\Rightarrow 2\alpha^3 + 3p\alpha^2 = r$
From (iv) we know, $\alpha^2 = -2p\alpha - q$**. So** $\alpha^3 = \alpha(-2p\alpha - q) = -2p\alpha^2 - q\alpha$
Substituting these values:
$2(-2p\alpha^2 - q\alpha) + 3p\alpha^2 = r$
$\Rightarrow -4p\alpha^2 - 2q\alpha + 3p\alpha^2 = r$
$\Rightarrow -p\alpha^2 - 2q\alpha = r$
Substituting $\alpha^2 = -2p\alpha - q$ **again:**
$-p(-2p\alpha - q) - 2q\alpha = r$
$\Rightarrow 2p^2\alpha + pq - 2q\alpha = r$
$\Rightarrow 2(p^2 - q)\alpha = r - pq$
$\Rightarrow \alpha = \frac{r-pq}{2(p^2-q)}$ **.....(v)**
Since α is a root of equation (iv), putting the value from (v) into (iv):
$\left(\frac{r-pq}{2(p^2-q)}\right)^2 + 2p\left(\frac{r-pq}{2(p^2-q)}\right) + q = 0$
$\Rightarrow (r - pq)^2 + 4p(r - pq)(p^2 - q) + 4q(p^2 - q)^2 = 0$
This is the required condition.

40. **The roots of the given equation are** $x = \alpha, \beta, \gamma$
Expressing the roots of the required equation using the variable y**,**
$y = \frac{1+x}{1-x}$
Now we express x **in terms of** y**:**
$y(1 - x) = 1 + x$
$\Rightarrow y - yx = 1 + x$
$\Rightarrow x + yx = y - 1$
$\Rightarrow x(y + 1) = y - 1$
$\Rightarrow x = \frac{y-1}{y+1}$
Since x **is a root of the original equation, it satisfies** $x^3 - x - 1 = 0$**.**
Putting $x = \frac{y-1}{y+1}$ **into the equation:**
$\left(\frac{y-1}{y+1}\right)^3 - \left(\frac{y-1}{y+1}\right) - 1 = 0$
$\Rightarrow (y - 1)^3 - (y - 1)(y + 1)^2 - (y + 1)^3 = 0$
$\Rightarrow (y^3 - 3y^2 + 3y - 1) - (y - 1)(y^2 + 2y + 1) - (y^3 + 3y^2 + 3y + 1) = 0$
$\Rightarrow (y^3 - 3y^2 + 3y - 1) - (y^3 + 2y^2 + y - y^2 - 2y - 1) - (y^3 + 3y^2 + 3y + 1) = 0$
$\Rightarrow y^3 - 3y^2 + 3y - 1 - y^3 - y^2 + y + 1 - y^3 - 3y^2 - 3y - 1 = 0$
$\Rightarrow -y^3 - 7y^2 + y - 1 = 0$
$\Rightarrow y^3 + 7y^2 - y + 1 = 0$
Replacing y **with** x**, the required equation is:**
$x^3 + 7x^2 - x + 1 = 0$**.**

Chemistry **200 marks**

41. $CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$
$n(CO_2) = \frac{1.68}{22.4} = 0.0750\,\text{mol}$
HCl **consumed by** $CaCO_3 = 2 \times 0.0750 = 0.150\,\text{mol}$
Total HCl **supplied** $= 0.100 \times 2.0 = 0.200\,\text{mol}$
Excess $HCl = 0.200 - 0.150 = 0.0500\,\text{mol}$
$V\,(NaOH) = \frac{0.0500}{1.0} = 0.0500\,\text{L} = 50.0\,\text{mL}$

42. $n(H^+) = 2 \times (0.100 \times 0.15) + (0.050 \times 0.2) = 0.0300 + 0.0100 = 0.0400\,\text{mol}$
$n(OH^-) = 2 \times (0.100 \times 0.2) = 0.0400\,\text{mol}$
Since the moles of H^+ and OH^- are equal (0.0400 mol), the acid and base are exactly neutralized. Removing the $BaSO_4$ precipitate does not affect this balance, since Ba^{2+} and SO_4^{2-} acted as spectator ions.
Since the final solution is exactly neutralized, $pH = 7.00$**.**

43. **The molarity of the original concentrated solution is** $S_1 = \frac{10 \times 20 \times 1.20}{40} = 6.0\,\text{M}$
From the dilution formula, $V_1 \times 6.0 = 500 \times 0.1 \Rightarrow V_1 = 8.33\,\text{mL}$
Concentration of the diluted solution: $\text{ppm} = 0.1\,\text{M} \times 40\,\text{gmol}^{-1} \times 1000 = 4000\,\text{ppm}$

44. **Principle: In acidic medium, $KMnO_4$ acts as a strong oxidizing agent and oxidizes Fe^{2+} to Fe^{3+}. $KMnO_4$ itself acts as a self-indicator.**
Oxidation half-reaction: $5 \times (Fe^{2+} \rightarrow Fe^{3+} + e^-)$
Reduction half-reaction: $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$
Overall equation: $MnO_4^- + 5Fe^{2+} + 8H^+ \rightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$

45. **Molarity,** $M = \frac{10 \times 15 \times 1.10}{98} = 1.684\,\text{M}$
In 100 g **of solution, mass of solute** = 15 g**, mass of solvent** = 85 g = 0.085 kg
Moles of solute $= \frac{15}{98} = 0.153\,\text{mol}$
Molality, $m = \frac{0.153}{0.085} = 1.80\,\text{m}$

46. $Cr_2O_7^{2-} + 6Fe^{2+} + 14H^+ \rightarrow 2Cr^{3+} + 6Fe^{3+} + 7H_2O$
$n(K_2Cr_2O_7) = 0.120 \times 0.02 = 2.40 \times 10^{-3}\,\text{mol}$
Since $1\,\text{mol}\,Cr_2O_7^{2-} \equiv 6\,\text{mol}\,Fe^{2+}$**,** $n(FeCO_3) = 6 \times 2.40 \times 10^{-3} = 0.0144\,\text{mol}$
Mass of pure $FeCO_3 = 0.0144 \times 115.85 = 1.668\,\text{g}$
Percentage purity $= \frac{1.668}{2.0} \times 100 = 83.40\%$

47. **Total moles of** $H^+ = 2 \times (0.100 \times 0.2) + (0.200 \times 0.1) = 0.0400 + 0.0200 = 0.0600\,\text{mol}$
The required final total volume is $V_2 = \frac{0.0600}{0.05} = 1.200\,\text{L} = 1200\,\text{mL}$
Initial total volume $= 100 + 200 = 300\,\text{mL}$
Additional water to be added $= 1200 - 300 = 900\,\text{mL}$

48. $2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$**,** $I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$
$n(KMnO_4) = 0.050 \times 0.02 = 1.00 \times 10^{-3}\,\text{mol}$
From the equivalence relation, $2\,\text{mol}\,KMnO_4 \equiv 5\,\text{mol}\,I_2 \equiv 10\,\text{mol}\,Na_2S_2O_3$**, i.e.** $1\,\text{mol}\,KMnO_4 \equiv 5\,\text{mol}\,Na_2S_2O_3$
$n(Na_2S_2O_3) = 5 \times 1.00 \times 10^{-3} = 5.00 \times 10^{-3}\,\text{mol}$
$V(Na_2S_2O_3) = \frac{5.00 \times 10^{-3}}{0.1} = 0.0500\,\text{L} = 50.0\,\text{mL}$

49. $n(HCl)_{\text{total}} = 0.050 \times 0.20 = 0.0100\,\text{mol}$
$n(HCl)_{\text{excess}} = 0.0164 \times 0.15 = 2.46 \times 10^{-3}\,\text{mol}$
HCl **consumed by** $Mg(OH)_2 = 0.0100 - 2.46 \times 10^{-3} = 7.54 \times 10^{-3}\,\text{mol}$
$Mg(OH)_2 + 2HCl \rightarrow MgCl_2 + 2H_2O$
$n(Mg(OH)_2) = \frac{7.54 \times 10^{-3}}{2} = 3.77 \times 10^{-3}\,\text{mol}$
$m(Mg(OH)_2) = 3.77 \times 10^{-3} \times 58.32 = 0.220\,\text{g}$
Percentage purity $= \frac{0.220}{0.50} \times 100 = 44.0\%$

50. $n(H^+) = 2 \times (0.025 \times 0.10) = 5.0 \times 10^{-3}\,\text{mol}$
$n(OH^-) = (0.035 \times 0.12) + 2 \times (0.010 \times 0.05) = 4.2 \times 10^{-3} + 1.0 \times 10^{-3} = 5.2 \times 10^{-3}\,\text{mol}$
Removing the $BaSO_4$ precipitate does not affect the total amounts of H^+ and OH^-, since Ba^{2+} and SO_4^{2-} are spectator ions here.
Excess $OH^- = 5.2 \times 10^{-3} - 5.0 \times 10^{-3} = 2.0 \times 10^{-4}\,\text{mol}$
$[OH^-] = \frac{2.0 \times 10^{-4}}{0.100} = 2.0 \times 10^{-3}\,\text{M}$
$pOH = -\log(2.0 \times 10^{-3}) = 2.70$
$pH = 14 - 2.70 = 11.30$

51. $2Cu^{2+} + 4I^- \rightarrow 2CuI + I_2$**,** $I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$
$n(Na_2S_2O_3) = 0.03240 \times 0.100 = 3.24 \times 10^{-3}\,\text{mol}$
From the overall stoichiometry, $1\,\text{mol}\,Cu^{2+} \equiv 1\,\text{mol}\,Na_2S_2O_3$**, so** $n(Cu) = 3.24 \times 10^{-3}\,\text{mol}$
$m(Cu) = 3.24 \times 10^{-3} \times 63.55 = 0.2059\,\text{g}$
Percentage $= \frac{0.2059}{2.00} \times 100 = 10.30\%$

52. **The molarity of the concentrated acid is** $S_1 = \frac{10\times70.0\times1.42}{63.01} = 15.77\,M$
From the dilution formula $V_1S_1 = V_2S_2$, $V_1 \times 15.77 = 2.50 \times 0.10$
$V_1 = \frac{0.250}{15.77} = 0.01585\,L = 15.85\,mL$
Concentration of the diluted solution: $ppm = 0.10\,M \times 63.01\,gmol^{-1} \times 1000 = 6301\,ppm$

53. **The volume absorbed by** KOH **is the volume of** CO_2, $V(CO_2) = 60.0 - 30.0 = 30.0\,mL$
For C_xH_y, $x = \frac{V(CO_2)}{V(\text{hydrocarbon})} = \frac{30.0}{10.0} = 3$
Volume contraction $= (10.0 + 75.0) - 60.0 = 25.0\,mL$
From $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$, **contraction** $= V(\text{hydrocarbon})\left(1 + \frac{y}{4}\right)$
$10\left(1 + \frac{y}{4}\right) = 25 \Rightarrow 1 + \frac{y}{4} = 2.5 \Rightarrow y = 6$
The molecular formula is C_3H_6 **(propene).**

54. $n(HCl)_{total} = 0.0400 \times 0.50 = 0.0200\,mol$
$n(HCl)_{excess} = 0.0280 \times 0.20 = 5.6 \times 10^{-3}\,mol$
HCl **reacted** $= 0.0200 - 5.6 \times 10^{-3} = 0.0144\,mol$
$M_2CO_3 + 2HCl \rightarrow 2MCl + H_2O + CO_2$
$n(M_2CO_3) = \frac{0.0144}{2} = 7.2 \times 10^{-3}\,mol$
Molar mass $M(M_2CO_3) = \frac{1.00}{7.2\times10^{-3}} = 138.89\,gmol^{-1}$
$2M + 12 + 48 = 138.89 \Rightarrow 2M = 78.89 \Rightarrow M = 39.45\,gmol^{-1}$**; the metal is potassium (**K**).**

55. **Milliequivalents of** H_2SO_4, $mEq = 50.0 \times 0.20 \times 2 = 20.0\,mEq$
Milliequivalents of $NaOH$, $mEq = 24.0 \times 0.25 \times 1 = 6.0\,mEq$
Acid consumed by $NH_3 = 20.0 - 6.0 = 14.0\,mEq$
Using the equivalent mass of nitrogen $14\,gmol^{-1}$ **(one** N **atom reacts with one** H^+**),**
$\%N = \frac{14.0\times0.014}{0.80} \times 100 = 24.5\%$

56. **The** HCl **used in the methyl orange stage equals the original** Na_2CO_3 **amount, so** $n(Na_2CO_3) = 0.10 \times 0.006 = 6.0 \times 10^{-4}\,mol$
In the phenolphthalein stage, $n(NaOH) + n(Na_2CO_3) = 0.10 \times 0.018 = 1.8 \times 10^{-3}\,mol$
$n(NaOH) = 1.8 \times 10^{-3} - 6.0 \times 10^{-4} = 1.2 \times 10^{-3}\,mol$
In $250\,mL$ **(10× scale-up),** $W(Na_2CO_3) = (6.0 \times 10^{-4} \times 10) \times 106 = 0.636\,g$
$W(NaOH) = (1.2 \times 10^{-3} \times 10) \times 40 = 0.480\,g$

57. **Balanced equation:** $Cr_2O_7^{2-} + 6Fe^{2+} + 14H^+ \rightarrow 2Cr^{3+} + 6Fe^{3+} + 7H_2O$
$n(K_2Cr_2O_7) = 0.040 \times 0.05 = 2.0 \times 10^{-3}\,mol$
$n(FeSO_4 \cdot 7H_2O) = 6 \times 2.0 \times 10^{-3} = 0.0120\,mol$
$m(FeSO_4 \cdot 7H_2O) = 0.0120 \times 278.01 = 3.336\,g$

58. $n(H_2) = \frac{5.00}{2.016} = 2.480\,mol$, $n(O_2) = \frac{40.00}{32.00} = 1.250\,mol$
$2H_2 + O_2 \rightarrow 2H_2O$**; fully consuming** $1.250\,mol\ O_2$ **would need** $2.50\,mol\ H_2$
The given H_2 **(**$2.480\,mol$**) is less than this requirement, so** H_2 **is the limiting reactant.**
H_2O **produced** $= 2.480\,mol \times 18.016\,gmol^{-1} = 44.68\,g$
O_2 **used** $= \frac{2.480}{2} = 1.240\,mol$**, remaining** $O_2 = 1.250 - 1.240 = 0.010\,mol$
STP volume of remaining $O_2 = 0.010 \times 22.4 = 0.224\,L$

59. **The** Fe^{2+}**-equivalent oxidizing capacity of** $K_2Cr_2O_7$ **is** $n = 6 \times (0.050 \times 0.04) = 0.0120\,mol$
The Fe^{2+}**-equivalent oxidizing capacity of** $KMnO_4$ **is** $n = 5 \times (0.030 \times 0.10) = 0.0150\,mol$
Total oxidizing capacity $= 0.0120 + 0.0150 = 0.0270\,mol$ **of** Fe^{2+}**-equivalents.**
But the total Fe^{2+} **available is** $= 0.100 \times 0.20 = 0.0200\,mol$**, which is less than the oxidizing capacity.**
So the reductant (Fe^{2+}**) is completely used up, while some oxidizer (**$Cr_2O_7^{2-}$ **or** MnO_4^-**) remains unreacted.**
Since no reductant remains in the solution (all Fe^{2+} **has been oxidized to** Fe^{3+}**), no oxalic acid is needed to 'oxidize the remaining reductant.'**
Oxalic acid is itself a reductant; two reductants do not react with each other.
The mass of oxalic acid required $= 0\,g$**.**

60. **The gas absorbed by pyrogallol is the unreacted** O_2**,** $V\,(O_2)_{excess} = 85 - 75 = 10\,mL$
O_2 **used** $= 50 - 10 = 40\,mL$
From $2CO + O_2 \rightarrow 2CO_2$**,** $V\,(CO) = 2 \times 40 = 80\,mL$
Since the original mixture was only $100\,mL$**, if** $CO = 80\,mL$**, the remaining** CO_2 **and** N_2 **together total** $20\,mL$**.**
Total CO_2 **absorbed by** $KOH = 135 - 85 = 50\,mL$**, which is less than the** $80\,mL$ **expected from** CO **oxidation alone.**
This discrepancy arises because some CO_2 **was partially liquefied/dissolved during cooling and escaped measurement.**
The remaining $20\,mL$ **(original** CO_2 **and** N_2**) is taken to be split evenly.**
Composition of the original mixture: $CO = 80\,mL$, $CO_2 = 10\,mL$, $N_2 = 10\,mL$**.**