Engineering Weekly Exam 03 [Set 02] EV

Complex Numbers + Polynomials and Polynomial Equations + Newtonian Mechanics + Quantitative Chemistry

Total Marks: 600 **Time:** 3 Hours

Physics: 200 Mathematics: 200 Chemistry: 200

Physics **200 marks**

P-1 | Q-1

1. Two blocks of 4 kg and 6 kg are connected by a light, inextensible string and lie on a rough horizontal surface. The coefficient of kinetic friction between both blocks and the surface is 0.20. A horizontal force of 40 N is applied to the right on the 4 kg block. Find the acceleration of the system and the tension in the string between the blocks. (Take $g = 9.8\ ms^{-2}$.)

P-1 | Q-2

2. A 2 kg block rests on a 30° incline and a 3 kg block hangs vertically. They are connected by a massless pulley and an inextensible string. The coefficients of static and kinetic friction between the block and the incline are 0.40 and 0.25. Check whether the system remains at rest, and if not, find the acceleration and the tension. (Take $g = 9.8\ ms^{-2}$.)

P-2 | Q-3

3. **A 0.50 kg particle is at position $\vec{r} = 2\hat{i}$ m relative to a rotating surface. The rotating frame's angular velocity is $\vec{\Omega} = 4\hat{k}$ rads^{-1}, angular acceleration $1.5\hat{k}$ rads^{-2}, and the particle's relative velocity is $\vec{v}_{rel} = 3\hat{j}$ ms^{-1}. Find the magnitude and direction of the centrifugal, Coriolis, and Euler pseudo-forces on the particle.**

P-2 | Q-4

4. **A packet is thrown forward from a truck moving horizontally at 15 ms^{-1}, at 20 ms^{-1} relative to the truck at 30° above the horizontal. The truck's height above the ground is 10 m. Find the time for the packet to reach the ground and the horizontal distance from the truck's initial position. (Take g = 9.8 ms^{-2}.)**

P-3 | Q-5

5. **A 1000 kg car moves inside a vertical circular track of radius 5.0 m, with speed 20 ms^{-1} at the bottom. Neglecting air resistance, find the speed at the top and the normal force from the track at both the bottom and the top. (Take g = 9.8 ms^{-2}.)**

P-3 | Q-6

6. **A 0.20 kg ball strikes a smooth vertical wall with velocity $(15\hat{i} + 6\hat{j})$ ms^{-1} and rebounds after 0.020 s with velocity $(-10\hat{i} + 6\hat{j})$ ms^{-1}. Find the average force from the wall on the ball, and the impulse on the ball.**

P-4 | Q-7

7. **A 240 kg boat floats at rest in still water. An 80 kg person on the boat walks 2.5 m relative to the boat from one end toward the other. The boat's length is 4.0 m and water resistance is negligible. Find the displacement of the boat and the displacement of the person relative to the ground.**

P-4 | Q-8

8. **A 2 kg particle moves right at 6 ms^{-1} under a force $\vec{F} = (6 - 2t)\hat{i}$ N. Find the impulse on the particle and the change in velocity from t = 0 to 3 s. If the initial velocity was 1 ms^{-1} to the right, find the velocity at t = 3 s.**

P-5 | Q-9

9. **A wheel has angular acceleration $\alpha = 4t$ rads^{-2}, with angular velocity 2 rads^{-1} at $t = 0$. Find the angular velocity and angular displacement from 0 to 3 s. If the wheel's radius is 0.30 m, also find the tangential and centripetal accelerations at $t = 3$ s.**

P-5 | Q-10

10. **Four point masses of 1 kg each are placed at the corners of a square of side 0.40 m. Find the moment of inertia of the system about an axis perpendicular to the plane through the centre. Starting from rest, a constant torque of 2 Nm acts for 5 s; find the final angular velocity and rotational kinetic energy.**

P-6 | Q-11

11. **A solid cylinder of mass 4 kg rolls without slipping down a 37° incline, losing 1.4 m of vertical height. Its radius is 0.20 m. Find the speed of the centre, the angular velocity, the acceleration, and the static friction on reaching the bottom. $\sin 37° = 0.60$, $\cos 37° = 0.80$. (Take $g = 9.8\ ms^{-2}$.)**

P-6 | Q-12

12. **A uniform ladder of length 5 m and mass 20 kg leans against a smooth vertical wall at 60° to a rough horizontal floor. The wall exerts no friction. Find the minimum coefficient of static friction between the floor and the ladder to prevent slipping.**

P-7 | Q-13

13. **A water jet of diameter 1.5 cm strikes a stationary flat plate perpendicularly at 30 ms^{-1} and rebounds in the opposite direction with the same speed. The density of water is 1000 kgm^{-3}. Find the average force of the water on the plate.**

P-7 | Q-14

14. **A 10 kg sand-carrying cart moves at 2 ms^{-1}. Stationary sand falls onto it at 0.50 kg per second and sticks to the cart. Find the cart's velocity and acceleration after 4 s. The external horizontal force is zero.**

P-8 | Q-15

15. **Two particles of equal mass undergo a perfectly elastic collision. The first was moving at $10\ ms^{-1}$ and the second was at rest. After the collision, the first particle moves at 30° to its original direction. Find the magnitude and direction of both particles' velocities after the collision.**

P-8 | Q-16

16. **A 6 kg wedge rests on a smooth horizontal floor. Its 30° incline slopes down to the right, and a 2 kg block rests on it. The wedge is pulled to the right by a force P. If the coefficient of static friction between the block and the incline is 0.20, find the minimum and maximum values of P for which the block stays stationary relative to the wedge. (Take $g = 9.8\ ms^{-2}$.)**

P-9 | Q-17

17. **A particle's position is $x(t) = 2t^3 - 3t^2 + 4$ m and its mass is 2 kg. Find the velocity, acceleration, and net force at $t = 2$ s. Find the work done by the net force from $t = 0$ to 2 s.**

P-9 | Q-18

18. **A 2 kg particle moves in a circle of radius 0.50 m at angular velocity 6 rads^{-1}. A constant tangential force of 4 N acts on it for 3 s. Find the angular velocity after 3 s, the change in angular momentum, and the work done by the force on the particle.**

P-10 | Q-19

19. **A system has three particles of mass 2 kg, 3 kg, and 5 kg at positions $(0, 0)$ m, $(4, 0)$ m, and $(0, 3)$ m respectively. External forces on them are $3\hat{i}$ N, $(-\hat{i} + 2\hat{j})$ N, and $4\hat{j}$ N respectively. Find the position and acceleration of the system's centre of mass.**

P-10 | Q-20

20. **A 100 kg cart moves right at 2 ms^{-1}. A 50 kg person on the cart jumps to the right at 4 ms^{-1} relative to the cart. Find the velocities of the cart and the person relative to the ground after separation, and the change in kinetic energy in the process.**

P-11 | Q-21

21. **Find the sixth roots of −729.**

P-11 | Q-22

22. **For a complex number z, if $|z - 5| - |z + 5| = 8$, prove that it represents a hyperbola. Find the equation and eccentricity of the hyperbola.**

P-12 | Q-23

23. If $(1+x)^n = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots + c_nx^n$, use complex numbers to prove that,

(i) $c_0 - c_2 + c_4 - \cdots = 2^{\frac{n}{2}} \cos\left(\frac{n\pi}{4}\right)$

(ii) $c_1 - c_3 + c_5 - \cdots = 2^{\frac{n}{2}} \sin\left(\frac{n\pi}{4}\right)$

P-12 | Q-24

24. Find the maximum and minimum distance of the origin from any point on the circle $|z - 4 - 3i| = 2$.

P-13 | Q-25

25. If $x + \frac{1}{x} = 2\cos\theta$, prove that $x^n + \frac{1}{x^n} = 2\cos n\theta$

P-13 | Q-26

26. For two complex numbers z_1 and z_2, $|z_1| = |z_2| = 1$ and $z_1 \neq z_2$. Prove that $\frac{z_1+z_2}{z_1-z_2}$ is a purely imaginary number.

P-14 | Q-27

27. If $\left|\frac{z-3}{z+3}\right| = 2$, prove that the locus of z is a circle. Find the centre and radius of the circle.

P-14 | Q-28

28. If a complex number z has $\text{Arg}(z) = \frac{\pi}{4}$ and $|z - 6| = 3\sqrt{2}$, find z.

P-15 | Q-29

29. For all real values of x, show that the value of $f(x) = \frac{x^2-2x+4}{x^2+2x+4}$ lies between $\frac{1}{3}$ and 3.

P-15 | Q-30

30. On the Argand plane, three points A, B and C represent the complex numbers z, iz and $z + iz$ respectively. Prove that ABC is an isosceles right triangle. If the area of the triangle is 18 square units and z is a complex number in the first quadrant with argument $\frac{\pi}{6}$, find z.

P-16 | Q-31

31. If the roots of $x^3 + 2x^2 - 3x + 4 = 0$ are α, β, γ, form a cubic equation whose roots are $\frac{\alpha}{\beta+\gamma}, \frac{\beta}{\gamma+\alpha}, \frac{\gamma}{\alpha+\beta}$ respectively.

P-16 | Q-32

32. If the equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root, prove that $a + b + c = 0$ or $a = b = c$

P-17 | Q-33

33. If the roots of $x^3 + 2x^2 + 3x + 4 = 0$ are α, β, γ, find the value of the symmetric expression $\sum \alpha^3$.

P-17 | Q-34

34. If the roots of $ax^2 + bx + c = 0$ are in the ratio $m : n$, prove that $mnb^2 = ac(m + n)^2$

P-18 | Q-35

35. If the roots of $x^3 - 12x^2 + 39x + k = 0$ are in Arithmetic Progression, find the value of k and solve the equation.

P-18 | Q-36

36. Find a cubic equation, one of whose roots is $2 - i\sqrt{3}$ and the sum of whose roots is 5 (the coefficients of the equation are real).

P-19 | Q-37

37. If α and β are the roots of $x^2 - 3x + 5 = 0$, form a quadratic equation whose roots are $\alpha^2 + \beta^2$ and $\alpha^{-2} + \beta^{-2}$

P-19 | Q-38

38. If a and b are the roots of $px^2 + qx + r = 0$, find the equation whose roots are $(a^2 + a)$ and $(b^2 + b)$. What will be the ratio of the discriminants of this equation and the given equation?

P-20 | Q-39

39. Find the condition for two roots of the equation $x^3 + 3px^2 + 3qx + r = 0$ to be equal.

P-20 | Q-40

40. If the roots of $x^3 - x - 1 = 0$ are α, β, γ, find an equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

P-21 | Q-41

41. **A 10.0 g impure limestone sample is dissolved in 100 mL of 2.0 M HCl. The CO_2 gas released, collected at STP, has a volume of 1.68 L. Find the volume of 1.0 M $NaOH$ needed to completely neutralize the unreacted excess HCl.**

P-21 | Q-42

42. **100 mL of 0.15 M H_2SO_4 and 50 mL of 0.2 M HCl are mixed, then 100 mL of 0.2 M $Ba(OH)_2$ is added. The $BaSO_4$ precipitate is filtered off, and the remainder is diluted to 500 mL. Find the pH of the final solution.**

P-22 | Q-43

43. What volume of a 20% (w/w) $NaOH$ solution with density 1.20 gmL^{-1}, diluted with water, gives 500 mL of 0.1 M $NaOH$? Express the concentration of the diluted solution in ppm.

P-22 | Q-44

44. Write down the chemical principle and ionic reactions involved in the estimation of Fe^{2+} using standard $KMnO_4$ solution in acidic medium.

P-23 | Q-45

45. **An aqueous H_2SO_4 solution with density $1.10\ gmL^{-1}$ contains 15% H_2SO_4 by mass. Find the molarity (M) and molality (m) of the solution.**

P-23 | Q-46

46. **A 2.0 g impure siderite ($FeCO_3$) sample is dissolved in acid, converting the iron to Fe^{2+}. Fully oxidizing this solution requires 120 mL of 0.02 M $K_2Cr_2O_7$. Find the percentage purity of $FeCO_3$ in the ore.**

P-24 | Q-47

47. **100 mL of 0.2 M H_2SO_4 and 200 mL of 0.1 M HCl are mixed. What additional volume of water must be added so that the final $[H^+]$ concentration becomes 0.05 M?**

P-24 | Q-48

48. **In acidic medium, 50 mL of 0.02 M $KMnO_4$ is reacted with excess KI. Find the volume of 0.1 M $Na_2S_2O_3$ needed to completely neutralize the liberated iodine.**

P-25 | Q-49

49. **A 0.50 g commercial antacid tablet (a mixture of $Mg(OH)_2$ and an inert ingredient) is dissolved in 50.0 mL of 0.20 M HCl. Neutralizing the excess acid requires 16.4 mL of 0.15 M NaOH. Find the mass of $Mg(OH)_2$ in the tablet and its percentage purity.**

P-25 | Q-50

50. **25.0 mL of 0.10 M H_2SO_4 and 35.0 mL of 0.12 M KOH are mixed, then 10.0 mL of 0.05 M $Ba(OH)_2$ is added. The precipitate formed is filtered off, and the remaining solution is made up to 100.0 mL. Find the pH of the final solution.**

P-26 | Q-51

51. **A 2.00 g chalcopyrite ore sample is dissolved in acid to convert copper to Cu^{2+}, and excess KI is added. Neutralizing the liberated iodine requires 32.40 mL of 0.100 M $Na_2S_2O_3$. Find the mass and percentage of copper in the ore.**

P-26 | Q-52

52. **A commercial acid bottle contains 70.0% (w/w) HNO_3 with density 1.42 gmL^{-1}. What volume of this concentrated acid, diluted with water, gives 2.50 L of 0.10 M HNO_3? Also find the concentration of the diluted solution in ppm.**

P-27 | Q-53

53. **10.0 mL of a gaseous hydrocarbon is mixed with 75.0 mL of O_2 and combusted. After cooling, the volume is 60.0 mL. Passing this through KOH reduces the volume to 30.0 mL. Find the molecular formula of the hydrocarbon.**

P-27 | Q-54

54. **A 1.00 g metal carbonate (M_2CO_3) is completely dissolved in 40.0 mL of 0.50 M HCl. Titrating the unreacted acid requires 28.0 mL of 0.20 M $NaOH$. Find the atomic mass of the metal (M) and identify it.**

P-28 | Q-55

55. **NH_3 gas obtained from a 0.80 g organic sample is absorbed in 50.0 mL of 0.20 M H_2SO_4. Neutralizing the excess acid requires 24.0 mL of 0.25 M $NaOH$. Find the percentage of nitrogen in the organic compound.**

P-28 | Q-56

56. **A 1.50 g mixture of $NaOH$, Na_2CO_3, and an inert substance is dissolved in water to make 250.0 mL of solution. A 25.0 mL portion requires 18.0 mL of 0.10 M HCl with phenolphthalein indicator, and a further 6.0 mL after adding methyl orange. Find the mass of Na_2CO_3 and $NaOH$.**

P-29 | Q-57

57. **Balance the oxidation of $FeSO_4$ by $K_2Cr_2O_7$ in acidic medium using the ion-electron method. Find the mass of $FeSO_4 \cdot 7H_2O$ that 40.0 mL of 0.05 M $K_2Cr_2O_7$ would oxidize.**

P-29 | Q-58

58. **A mixture of 5.00 g H_2 and 40.00 g O_2 is combusted in a sealed container to form water. Which is the limiting reactant? Find the mass of water produced and the volume of the remaining gas at STP.**

P-30 | Q-59

59. In acidic medium, 50 mL of 0.04 M $K_2Cr_2O_7$ and 30 mL of 0.10 M $KMnO_4$ are mixed together, then 100 mL of 0.20 M $FeSO_4$ is added. After the reaction, how many grams of oxalic acid ($H_2C_2O_4 \cdot 2H_2O$) must be added to completely oxidize the reductant remaining in the solution?

P-30 | Q-60

60. 50 mL of O_2 is added to a 100 mL mixture of CO, CO_2, and N_2 in a vessel and exploded. After cooling, the volume is 135 mL. Passing this through KOH reduces the volume to 85 mL, and passing the remainder through alkaline pyrogallol solution gives 75 mL. Find the molar composition of the original mixture.