Engineering Weekly Exam 03 [Set 01] EV

Complex Numbers + Polynomials and Polynomial Equations + Newtonian Mechanics + Quantitative Chemistry

Total Marks: 600 **Negative Marks:** 0.25 **Time:** 3 Hours

Physics

200 marks

P-1 | Q-1

1. A 4 kg block A rests on a 37° incline. A uniform solid disk-shaped pulley of mass 2 kg and radius 0.20 m is mounted (frictionless axle) at the top of the incline. A light, inextensible string runs from block A up the incline, over the pulley, and down vertically to a hanging mass B on the other side. The string does not slip on the pulley. The coefficients of static and kinetic friction between block A and the incline are 0.50 and 0.25. For what range of B's mass will the system remain in equilibrium? If B = 5 kg, find the acceleration, the tension on each side of the string, and the magnitude and direction of the force on the pulley's axle. Starting from rest, verify the result using the total kinetic energy after B falls 2 m.

P-1 | Q-2

2. A 10 kg box rests on a horizontal floor. The coefficients of static and kinetic friction with the floor are 0.50 and 0.25. A 50 N tension is applied at 37° above the horizontal. Determine whether the box moves, and if so, its acceleration. Also find the minimum tension in the same direction needed to just start the motion. Take sin 37° = 0.60 and cos 37° = 0.80.

P-2 | Q-3

3. A 2 kg block A rests on top of a 3 kg block B on a frictionless horizontal floor. The coefficients of static and kinetic friction between A and B are 0.40 and 0.30. A horizontal force F is applied only to B, pulling it to the right. Find the maximum F for which the blocks move together without slipping. Then, starting from rest with $F = 25$ N, find the acceleration of each block and the time for A to reach the left end of B, given that A must slide 0.60 m relative to B to get there.

P-2 | Q-4

4. A 4 kg block rests on a 30° incline. A light, inextensible string over a massless, frictionless pulley at the top of the incline connects it to a hanging 3 kg mass. The coefficients of static and kinetic friction between the block and the incline are both $\frac{1}{4\sqrt{3}}$. If the system is released from rest, find the direction of motion, the acceleration, and the tension in the string.

P-3 | Q-5

5. A 0.99 kg ball hangs from a light, inextensible string of length 1.0 m. At the lowest point, a 0.010 kg bullet moving horizontally at 600 ms^{-1} embeds itself in the ball. Treat the combined body as a particle after the collision and neglect air resistance. Determine whether a complete vertical circle is possible. If not, find the angle (measured from the lowest point) at which the string goes slack, and the maximum height the combined body then reaches. Also find the minimum initial bullet speed required for a complete circle.

P-3 | Q-6

6. A lift accelerates upward at 2 ms^{-2}. A massless pulley hangs from a spring balance fixed to the ceiling of the lift. Light strings over the pulley carry hanging masses of 3 kg and 2 kg. Neglecting friction, find the acceleration of the masses relative to the lift, the string tension, and the spring-balance reading. Also state the direction of the 3 kg mass's acceleration relative to the ground.

P-4 | Q-7

7. A 0.20 kg ball is dropped from rest from a height of 5.0 m, strikes a horizontal floor, and rebounds to a height of 3.2 m. The contact time with the floor is 0.040 s. Find the impulse of the net force on the ball, the average contact force from the floor, and the coefficient of restitution. Do not neglect the ball's weight during the contact time.

P-4 | Q-8

8. A rocket has an initial total mass of 1000 kg, of which 200 kg is fuel. It is launched vertically from rest. Gas is expelled downward at 10 kg per second at a speed of 2000 ms^{-1} relative to the rocket. Take g as constant and neglect air resistance. Find the time at which the fuel runs out, the initial and final accelerations, and the velocity just as the fuel runs out.

P-5 | Q-9

9. On a frictionless horizontal surface, two particles of mass 2 kg and 1 kg move with velocities $(6\hat{i} + 2\hat{j})\ \text{ms}^{-1}$ and $(-3\hat{i} + 4\hat{j})\ \text{ms}^{-1}$ respectively. They stick together after colliding. Find the combined velocity vector, its magnitude, and the kinetic energy lost.

P-5 | Q-10

10. On a frictionless straight track, particle A (mass 2 kg) starts from $x = 0$ with velocity $+5\ \text{ms}^{-1}$ and particle B (mass 3 kg) starts from $x = 12$ m with velocity $-1\ \text{ms}^{-1}$ at the same instant. Their head-on collision is perfectly elastic. Find the time and position of the collision, the velocities of both particles after the collision, and the velocity of the system's centre of mass. Verify the collision position using the motion of the centre of mass.

P-6 | Q-11

11. **Two blocks of mass 1.00 kg and 1.98 kg rest in a line on a frictionless horizontal table. A 0.020 kg bullet passes through the first block and embeds in the second. The first block's final speed is 2.0 ms^{-1}, and the combined speed of the second block and bullet is 3.0 ms^{-1}, both in the bullet's original direction of motion. Find the bullet's speed after leaving the first block, its initial speed, and the kinetic energy lost in each collision.**

P-6 | Q-12

12. **A uniform thin rod of length 2 m and mass 3 kg can rotate about a fixed axis perpendicular to it through one end. A 1 kg point mass is fixed at the other end and a 2 kg point mass at the midpoint. Starting from rest, a constant net torque of 5 Nm acts about the axis for 4 s. Find the total moment of inertia, the radius of gyration, the final angular velocity, and the kinetic energy.**

P-7 | Q-13

13. **A solid sphere of mass 2 kg and radius 0.20 m rolls without slipping from rest down a sufficiently rough incline, losing 1.4 m of vertical height. There is no energy dissipation. Find the speed of the centre, the angular velocity, and the translational and rotational kinetic energies separately on reaching the bottom.**

P-7 | Q-14

14. **A 4 kg spool rests on a rough horizontal floor. Its outer radius is $R = 0.40$ m, inner (hub) radius $r = 0.20$ m, and radius of gyration about the centre $k = 0.30$ m. A light string wound on the inner hub comes off tangentially from below and is pulled to the right at 30° above the horizontal with force P. Take counter-clockwise rotation as positive; the coefficient of static friction is 0.20. Find the maximum P for rolling without slipping. For $P = 10$ N, find the acceleration and friction force, and show at what angle the direction of the acceleration reverses.**

P-8 | Q-15

15. A road curve of radius 80 m is banked at angle θ, where $\tan\theta = \frac{1}{4}$. Assuming zero lateral friction between the tyres and the road, find the safe speed and the total normal reaction for an 800 kg car. If the distance between the two edges of the road, measured along the slope, is 1.50 m, how much higher is the outer edge?

P-8 | Q-16

16. A 0.50 kg particle tied to a light string of length 1.0 m moves in a horizontal circle at constant speed. The string makes a 60° angle with the vertical. Find the tension in the string, the speed of the particle, the period of revolution, and the centripetal acceleration.

P-9 | Q-17

17. A person stands on a frictionless rotating platform about a vertical axis. The combined moment of inertia of the person and the platform is initially $6\,\text{kg}\,\text{m}^2$ with angular velocity $2\,\text{rad}\,\text{s}^{-1}$. The person pulls in their arms, reducing the moment of inertia to $2\,\text{kg}\,\text{m}^2$; the external torque is zero during this phase. Then, keeping the moment of inertia the same, a constant opposing torque of $3\,\text{Nm}$ brings the platform to rest. Find the angular velocity after pulling in the arms, the work done by the person, the braking time, the angular displacement, and the energy dissipated by the brake.

P-9 | Q-18

18. A water jet of diameter $2.0\,\text{cm}$ strikes a stationary, smooth curved vane at $20\,\text{m}\,\text{s}^{-1}$ along $+\text{x}$. The entire flow leaves at the same speed at $120°$ from the $+\text{x}$ axis, exiting on the $+\text{y}$ side. The density of water is $1000\,\text{kg}\,\text{m}^{-3}$. Pressure is equal at entry and exit; the weight of the jet is negligible. Find the two components of the force of the water on the vane, the magnitude and direction of the resultant, and the force needed to hold the vane in place.

P-10 | Q-19

19. A 0.50 kg particle has position $\vec{r} = (3\cos 2t)\hat{i} + (2\sin 2t)\hat{j}$ metres, where t is in seconds. Find the angular momentum and torque about the origin. Show whether the force is central, and, taking the potential energy at the origin to be zero, find the total mechanical energy.

P-10 | Q-20

20. A 3 kg wedge rests on a frictionless horizontal floor. Its smooth incline slopes down to the right at 30° to the horizontal. A 1 kg block rests on the incline; both start at rest. Find the wedge's displacement, the ground-frame velocities of both the wedge and the block, the wedge's acceleration, and the normal reaction, at the moment the block has slid 2.0 m down the incline relative to the wedge. The block does not reach the edge of the incline in this interval.

P-11 | Q-21

21. For a complex number z, if $|z-3|+|z+3|=10$, prove that it represents an ellipse. Find the area of the ellipse.

P-11 | Q-22

22. If $z=16\sqrt{3}+16i$, find the fifth roots of z and express the roots obtained in polar form.

P-12 | Q-23

23. If a complex number z satisfies the inequality $|z - 4\sqrt{3} - 4i| \leq 4$, find the maximum and minimum principal argument of z.

P-12 | Q-24

24. Solve: $z^2 + \bar{z} = 0$

P-13 | Q-25

25. z_1 **and** z_2 **are two complex numbers with** $|z_1| = |z_2| = 1$**. If** $1 + z_1 z_2 \neq 0$**, show that** $\frac{z_1+z_2}{1+z_1 z_2}$ **is a real number.**

P-13 | Q-26

26. If $|z + 2i| \leq 1$**, find the maximum and minimum value of** $|z - 3 + 4i|$**.**

P-14 | Q-27

27. If $x = -1 + i\sqrt{2}$, find the value of $x^4 + 4x^3 + 6x^2 + 4x + 9$.

P-14 | Q-28

28. On the Argand plane, $\text{Arg}\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a locus. Find the equation of the locus and state what kind of curve it is.

P-15 | Q-29

29. Let $P(x) = x^4 + ax^3 + bx^2 + ax + 1$, where $a, b \in \mathbb{R}$. The polynomial is divisible by $x^2 - 3x + 1$, and the equation $P(x) = 0$ has exactly three distinct real roots. Find the possible values of a, b and determine the repeated root in each case.

P-15 | Q-30

30. If $z = \cos\theta + i\sin\theta$, prove that,

(i) $\frac{2}{1+z} = 1 - i\tan\frac{\theta}{2}$

(ii) $(1+z)^n + (1+\bar{z})^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$

P-16 | Q-31

31. If the equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ ($a \neq c$) have a common root, prove that $a + b + c = 0$ or $a - b + c = 0$.

P-16 | Q-32

32. For all real values of x, the function $f(x) = \frac{x^2+34x-71}{x^2+2x-7}$ can never take a value between 5 and 9 — verify the truth of this statement mathematically.

P-17 | Q-33

33. Prove that, for all real values of a, b, c, the roots of the equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are always real. When will the roots be equal?

P-17 | Q-34

34. If α and β are the roots of the equation $x^2 - px + q = 0$, express the roots of the equation $q^2x^2 - (p^3 - 3pq)x + q = 0$ in terms of α and β.

P-18 | Q-35

35. If the roots of the equation $x^3 + 3px^2 + 3qx + r = 0$ are in geometric progression, express r in terms of p and q.

P-18 | Q-36

36. If one root of the equation $ax^2 + bx + c = 0$ is the n-th power of the other, find the value of $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}}$.

P-19 | Q-37

37. If the roots of the equation $x^2 - 2cx + ab = 0$ are real and unequal, prove that the roots of the equation $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ are complex (imaginary).

P-19 | Q-38

38. If the roots of the equation $x^3 - 2x^2 + 3x - 4 = 0$ are α, β, γ, find the value of the symmetric expressions $\sum \alpha^2\beta$ and $\sum \alpha^3\beta$.

P-20 | Q-39

39. If the roots of the equation $32x^3 - 48x^2 + 22x - k = 0$ are in arithmetic progression, find the value of k and solve the equation.

P-20 | Q-40

40. If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root (where $p \neq q$), prove that $p + q + 1 = 0$. Find the new equation formed by the other roots of the two equations.

P-21 | Q-41

41. Balance the following redox equation by the ion-electron method: $MnO_4^- + C_2O_4^{2-} + H^+ \rightarrow Mn^{2+} + CO_2 + H_2O$

P-21 | Q-42

42. 0.63 g of a dibasic organic acid requires 100 mL of 0.1 M NaOH for complete neutralization. Calculate the molecular mass of the organic acid.

P-22 | Q-43

43. A sample of gas occupies 450 mL at 27°C and 750 mmHg pressure. What will be its volume at STP?

P-22 | Q-44

44. How many molecules of CO_2 are present in 1.12 L of CO_2 gas measured at STP?

P-23 | Q-45

45. Explain why phenolphthalein is not a suitable indicator for titrating a weak base (NH_4OH) against a strong acid (HCl).

P-23 | Q-46

46. Calculate the volume of O_2 gas at 25°C and 1 atm pressure produced by the complete decomposition of 12.25 g of $KClO_3$.

47. Equal volumes of 0.1 M HCl and 0.1 M $Ba(OH)_2$ solutions are mixed together. Determine the nature of the mixed solution and calculate the concentration of excess OH^- ions.

48. A compound contains 40% carbon, 6.67% hydrogen, and 53.33% oxygen by mass. If its vapor density is 30, determine its empirical and molecular formula.

P-25 | Q-49

49. What is the strength of a '20 Volume' H_2O_2 solution in percentage (w/v) and Molarity?

P-25 | Q-50

50. A 2.5 g mixture of Na_2CO_3 and $NaOH$ is dissolved in water to make 250 mL of solution. A 25 mL portion requires 15 mL of 0.1 M HCl to reach the phenolphthalein endpoint. Adding methyl orange and continuing the titration requires a further 10 mL of HCl. Find the percentage of Na_2CO_3, $NaOH$, and inert impurity in the original mixture.

51. A 1.74 g pyrolusite ore sample is heated with excess concentrated HCl. The Cl_2 gas produced is passed completely into excess KI solution. Titrating the liberated I_2 requires 80.0 mL of 0.200 M $Na_2S_2O_3$. Write the three relevant reactions, find the percentage of pure MnO_2 in the ore, and the mass of $Na_2S_2O_3 \cdot 5H_2O$ used.

52. A 3.2 g impure copper ore is dissolved in acid, and excess KI is added. Neutralizing the liberated iodine requires 40 mL of 0.15 M $Na_2S_2O_3$ solution. Find the percentage purity of copper in the ore.

P-27 | Q-53

53. A 98% (by mass) H_2SO_4 solution has density 1.84 gmL^{-1}. A 10.0 mL portion is diluted with water to make 500 mL of solution. Find the molarity of the concentrated solution, the molarity of the diluted solution, the volume of 0.500 M $NaOH$ needed to completely neutralize 25.0 mL of the diluted solution, and the total millimoles of H^+ in that 25.0 mL sample.

P-27 | Q-54

54. 100 mL of 0.2 M H_2SO_4 and 200 mL of 0.15 M KOH are mixed, and water is added to make a total volume of 1 L. Find the pH of the final solution.

P-28 | Q-55

55. A seawater sample has density 1.025 gmL^{-1}, containing 3.5% $NaCl$ and 0.13% Mg^{2+} ion by mass. Express the concentration of Mg^{2+} in ppm and molarity.

P-28 | Q-56

56. A 500 mL water sample contains 10.0 mg dissolved As_2O_3. Taking the solution density as 1.00 gmL^{-1} and As = 75, O = 16, find the concentration of As_2O_3 in ppm, the concentration of As alone in ppm, and the total volume the sample must be diluted to in order to reduce the As concentration to 10.0 ppb.

P-29 | Q-57

57. A 0.265 g sample of pure, dry Na_2CO_3 requires 48.0 mL of an HCl solution for complete neutralization. Then, 25.0 mL of that same HCl completely reacts with a 0.150 g impure $CaCO_3$ sample. Find the molarity of the HCl solution, the mass of pure $CaCO_3$ in the sample, and the percentage purity of the sample.

P-29 | Q-58

58. A 1.00 g sample of bleaching powder is reacted with excess KI and dilute acid. Titrating the liberated I_2 requires 28.0 mL of 0.200 M $Na_2S_2O_3$. Write the iodometric reactions, find the percentage available chlorine of the sample, and the volume the equivalent amount of Cl_2 would occupy at STP.

P-30 | Q-59

59. A 13.70 g mixture of Na_2CO_3 and $NaHCO_3$ is heated to constant mass, losing 3.10 g. The residue is dissolved in water to make 250 mL of solution. Find the mass of each component in the original mixture. Find the volume of 0.200 M HCl needed to titrate a 25.0 mL portion to the phenolphthalein endpoint, and the additional volume needed to reach the methyl orange endpoint.

P-30 | Q-60

60. An acidified 50.0 mL, 0.0200 M $K_2Cr_2O_7$ solution was mixed with 80.0 mL, 0.100 M $FeSO_4$ solution. After completion of the reaction, the remaining Fe^{2+} ions were titrated with 0.0200 M acidified $KMnO_4$ solution. Determine the limiting reactant in the first reaction and the volume of $KMnO_4$ required for the second reaction.