Engineering Weekly Exam 03 [Home Practice] EV

Complex Numbers + Polynomials and Polynomial Equations + Newtonian Mechanics + Quantitative Chemistry

Total Marks: 600 **Time:** 3 Hours

Physics: 200 Mathematics: 200 Chemistry: 200

Physics

200 marks

P-1 | Q-1

1. **An 8 kg block rests on a 30° incline that rises to the right. A horizontal force of 60 N is applied to the block, directed to the right (into the slope). The coefficients of static and kinetic friction are 0.25 and 0.20. Determine whether the block moves. Also find the minimum horizontal force (in the same direction) needed to start it moving up the incline. (Take g = 9.8 ms^{-2}.)**

P-1 | Q-2

2. **A rocket has an initial total mass of 1200 kg, of which 300 kg is fuel. It rises vertically from rest, expelling gas downward at 15 kg per second at 2100 ms^{-1} relative to the rocket. Neglecting air resistance, find the time for the fuel to run out, the initial and final accelerations, and the velocity just as the fuel runs out. (Take g = 9.8 ms^{-2}.)**

P-1 | Q-3

3. **Two masses of 2 kg and 6 kg are tied to the ends of a light, inextensible string passing over a solid disk pulley of mass 2 kg and radius 0.10 m. The axle is frictionless and the string does not slip. Find the acceleration of the system and the tension on both sides of the string. (Take g = 9.8 ms^{-2}.)**

P-1 | Q-4

4. **A 60 kg person stands on a spring balance in a lift. The lift first accelerates upward at 1.5 ms^{-2}, then downward at 2.0 ms^{-2}. Find the balance reading in both cases and the percentage change from the reading at rest. (Take g = 9.8 ms^{-2}.)**

P-1 | Q-5

5. **A ball is projected from the edge of a 20 m high cliff at 28 ms^{-1}, at 37° above the horizontal. Taking sin 37° = 0.60 and cos 37° = 0.80, find the time to reach the ground, the horizontal range, the velocity at impact, and the angle of impact with the horizontal. (Take g = 9.8 ms^{-2}.)**

P-1 | Q-6

6. A 3 kg block rests on a 37° incline and a 2 kg block rests on a horizontal surface. They are connected by the same string over a solid disk pulley of mass 1 kg and radius 0.20 m. The coefficients of kinetic friction on the incline and the horizontal surface are 0.10 and 0.20 respectively. Taking the 3 kg block as sliding down the incline, find the acceleration and the tension on each side. $\sin 37° = 0.60$, $\cos 37° = 0.80$. (Take $g = 9.8\ \text{ms}^{-2}$.)

P-1 | Q-7

7. A road curve of radius 90 m is banked at angle θ, where $\tan\theta = \frac{1}{3}$. Assuming zero lateral friction, find the safe speed and the total normal reaction on the road for a 1000 kg car. If the distance between the two edges, measured along the slope, is 1.50 m, how much higher is the outer edge? (Take $g = 9.8\ \text{ms}^{-2}$.)

P-1 | Q-8

8. A boat of mass 180 kg floats at rest in water. A 60 kg person on the boat walks 3.0 m relative to the boat from one end toward the other. Both were initially at rest, and the water exerts no horizontal force on the boat. Find the displacement of the boat and the displacement of the person relative to the ground.

P-1 | Q-9

9. A 2 kg particle moves right at $8\ \text{ms}^{-1}$ and a 3 kg particle moves left at $2\ \text{ms}^{-1}$. Their 1D collision has a coefficient of restitution of 0.50. Find the velocities of both particles after the collision, the impulse on each particle, and the kinetic energy lost.

P-1 | Q-10

10. A force $\vec{F}(t) = (4t\hat{i} + 6\hat{j})$ N acts on a 2 kg particle. At $t = 0$, its velocity is $(2\hat{i} - \hat{j})\ \text{ms}^{-1}$ and its position is at the origin. Find the velocity, position, and speed at $t = 3$ s.

P-1 | Q-11

11. A particle at position $\vec{r} = (2\hat{i} + 3\hat{j})$ m experiences a force $\vec{F} = (4\hat{i} - \hat{j} + 2\hat{k})$ N. Find the torque about the origin. If the particle is part of a rigid body with moment of inertia $I = 5\ \text{kg m}^2$ about the z-axis, and the initial angular velocity is $10\ \text{rads}^{-1}$, find the angular velocity after 2 s and the work done by the torque.

P-1 | Q-12

12. A uniform thin rod of length 2 m and mass 3 kg has a fixed axis perpendicular to it through one end. Point masses of 1 kg each are attached at 0.50 m and 2.0 m from the axis. Starting from rest, a constant torque of 10 Nm acts for 2 s. Find the total moment of inertia, the angular acceleration, the final angular velocity, and the final kinetic energy.

P-1 | Q-13

13. A thin ring of mass 3 kg and radius 0.25 m rolls without slipping down a 30° incline, losing 2.45 m of vertical height. Find the speed of the centre, the angular velocity, the acceleration, and the magnitude and direction of the static friction on reaching the bottom. (Take $g = 9.8\ ms^{-2}$.)

P-1 | Q-14

14. A solid disk of mass 4 kg and radius 0.40 m is at rest, free to rotate about a frictionless axis. A 0.010 kg bullet strikes the edge of the disk tangentially at $400\ ms^{-1}$ and embeds itself there. Find the angular velocity after the collision and the kinetic energy lost.

P-1 | Q-15

15. A jet of water, 4.0 cm in diameter, strikes a stationary, smooth curved vane at $25\ ms^{-1}$ along +x. The water leaves at the same speed along +y. The density of water is $1000\ kgm^{-3}$. Find the components of the force of the water on the vane, and the resultant with its direction.

P-1 | Q-16

16. A 2 kg particle moves under a central force $\vec{F} = -6x\hat{i} - 6y\hat{j}$ N. At $t = 0$, $\vec{r} = (2\hat{i} + \hat{j})$ m and $\vec{v} = (3\hat{i} - 4\hat{j})\ ms^{-1}$. Find the torque and angular momentum about the origin, and the total mechanical energy.

P-1 | Q-17

17. Two blocks of 2 kg and 3 kg are connected by a light spring and rest on a frictionless horizontal surface. The spring constant is $200\ Nm^{-1}$. The first block moves right at $4\ ms^{-1}$ while the second is at rest. Find the maximum compression, the velocity of each block at that instant, and the velocity of the centre of mass.

P-1 | Q-18

18. A small bead moves inside a smooth vertical circular track of radius 2.0 m. It is released from rest at height h above the bottom point. Find the minimum h needed to maintain contact throughout, and, in that case, the normal force on a 0.50 kg bead at the bottom point. (Take $g = 9.8\ ms^{-2}$.)

P-1 | Q-19

19. A stationary shell of mass 10 kg explodes into three fragments. A 2 kg fragment moves at $6\hat{i}\ ms^{-1}$ and a 3 kg fragment at $-2\hat{j}\ ms^{-1}$. The third fragment has mass 5 kg. Find the velocity of the third fragment and the total kinetic energy produced by the explosion.

P-1 | Q-20

20. A motorcycle takes a flat curve of radius 50 m at $12\ ms^{-1}$. The coefficient of friction between the tyres and the road is 0.30. Find the minimum radius for taking the turn without slipping, and the angle from the vertical the motorcycle must lean. (Take $g = 9.8\ ms^{-2}$.)

Mathematics

200 marks

P-1 | Q-21

21. If $z^2 + 4z + (1 - 12i) = 0$ has two distinct roots z_1 and z_2, find the value of $|z_1|^2 + |z_2|^2$.

P-1 | Q-22

22. z is a complex number satisfying $|z + 3| = |z - 3|$ and $\arg\left(\frac{z+4}{z-2i}\right) = \frac{\pi}{4}$. Find the value of $|z|^2$.

P-1 | Q-23

23. If z_1 and z_2 are the roots of $z^2 + 6z + 36 = 0$, find the value of $|z_1 + 3\sqrt{3}i|^2 + |z_2 + 3\sqrt{3}i|^2$.

P-1 | Q-24

24. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}$, find the value of $(z^{105} - i)^6$.

P-1 | Q-25

25. $S = \{z \in \mathbb{C} : 2z^2 + \bar{z} = 0\}$ is a set of complex numbers. Find the sum of the squares of the moduli of the nonzero roots in S, i.e. $\left(\sum_{z \in S, z \neq 0} |z|^2\right)$.

P-1 | Q-26

26. $C_1 : |z| = 4$ and $C_2 : |z - 5 - 12i| = 6$ are two circles. If $z_1 \in C_1$ and $z_2 \in C_2$, find the value of $\max |z_1 - z_2| + \min |z_1 - z_2|$.

P-1 | Q-27

27. z is a complex number satisfying both $|z - 8| = 5$ and $|z - 2 - 8i| = 5$. Find the value of $z^2 - 10z + 45$.

P-1 | Q-28

28. x and y are real numbers satisfying $5\left(\frac{x}{1+2i} + \frac{y}{2-i}\right) = 11 - 2i$. Find the value of $x + y$.

P-1 | Q-29

29. Subject to $|z - 4| \leq 2$, find the complex number that produces the maximum principal argument of z .

P-1 | Q-30

30. Subject to $|2z - 4(1 + i)| \leq 6$, find the minimum value of $|z + 2 - i|$.

P-1 | Q-31

31. α and β are the roots of $x^2 - 7x + r = 0$. Also, 2α and $\frac{\beta}{2}$ are the roots of $x^2 - 5x + r = 0$. Find the value of r.

P-1 | Q-32

32. α and β are the roots of $\lambda x^2 - (\lambda + 4)x + 4 = 0$ where $\alpha < \beta$. If $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{2}$, find the sum of all possible values of λ.

P-1 | Q-33

33. If α and β are the roots of $x^2 - \sqrt{3}x + 3 = 0$, find the value of $\alpha^{10} + \beta^{10}$.

P-1 | Q-34

34. The roots of $x^3 + px + q = 0$ (where p, q are real numbers) are α, β, γ. If $\beta\gamma = 2 = -\alpha$, find the value of $p^3 + q^3 - 3\alpha\beta\gamma$.

P-1 | Q-35

35. If both roots of the quadratic equation $(k-2)x^2 - 4kx + 5k = 0$ are real, distinct and positive, find how many integer values of k are possible.

P-1 | Q-36

36. Find the sum of the squares of the real roots of the equation $x^2 - 7|x| + 12 = 0$.

P-1 | Q-37

37. If the solution set of the equation $|x^2 + 2x - 15| = |x^2 - x| + |3x - 15|$ is $[\alpha, \beta] \cup [\gamma, \infty)$, find the value of $\alpha + \beta + \gamma$.

P-1 | Q-38

38. The equations $x^2 + cx + d = 0$ and $x^2 + dx + c = 0$ (where $c \neq d$) have a common root. Find the new equation formed by the two distinct (non-common) roots of the two equations.

P-1 | Q-39

39. If the roots of $x^3 - 5x^2 + 8x - 4 = 0$ are α, β, γ, find the value of $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha)$.

P-1 | Q-40

40. If the roots of $x^3 - 9x^2 + kx - 24 = 0$ are in arithmetic progression, find the value of k.

Chemistry

200 marks

P-1 | Q-41

41. A 2.00 g mixture of Na_2CO_3, $NaOH$, and an inert (acid-unreactive) impurity is dissolved to make 200 mL of solution. A 20.0 mL portion requires 12.0 mL of 0.150 M HCl to reach the phenolphthalein endpoint, and a further 8.00 mL to reach the methyl orange endpoint. Find the percentage composition of all three components.

P-1 | Q-42

42. A 1.26 g pure divalent metal carbonate MCO_3 is dissolved in 50.0 mL of 0.800 M HCl. Neutralizing the excess acid requires 25.0 mL of 0.400 M $NaOH$. Find the atomic mass and identity of the metal, and the STP volume of CO_2 produced.

P-1 | Q-43

43. In acidic medium, 40.0 mL of 0.0250 M $K_2Cr_2O_7$ is mixed with 60.0 mL of 0.150 M $FeSO_4$. Find the limiting reactant, the moles of unreacted Fe^{2+}, and the volume of 0.0250 M acidic $KMnO_4$ needed to oxidize it.

P-1 | Q-44

44. A 2.61 g pyrolusite ore sample is heated with excess concentrated HCl. The Cl_2 produced is passed into excess KI solution, and titrating the liberated I_2 requires 96.0 mL of 0.250 M $Na_2S_2O_3$. Find the percentage purity of the ore (as MnO_2) and the mass of $Na_2S_2O_3 \cdot 5H_2O$ used.

P-1 | Q-45

45. Heating 2.50 g of $CuSO_4 \cdot xH_2O$ to constant mass gives 1.60 g of $CuSO_4$. Find x, the percentage of crystal water, and the mass of anhydrous salt obtainable from 12.50 g of the hydrated salt.

P-1 | Q-46

46. A 0.300 g mixture of $NaCl$ and $NaBr$ is reacted with excess $AgNO_3$, giving 0.600 g of mixed $AgCl$ and $AgBr$ precipitate. Find the mass and percentage composition of both salts. ($NaCl$ = 58.5, $NaBr$ = 103, $AgCl$ = 143.5, $AgBr$ = 188)

P-1 | Q-47

47. 15.0 mL of a gaseous hydrocarbon is exploded with 90.0 mL of O_2. After cooling, the volume is 75.0 mL. Passing this through excess KOH solution reduces the volume by a further 30.0 mL. Find the formula of the hydrocarbon and the volume of O_2 used.

P-1 | Q-48

48. 50.0 mL of 0.200 M H_3PO_4 is mixed with 75.0 mL of 0.200 M NaOH. Given pK_{a2} = 7.20, find the dominant phosphate species, the pH of the mixture, and the additional volume of 0.200 M NaOH needed to reach the second equivalence point.

P-1 | Q-49

49. A 2.00 g impure $FeC_2O_4 \cdot 2H_2O$ sample is dissolved in dilute H_2SO_4 and 50.0 mL of 0.200 M $KMnO_4$ is added. Neutralizing the excess $KMnO_4$ requires 100.0 mL of 0.200 M $FeSO_4$. Find the purity of the sample, the STP volume of CO_2 produced, and the moles of Fe^{3+} produced from the oxidation of FeC_2O_4.

P-1 | Q-50

50. A solution contains $FeSO_4$ and $Fe_2(SO_4)_3$. A 25.0 mL portion titrated directly requires 10.0 mL of 0.0200 M acidic $KMnO_4$. Another 25.0 mL portion, after reducing all Fe^{3+} to Fe^{2+}, requires 25.0 mL of the same $KMnO_4$. Find the molar ratio of Fe^{2+} : Fe^{3+}, and the concentrations of $FeSO_4$ and $Fe_2(SO_4)_3$ in gL^{-1}. ($FeSO_4$ = 152, $Fe_2(SO_4)_3$ = 400)

P-1 | Q-51

51. A 0.500 g impure $KMnO_4$ sample is titrated in acidic medium, requiring 150 mL of 0.100 M Fe^{2+}. Find the purity of the sample, the mass of impurity, and the moles of Mn^{2+} produced.

P-1 | Q-52

52. A 98.0% (by mass) H_2SO_4 solution has density 1.80 gmL^{-1}. A 12.0 mL portion is used to make 600 mL of solution. Find the molarity of the concentrated and diluted solutions, and the volume of 0.450 M NaOH needed to neutralize 25.0 mL of the diluted solution.

P-1 | Q-53

53. A 50.0 mL hard water sample requires 12.5 mL of 0.0100 M EDTA for titration. Find the hardness in ppm (as $CaCO_3$), in French degrees, and the total $CaCO_3$-equivalent mass in 2.00 L of sample. ($1°f = 10\ mgL^{-1}$)

P-1 | Q-54

54. Combustion of 0.120 g of a compound containing C, H, O gives 0.264 g CO_2 and 0.108 g H_2O. The molar mass of the compound is 160 $gmol^{-1}$. Find the percentage composition, the empirical formula, and the molecular formula.

P-1 | Q-55

55. Find the equivalent mass of H_3PO_4 (= 98) for its conversion to NaH_2PO_4, Na_2HPO_4, and Na_3PO_4 from a 4.90 g sample. Also find the separate volumes of 0.500 M NaOH needed to reach the Na_2HPO_4 and Na_3PO_4 stages.

P-1 | Q-56

56. A mixture contains 8.00 g O_2 and 14.0 g N_2. Find the mole fraction of each gas, the partial pressures if the total pressure is 2.00 atm, the average molar mass of the mixture, and its density at STP.

P-1 | Q-57

57. A 1.20 g sample of bleaching powder reacts with excess KI and dilute acid. Titrating the liberated I_2 requires 32.0 mL of 0.250 M $Na_2S_2O_3$. Find the percentage available chlorine and the STP volume of the equivalent Cl_2.

P-1 | Q-58

58. Neutralizing 0.5105 g of potassium hydrogen phthalate, $KHC_8H_4O_4$, requires 25.00 mL of NaOH. Neutralizing 20.00 mL of this standardized NaOH requires 18.50 mL of HCl. Find the molarity of both NaOH and HCl. (Molar mass of KHP = 204.2 $gmol^{-1}$)

P-1 | Q-59

59. **A 16.44 g mixture of Na_2CO_3 and $NaHCO_3$ is heated to constant mass, losing 3.72 g. The residue is dissolved to make 200 mL of solution. Find the mass of each component in the original mixture, and the volume of 0.150 M HCl needed to titrate a 20.0 mL portion to the phenolphthalein and methyl orange endpoints.**

P-1 | Q-60

60. **A 10.20 g pure mixture of $CaCO_3$ and $MgCO_3$ is strongly heated to constant mass, losing 4.84 g. Find the mass and percentage of each carbonate in the original mixture, the mass of CaO and MgO residue, and the volume of 1.00 M HCl needed to dissolve the entire residue.**