MCQ 50 marks

1. **A.** $2.88 \times 10^7\ \text{NC}^{-1}$

Explanation: A and C are at opposite ends of a diameter, so their fields cancel

B and D are at opposite ends of a diameter; both fields point towards D

$E = 2 \times \frac{9\times10^9\times4\times10^{-6}}{(0.05)^2} = 2.88 \times 10^7\ \text{NC}^{-1}$

2. **C.** $-\frac{9q}{16}$**, at** $\frac{1}{4}$ **from q**

Explanation: Point of zero field: $\frac{x}{1-x} = \sqrt{\frac{1}{9}} \Rightarrow x = \frac{1}{4}$

Equilibrium of q: $\frac{kQq}{\left(\frac{1}{4}\right)^2} = \frac{k(9q)q}{l^2}$

$Q = \frac{9q}{16}$, and it must be negative: $Q = -\frac{9q}{16}$

3. **A.** $6.32 \times 10^6\ \text{ms}^{-1}$

Explanation: $a = \frac{eE}{m} = \frac{1.6\times10^{-19}\times910}{9.1\times10^{-31}} = 1.6 \times 10^{14}\ \text{ms}^{-2}$

Maximum allowed deflection $y = 0.5\ \text{cm}$

$y = \frac{1}{2}a\left(\frac{L}{v}\right)^2 \Rightarrow v = L\sqrt{\frac{a}{2y}}$

$v = 0.05 \times \sqrt{\frac{1.6\times10^{14}}{0.01}} = 6.32 \times 10^6\ \text{ms}^{-1}$

4. **B.** $6.44 \times 10^5\ \text{NC}^{-1}$

Explanation: The fields of the charges at the two ends of that side are equal and opposite, so they cancel

The other two charges are at distance $r = \sqrt{a^2 + \frac{a^2}{4}} = \frac{\sqrt{5}a}{2}$

The components parallel to the side cancel and the perpendicular components add: $\cos\alpha = \frac{a}{r} = \frac{2}{\sqrt{5}}$

$E = 2 \times \frac{4kq}{5a^2} \times \frac{2}{\sqrt{5}} = \frac{16kq}{5\sqrt{5}a^2}$

$= \frac{16\times9\times10^9\times2\times10^{-6}}{5\sqrt{5}\times0.04} = 6.44 \times 10^5\ \text{NC}^{-1}$

5. **A.** $20\ \mu\text{C}, 5\ \mu\text{C}$

Explanation: $q_1q_2 = \frac{Fr^2}{k} = \frac{0.1\times9}{9\times10^9} = 10^{-10}\ \text{C}^2 = 100\ (\mu\text{C})^2$

$q_1 + q_2 = 25\ \mu\text{C}$

$(q_1 - q_2)^2 = 625 - 400 = 225 \Rightarrow q_1 - q_2 = 15$

$\therefore q_1 = 20\ \mu\text{C}, q_2 = 5\ \mu\text{C}$

6. **C.** $\frac{2kQq}{r^2}$

Explanation: If all six vertices carried +q, the force at the centre would be zero by symmetry. Changing one vertex from +q to −q is the same as adding an extra −2q there.

$\therefore F = \frac{kQ(2q)}{r^2} = \frac{2kQq}{r^2}$

7. **D.** 1.24×10^{36}

Explanation: $\frac{F_E}{F_G} = \frac{\frac{ke^2}{r^2}}{\frac{Gm_p^2}{r^2}} = \frac{ke^2}{Gm_p^2}$ (independent of the distance)

$= \frac{9\times10^9\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times(1.67\times10^{-27})^2}$

$= 1.24 \times 10^{36}$

8. **D.** $0.036\ \text{N}$

Explanation: With no gravity, the two threads lie straight, at 180° to each other

Distance between the balls = $2L = 1\ \text{m}$

$T = \frac{kq^2}{(2L)^2} = \frac{9\times10^9\times4\times10^{-12}}{1} = 0.036\ \text{N}$

9. **D.** $\frac{1}{27}$

Explanation: $F \propto \frac{1}{Kr^2}$

$\frac{F'}{F} = \frac{1}{3\times3^2} = \frac{1}{27}$

10. **B.** $\sqrt{\frac{m_p}{m_e}}$

Explanation: $s = \frac{1}{2}\frac{eE}{m}t^2 \Rightarrow t \propto \sqrt{m}$

$\frac{t_p}{t_e} = \sqrt{\frac{m_p}{m_e}}$

11. **B.** $2.03 \times 10^{-7}\ \text{NC}^{-1}$

Explanation: $2eE = m_\alpha g$

$E = \frac{6.64\times10^{-27}\times9.8}{3.2\times10^{-19}} = 2.03 \times 10^{-7}\ \text{NC}^{-1}$

12. **D.** $5\ \text{Nm}^2\text{C}^{-1}$

Explanation: Eight cubes around the vertex enclose the charge completely

$\phi = \frac{q}{8\epsilon_0} = \frac{3.542\times10^{-10}}{8\times8.854\times10^{-12}} = 5\ \text{Nm}^2\text{C}^{-1}$

13. **A.** $8.57\ \text{cm}$

Explanation: $\frac{x}{30-x} = \sqrt{\frac{4}{25}} = \frac{2}{5}$

$5x = 60 - 2x \Rightarrow x = \frac{60}{7} = 8.57\ \text{cm}$

14. **A.** $20\ \text{Nm}^2\text{C}^{-1}$

Explanation: $A = (0.2)^2 = 0.04\ \text{m}^2$, and the area vector is along the x-axis

$\phi = EA = 500 \times 0.04 = 20\ \text{Nm}^2\text{C}^{-1}$

15. **D.** $3.39 \times 10^5\ \text{Nm}^2\text{C}^{-1}$

Explanation: Only the enclosed charge matters: $q = 5 - 2 = 3\ \mu\text{C}$

$\phi = \frac{3\times10^{-6}}{8.854\times10^{-12}} = 3.39 \times 10^5\ \text{Nm}^2\text{C}^{-1}$

16. **C.** $0.2\ \mu\text{C m}^{-1}$

Explanation: $E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$

$\lambda = \frac{Er}{2k} = \frac{7.2\times10^4\times0.05}{1.8\times10^{10}} = 2 \times 10^{-7}\ \text{Cm}^{-1}$

17. **D.** $2 : 1$

Explanation: Inside, $E = \frac{kQr}{R^3} \Rightarrow E\left(\frac{R}{2}\right) = \frac{kQ}{2R^2}$

Outside, $E(2R) = \frac{kQ}{4R^2}$

Ratio $= \frac{\frac{1}{2}}{\frac{1}{4}} = 2 : 1$

18. **D.** $90\ \text{V}$

Explanation: $V = \frac{k\sum q}{r} = \frac{9\times10^9\times5\times10^{-9}}{0.5} = 90\ \text{V}$

19. **D.** $20\ \text{cm}$

Explanation: $E = 0$ outside the $-q$ charge:

$\frac{4}{x^2} = \frac{1}{(x-30)^2} \Rightarrow x = 60\ \text{cm}$

$V = 0$: between the charges, $\frac{4}{x} = \frac{1}{30-x} \Rightarrow x = 24\ \text{cm}$; outside them, $\frac{4}{x} = \frac{1}{x-30} \Rightarrow x = 40\ \text{cm}$

Nearest distance $= 60 - 40 = 20\ \text{cm}$

20. **C.** $-9\ \text{C}$

Explanation: All corners are equidistant from the centre, so $\sum q = 0$

$q_4 = -(6 - 9 + 12) = -9\ \text{C}$

21. **C.** $500\ \text{V}$

Explanation: $V_B = N^{\frac{2}{3}}V_s = 125^{\frac{2}{3}} \times 20 = 25 \times 20 = 500\ \text{V}$

22. **A.** $1.69 \times 10^7\ \text{V}$

Explanation: $q = 82 \times 1.6 \times 10^{-19}\ \text{C}$, $r = 7 \times 10^{-15}\ \text{m}$

$V = \frac{9\times10^9\times82\times1.6\times10^{-19}}{7\times10^{-15}}$

$= 1.69 \times 10^7\ \text{V}$

23. **C.** $2 \times 10^6\ \text{NC}^{-1}$

Explanation: $E = \frac{kQ}{R^2} = \frac{9\times10^9\times5\times10^{-6}}{(0.15)^2} = 2 \times 10^6\ \text{NC}^{-1}$

24. **D.** $0.09\ \text{J}$

Explanation: There are three pairs of charges:

$U = \frac{3kq^2}{a}$

$= \frac{3\times9\times10^9\times10^{-12}}{0.3} = 0.09\ \text{J}$

25. **D.** $1.76 \times 10^{11}\ \text{Ckg}^{-1}$

Explanation: $eV = \frac{1}{2}mv^2 \Rightarrow \frac{e}{m} = \frac{v^2}{2V}$

$= \frac{(8.4\times10^7)^2}{2\times2\times10^4} = 1.76 \times 10^{11}\ \text{Ckg}^{-1}$

26. **A.** $1.44 \times 10^{-13}\ \text{J}$

Explanation: $|W| = e(V_2 - V_1) = ekQ\left(\frac{1}{0.1} - \frac{1}{0.3}\right)$

$= 1.6 \times 10^{-19} \times 9 \times 10^9 \times 15 \times 10^{-6} \times 6.667 = 1.44 \times 10^{-13}\ \text{J}$

27. **B.** $3.2 \times 10^{-16}\ \text{J}$

Explanation: $K = eV = 1.6 \times 10^{-19} \times 2000 = 3.2 \times 10^{-16}\ \text{J}$

28. **C.** $x = \pm2$

Explanation: $E = -\frac{dV}{dx} = -(12 - 3x^2)$

$12 - 3x^2 = 0 \Rightarrow x = \pm2$

29. **A.** 7

Explanation: $\vec{E} = -\nabla V = 2\hat{i} - 3\hat{j} - 6\hat{k}$

$|\vec{E}| = \sqrt{4 + 9 + 36} = 7$ units

30. **B.** $-7\ \text{V}$

Explanation: $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r} = -\int_0^4 2x\,dx - \int_3^0 3\,dy$

$= -[x^2]_0^4 - [3y]_3^0 = -16 + 9 = -7\ \text{V}$

31. **C.** $5 \times 10^4\ \text{Vm}^{-1}$

Explanation: $E = \frac{V}{d} = \frac{250}{5\times10^{-3}} = 5 \times 10^4\ \text{Vm}^{-1}$

32. **B.** $5000\ \text{NC}^{-1}$

Explanation: $p = 3 \times 10^{-6} \times 0.04 = 1.2 \times 10^{-7}\ \text{Cm}$

$E = \frac{kp}{r^3} = \frac{9\times10^9\times1.2\times10^{-7}}{(0.6)^3} = 5000\ \text{NC}^{-1}$

33. **C.** $-5 \times 10^{-4}\ \text{J}$

Explanation: $U = -pE\cos\theta = -2 \times 10^{-8} \times 5 \times 10^4 \times 0.5 = -5 \times 10^{-4}\ \text{J}$

34. **B.** $\sqrt{2}\,qa$

Explanation: Treating $-2q$ as two charges of $-q$ gives two dipoles, each of moment qa

The two dipoles lie along two sides of the square, so they are perpendicular to each other

$p = \sqrt{(qa)^2 + (qa)^2} = \sqrt{2}\,qa$

35. **B. Net force zero, torque non-zero**

Explanation: The forces on $+q$ and $-q$ are equal and opposite, so the net force is zero

The two forces do not act along the same line, so a couple is produced: $\tau = pE\sin\theta \neq 0$

36. **A.** $\frac{1}{2}$

Explanation: Work done by the battery = QV
Stored energy = $\frac{1}{2}$QV
Ratio = $\frac{1}{2}$ (the rest is lost as heat and radiation)

37. **C.** 1.125×10^{-3} J

Explanation: $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 10 \times 10^{-6} \times 225 = 1.125 \times 10^{-3}$ J

38. **B.** $\frac{3}{16}$

Explanation: $C_s = \frac{2\times6}{8} = 1.5\ \mu F$, $C_p = 8\ \mu F$
V is the same, so $\frac{U_s}{U_p} = \frac{C_s}{C_p} = \frac{1.5}{8} = \frac{3}{16}$

39. **B.** 6 μF, 4 μF

Explanation: $C_1 + C_2 = 10$, $\frac{C_1C_2}{C_1+C_2} = 2.4 \Rightarrow C_1C_2 = 24$
$(C_1 - C_2)^2 = 100 - 96 = 4 \Rightarrow C_1 - C_2 = 2$
$C_1 = 6\ \mu F, C_2 = 4\ \mu F$

40. **B.** 0.0125 J

Explanation: $C_p = 5 \times 2 = 10\ \mu F$
$U = \frac{1}{2} \times 10 \times 10^{-6} \times 50^2 = 0.0125$ J

41. **A.** 0.5 μF

Explanation: $\frac{1}{C} = 1 + \frac{1}{2} + \frac{1}{4} + \cdots$ (geometric series, $r = \frac{1}{2}$)
$\frac{1}{C} = \frac{1}{1-\frac{1}{2}} = 2 \Rightarrow C = 0.5\ \mu F$

42. **C.** 60 μC

Explanation: $\frac{1}{C_s} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1 \Rightarrow C_s = 1\ \mu F$
$Q = 1 \times 60 = 60\ \mu C$ (the same on every capacitor)

43. **A.** $\frac{U}{9}$

Explanation: Charge is conserved: $V' = \frac{Q}{3C} = \frac{V}{3}$
For the first capacitor, $U_1 = \frac{1}{2}C\left(\frac{V}{3}\right)^2 = \frac{U}{9}$

44. **B.** 20 pF

Explanation: $C = \frac{R}{k} = \frac{0.18}{9\times10^9} = 2 \times 10^{-11}$ F = 20 pF

45. **B.** 6

Explanation: $C_1 = \frac{\epsilon_0 A}{d} = 2$ pF
$C_2 = \frac{K\epsilon_0 A}{3d} = \frac{K}{3} \times 2 = 4$
$K = 6$

46. **D.** 6.67 pF

Explanation: $C' = \frac{K\epsilon_0 A}{3d} = \frac{4}{3} \times 5 = 6.67$ pF

47. **A.** 1.5

Explanation: The two parts are in series: $C = \frac{\epsilon_0 A}{\frac{d}{2}+\frac{d}{2K}}$
$\frac{C}{C_0} = \frac{2K}{K+1} = \frac{6}{4} = 1.5$

48. **D.** 4.43×10^{-7} C

Explanation: $C = \frac{\epsilon_0 A}{d} = \frac{8.854\times10^{-12}\times0.5}{10^{-3}} = 4.43 \times 10^{-9}$ F
$Q = CV = 4.43 \times 10^{-7}$ C

49. **C.** 0.797 mm

Explanation: Sphere: $C = \frac{R}{k} = \frac{0.08}{9\times10^9} = 8.89 \times 10^{-12}$ F
$\frac{K\epsilon_0 A}{d} = C \Rightarrow d = \frac{4\times8.854\times10^{-12}\times2\times10^{-4}}{8.89\times10^{-12}}$
$d = 7.97 \times 10^{-4}$ m = 0.797 mm

50. **A.** $-\frac{cx^3}{3}$

Explanation: We know, $V = -\int E\,dx$
Here, $E = cx^2$
So, $V = -\int cx^2\,dx$
$= -c\frac{x^3}{3}$
Therefore, $V = -\frac{cx^3}{3}$