MCQ 50 marks

1. **D.** $90°$

Explanation: Let the angle between the forces be α

Given, $10^2 = 6^2 + 8^2 + 2 \cdot 6 \cdot 8 \cos\alpha$

$\Rightarrow 96\cos\alpha = 0$

Therefore, $\alpha = 90°$

2. **B.** $8\sqrt{2}\,\text{N}$

Explanation: Let the oblique force be F

For vertical equilibrium, $F\sin 45° = 8$

Therefore, $F = 8\sqrt{2}\,\text{N}$

3. **A.** $6\sqrt{2}\,\text{N}$

Explanation: Let the smaller force be P and the greater force $Q = 12\,\text{N}$

The resultant is perpendicular to P, so $P + Q\cos\alpha = 0$

$P = -12\cos 135° = 12 \times \frac{1}{\sqrt{2}}$

Therefore, $P = 6\sqrt{2}\,\text{N}$

4. **A.** $4 : 5$

Explanation: Let the forces be P and Q; the resultant R is perpendicular to P and $R = \frac{3}{5}Q$

P, R, Q form a right-angled triangle with hypotenuse Q: $Q^2 = P^2 + R^2$

$P^2 = Q^2 - \frac{9}{25}Q^2 = \frac{16}{25}Q^2 \Rightarrow P = \frac{4}{5}Q$

Therefore, $P : Q = 4 : 5$

5. **C.** $2\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Explanation: Let the angle between the forces be α; taking the bisector as the axis, the forces lie at $\frac{\alpha}{2}$ on either side of it

The component along the axis is $(P + Q)\cos\frac{\alpha}{2}$ and perpendicular to it $(P - Q)\sin\frac{\alpha}{2}$

$\tan\phi = \frac{P-Q}{P+Q}\tan\frac{\alpha}{2} \Rightarrow \tan 30° = \frac{10}{20}\tan\frac{\alpha}{2}$

$\Rightarrow \tan\frac{\alpha}{2} = \frac{2}{\sqrt{3}}$

Therefore, $\alpha = 2\tan^{-1}\left(\frac{2}{\sqrt{3}}\right) \approx 98.21°$

6. **C.** $2 : \sqrt{3}$

Explanation: Here, the angle between R and P is $360° - 120° - 150° = 90°$

Lami's theorem: $\frac{P}{\sin 150°} = \frac{Q}{\sin 90°} = \frac{R}{\sin 120°}$

$Q : R = 1 : \frac{\sqrt{3}}{2}$

Therefore, $Q : R = 2 : \sqrt{3}$

7. **B.** $135°$

Explanation: The resultant of the first two forces equals the third in magnitude: $P^2 = P^2 + 2P^2 + 2\sqrt{2}P^2\cos\alpha$

$\Rightarrow \cos\alpha = -\frac{2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$

Therefore, $\alpha = 135°$

8. **D. a couple of moment** $96\,\text{N m}$

Explanation: The forces form a closed polygon, so the resultant force is zero; the system is equivalent to a couple

Moment about A: the moments of AB and DA are zero; BC : $6 \times 8 = 48$, CD : $8 \times 6 = 48$

Therefore, the total moment $= 96\,\text{N m}$

9. **B.** $\sqrt{17}\,\text{N}$

Explanation: The diagonals of a rhombus are perpendicular; OA and OC are opposite, and so are OB and OD

Along OC: $7 - 3 = 4$

Along OB: $5 - 4 = 1$

Therefore, $R = \sqrt{16 + 1} = \sqrt{17}\,\text{N}$

10. **B.** $\frac{1}{6}\,\text{m}$

Explanation: When the scale reads zero the moments about the pivot balance. The board's centre of gravity lies to the right of the pivot by $1.5 - 1 = 0.5\,\text{m}$

If the child sits at distance x to the left of the pivot: $30x = 50 \times 0.5 \Rightarrow x = \frac{5}{6}\,\text{m}$

Therefore, the distance from the left end $= 1 - \frac{5}{6} = \frac{1}{6}\,\text{m}$

11. **D.** $77.13\,\text{N}$

Explanation: The directions of the resultant and of T are fixed; P is the distance of the tip of the resultant from the line of T

P is least when $P \perp T$: $P_{min} = 120\sin 40° = 120 \times 0.6428$

Therefore, $P_{min} \approx 77.13\,\text{N}$

12. **D.** $86\,\text{lb}$ **and** $94\,\text{lb}$

Explanation: Moments about A: $R_B \times 10 = 40 \times 5 + 60 \times 3 + 80 \times 7 = 940$
$R_B = 94$ lb, $R_A = 180 - 94 = 86$ lb
Therefore, the reactions are 86 lb and 94 lb respectively

13. **A.** 7.17 N
Explanation: Equilibrium along the plane: $3 + F\cos\alpha = W\sin\alpha$
Here, $\cos\alpha = \frac{12}{13}$, $\sin\alpha = \frac{5}{13}$
$3 + \frac{12}{13}F = 25 \times \frac{5}{13} = 9.615$
Therefore, $F = \frac{(9.615-3)\times 13}{12} \approx 7.17$ N

14. **A.** 62.45 N
Explanation: In equilibrium, $T\cos\theta = W$, $T\sin\theta = F \Rightarrow T^2 = W^2 + F^2$
For $T \le 80$, $F \le \sqrt{80^2 - 50^2} = \sqrt{3900}$
Therefore, $F_{max} \approx 62.45$ N

15. **C.** 4.62 m
Explanation: Let the support be at distance x from A; the moment of all loads about A equals total load $\times x$
$x = \frac{50\times 0 + 120\times 4 + 90\times 8}{50+120+90} = \frac{1200}{260}$
Therefore, $x \approx 4.62$ m

16. **C.** 48 kg-wt
Explanation: Here, $7^2 + 24^2 = 25^2 \Rightarrow \angle ACB = 90°$
The strings are perpendicular; at angle A, $\sin A = \frac{24}{25}$ and $\cos A = \frac{7}{25}$
Resolving the weight along the two strings, $T_{CA} = W\sin A$
Therefore, $T_{CA} = 50 \times \frac{24}{25} = 48$ kg-wt

17. **D.** 287.88 N
Explanation: The resultant is vertical; AB is inclined at 50° to it and AC at 35° on the other side
Sine rule: $\frac{F_{AB}}{\sin 35°} = \frac{F_{AC}}{\sin 50°} = \frac{500}{\sin 85°}$
Therefore, $F_{AB} = \frac{500\times 0.5736}{0.9962} \approx 287.88$ N

18. **D.** 4
Explanation: Let the total weight be W; by Lami's theorem: $T_{40} = \frac{W\sin 60°}{\sin 100°}$, $T_{60} = \frac{W\sin 40°}{\sin 100°}$
The string nearer the vertical (40°) carries the greater tension: $T_{40} = 0.8794W \le 300 \Rightarrow W \le 341.1$ N
Mass $m \le \frac{341.1}{9.8} = 34.81$ kg
Extra mass $\le 34.81 - 25 = 9.81$ kg
Number of discs $\le \frac{9.81}{2} = 4.9$
Therefore, at most 4

19. **C.** 120°
Explanation: Let each force be P; the resultants are $2P\cos\alpha$ and $2P\cos\beta$
Given, $\cos\beta = \sqrt{3}\cos\alpha$, $\beta = \alpha - 30°$
$\frac{\sqrt{3}}{2}\cos\alpha + \frac{1}{2}\sin\alpha = \sqrt{3}\cos\alpha \Rightarrow \tan\alpha = \sqrt{3}$
So $\alpha = 60°$ and the angle between the forces $2\alpha = 120°$
Therefore, the angle between the forces is 120°

20. **A.** 3.47 N
Explanation: Let k = 10; for forces taken in the same order along the sides, the cosines of the angles between the pairs are $-\cos C$, $-\cos A$, $-\cos B$
$R^2 = k^2\left[\cos^2 A + \cos^2 B + \cos^2 C - 6\cos A\cos B\cos C\right]$
Using $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$, $R^2 = k^2(1 - 8\cos A\cos B\cos C)$
$\cos 50°\cos 60°\cos 70° = 0.6428 \times 0.5 \times 0.3420 = 0.10992$
Therefore, $R = 10\sqrt{1 - 0.8794} = 10 \times 0.3473 \approx 3.47$ N

21. **B.** 6.67 ton-wt **and** 5.33 ton-wt
Explanation: Moments about A: $R_B \times 30 = 8 \times 15 + 4 \times 10 = 160 \Rightarrow R_B = 5.33$ ton-wt
$R_A = 12 - 5.33 = 6.67$ ton-wt
Therefore, the pressures are 6.67 ton-wt and 5.33 ton-wt respectively

22. **C.** 3.75 m
Explanation: Let the shoulder be at distance x from the 18 kg end; moments about the shoulder: $18x = 30(6 - x) \Rightarrow 48x = 180$
Therefore, $x = 3.75$ m

23. **D.** 8 kg
Explanation: The rod's centre of gravity is 5 m from the end, that is from the peg $5 - 2 = 3$ m
Let the mass of the rod be m; moments about the peg: $12 \times 2 = m \times 3$
Therefore, $m = 8$ kg

24. **D.** 20 N

Explanation: The ring is smooth, so the tension is the same (T) in both parts of the string
Vertical equilibrium: $T + T\cos 60° = 30 \Rightarrow \frac{3}{2}T = 30$
Therefore, $T = 20\,\text{N}$

25. **A.** $13.09\,\text{h}$
Explanation: Relative speed = $\sqrt{20^2 + 25^2 - 2\cdot 20\cdot 25\cos 60°} = \sqrt{525} \approx 22.91\,\text{kmh}^{-1}$
The distance is proportional to time: $22.91t \leq 300$
Therefore, $t \leq 13.09\,\text{h}$

26. **B.** $5\,\text{kmh}^{-1}$
Explanation: Here, $\vec{v}_{\text{boat,ground}} = \vec{v}_{\text{boat,water}} + \vec{v}_{\text{water}}$
Moving in the opposite direction, $8 - 3 = 5\,\text{kmh}^{-1}$
Therefore, the speed relative to the bank is $5\,\text{kmh}^{-1}$

27. **A.** $4\,\text{N}$**; parallel to** AB**,** $3.75\,\text{m}$ **from** AB **(on the side opposite** C**)**
Explanation: Take $A = (0,0)$, $B = (5,0)$, $C = (0,5)$ and the force $\vec{F} = (F_x, F_y)$; the moment about Q is $M_Q = M_A + (\vec{A} - \vec{Q}) \times \vec{F}$
$M_B = 15 + (-5)F_y = 15 \Rightarrow F_y = 0$
$M_C = 15 + 5F_x = 35 \Rightarrow F_x = 4\,\text{N}$
So the force is parallel to AB; if its line of action is $y = y_0$, $M_A = -y_0F_x = 15 \Rightarrow y_0 = -3.75\,\text{m}$
Therefore, the force is $4\,\text{N}$, parallel to AB and $3.75\,\text{m}$ from AB on the side opposite C

28. **C.** $\frac{20}{7}\,\text{kmh}^{-1}$
Explanation: Displacement = $\sqrt{8^2 + 6^2} = 10\,\text{km}$
Total time = $\frac{8}{4} + 1.5 = 3.5\,\text{h}$
Average velocity = $\frac{10}{3.5} = \frac{20}{7} \approx 2.86\,\text{kmh}^{-1}$
Therefore, the average velocity is $\frac{20}{7}\,\text{kmh}^{-1}$

29. **A.** $\tan^{-1}\left(\frac{4}{3}\right)$
Explanation: Let the maximum height be H and the range R: $\frac{H}{R} = \frac{\frac{u^2\sin^2\alpha}{2g}}{\frac{2u^2\sin\alpha\cos\alpha}{g}} = \frac{\tan\alpha}{4}$
Therefore, $\tan\alpha = \frac{4H}{R} = \frac{4}{3} \Rightarrow \alpha = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.13°$

30. **C.** $\tan^{-1}(1.47)$
Explanation: Let the initial speed be u and the angle of projection α: $T = \frac{2u\sin\alpha}{g}$, $R = u\cos\alpha \cdot T$
$u\sin\alpha = \frac{gT}{2}$, $u\cos\alpha = \frac{R}{T}$
$\tan\alpha = \frac{gT^2}{2R} = \frac{9.8\times 36}{240} = 1.47$
Therefore, $\alpha = \tan^{-1}(1.47) \approx 55.77°$

31. **A.** $3.6\,\text{km}$
Explanation: Let the current be u, the boat's speed 3u and the width d; the crossing time is $t = \frac{d}{3u}$
Downstream drift $x = ut = \frac{d}{3} = 1.2$
Therefore, $d = 3.6\,\text{km}$

32. **C.** $0.6\,\text{ms}^{-1}$
Explanation: Swimmer's speed $v = \frac{120}{2} = 60\,\text{m min}^{-1}$
Effective speed straight across with the current $\frac{120}{2.5} = 48 = \sqrt{v^2 - u^2}$
$u = \sqrt{3600 - 2304} = 36\,\text{m min}^{-1}$
Therefore, $u = 0.6\,\text{ms}^{-1}$

33. **D.** $125\,\text{m}$
Explanation: Crossing time $t = \frac{0.2}{4} = 0.05\,\text{h}$
Downstream drift $= 2.5 \times 0.05 = 0.125\,\text{km}$
Therefore, the distance $= 125\,\text{m}$

34. **B.** $14\,\text{ms}^{-1}$
Explanation: In the next 4 s the motion is uniform: $v = \frac{64}{4} = 16\,\text{ms}^{-1}$
$u + 4a = 16$, $60 = 4u + 8a$
$60 = 4(16 - 4a) + 8a = 64 - 8a \Rightarrow a = 0.5\,\text{ms}^{-2}$
Therefore, $u = 16 - 2 = 14\,\text{ms}^{-1}$

35. **B.** $1118\,\text{m}$
Explanation: Here, $u = 180\,\text{kmh}^{-1} = 50\,\text{ms}^{-1}$
Time of fall: $2450 = \frac{1}{2}(9.8)t^2 \Rightarrow t = \sqrt{500} \approx 22.36\,\text{s}$
Therefore, $x = 50 \times 22.36 \approx 1118\,\text{m}$

36. **A.** $\frac{1}{2}\sin^{-1}\left(\frac{8}{9}\right)$
Explanation: $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = (3u\cos\theta - u,\ 3u\sin\theta)$
For a north-east direction the two components are equal: $3\cos\theta - 1 = 3\sin\theta \Rightarrow \cos\theta - \sin\theta = \frac{1}{3}$
$1 - \sin 2\theta = \frac{1}{9} \Rightarrow \sin 2\theta = \frac{8}{9}$

Since $\cos\theta > \sin\theta$, $\theta < 45°$: $\theta = \frac{1}{2}\sin^{-1}\left(\frac{8}{9}\right) \approx 31.37°$

37. **C.** $38\,\text{kmh}^{-1}$

Explanation: The actual velocity of the rain is $(30, -8)$; the car's velocity is $(v, 0)$
The relative velocity is $(30 - v, -8)$; for 45° from the front, $v - 30 = 8$
Therefore, $v = 38\,\text{kmh}^{-1}$

38. **D.** 2

Explanation: Let the initial speed be u, the plank thickness d and the retardation a; through one plank $\left(\frac{4}{5}u\right)^2 = u^2 - 2ad \Rightarrow 2ad = \frac{9}{25}u^2$
Total stopping distance $s = \frac{u^2}{2a} = \frac{25}{9}d \approx 2.78d$
Therefore, the bullet pierces 2 planks completely and stops inside the third

39. **C.** 25 m

Explanation: Let the distance be D; the condition for catching is $10t = D + t^2 \Rightarrow t^2 - 10t + D = 0$
For a real root the discriminant must be ≥ 0: $100 - 4D \geq 0$
Therefore, $D \leq 25\,\text{m}$

40. **B.** $6\,\text{ms}^{-1}$

Explanation: Let the bus's acceleration be a and the man's speed v: $vt = 72 + \frac{1}{2}at^2$
Catching at the minimum speed means the bus's speed equals v at the moment of catching: $v = at = 24a$, $24v = 72 + 288a$
$576a = 72 + 288a \Rightarrow a = 0.25\,\text{ms}^{-2}$
Therefore, $v = 6\,\text{ms}^{-1}$

41. **D.** 18 m

Explanation: $120 = 4u + 8a$, $192 = 8u + 32a$ $\Rightarrow u + 2a = 30$, $u + 4a = 24 \Rightarrow a = -3$, $u = 36$
At $t = 8$ s the speed is $36 - 24 = 12\,\text{ms}^{-1}$
In the next 2 s (the body stops at $t = 12$ s, so it is still moving) $12 \times 2 - \frac{1}{2}(3)(4) = 18$ m
Therefore, the distance is 18 m

42. **D.** 7 s

Explanation: Let the retardation be a and the initial speed u; the total distance is $S = \frac{u^2}{2a}$ and the distance in the last second is $\frac{a}{2}$
$\frac{a}{2} = \frac{1}{49} \cdot \frac{u^2}{2a} \Rightarrow u = 7a$
In the 3rd second: $u - \frac{a}{2}(2 \cdot 3 - 1) = 18 \Rightarrow 7a - 2.5a = 18 \Rightarrow a = 4$, $u = 28$
Therefore, the total time $= \frac{u}{a} = 7$ s

43. **A.** $2\,\text{ms}^{-2}$

Explanation: $s(4.5) - s(4) = 0.5u + \frac{a}{2}(20.25 - 16) = 0.5u + 2.125a = 6.25$
In the 8th second: $u + \frac{a}{2}(15) = u + 7.5a = 19$
Multiplying the first by 2: $u + 4.25a = 12.5$
Subtracting, $3.25a = 6.5$
Therefore, $a = 2\,\text{ms}^{-2}$

44. **B.** 3

Explanation: Let the maximum speed be V; the distance $= \frac{1}{2}V \times 3 = 1.5 \Rightarrow V = 1\,\text{km min}^{-1}$
$t_1 = \frac{V}{x}$, $t_2 = \frac{V}{y}$, $t_1 + t_2 = 3$
$V\left(\frac{1}{x} + \frac{1}{y}\right) = 3$
Therefore, $\frac{1}{x} + \frac{1}{y} = 3$

45. **A.** 24.81 m

Explanation: Let the speed at the upper point be v: $44.1 = 1.5v + \frac{1}{2}(9.8)(1.5)^2 = 1.5v + 11.025 \Rightarrow v = 22.05\,\text{ms}^{-1}$
Height $h = \frac{v^2}{2g} = \frac{486.2}{19.6}$
Therefore, $h \approx 24.81$ m

46. **B.** $55.53\,\text{ms}^{-1}$

Explanation: First particle: $t_{max} = \frac{49}{9.8} = 5$ s, $H = \frac{49^2}{19.6} = 122.5$ m
The second particle reaches height H in $5 - 2 = 3$ s: $122.5 = 3v - 4.9 \times 9$
Therefore, $v = \frac{122.5 + 44.1}{3} \approx 55.53\,\text{ms}^{-1}$

47. **A.** 1.0 s

Explanation: The ball's acceleration relative to the lift $= g + f = 9.8 + 1.2 = 11\,\text{ms}^{-2}$, downward
Time to return $t = \frac{2v}{g+f} = \frac{2 \times 5.5}{11}$
Therefore, $t = 1$ s

48. **B.** 34.79 N

Explanation: Work–energy principle: the work done by gravity over the total fall of $(4.9 + 0.07)$ m equals the work done against the resistance
$F \times 0.07 = mg(4.97) = 0.05 \times 9.8 \times 4.97$
Therefore, $F = \frac{2.4353}{0.07} \approx 34.79$ N

49. **B.** 20.92 m

Explanation: By symmetry, if the first post is at distance x_1 from the point of projection,
$R = 2x_1 + 6$
From the path $y = x\tan 45° \left(1 - \frac{x}{R}\right)$, $x_1 \cdot \frac{R - x_1}{R} = 4.8 \Rightarrow x_1(x_1 + 6) = 4.8(2x_1 + 6)$
$x_1^2 - 3.6x_1 - 28.8 = 0 \Rightarrow x_1 = \frac{3.6 + \sqrt{12.96 + 115.2}}{2} \approx 7.46\,\text{m}$
Therefore, $R = 2(7.46) + 6 \approx 20.92\,\text{m}$

50. **C.** $\tan^{-1}\left(\frac{1}{2}\right)$

Explanation: $\frac{u^2 \sin 2\alpha}{g} = \frac{4}{5} \cdot \frac{u^2}{g} \Rightarrow \sin 2\alpha = \frac{4}{5}$
$\frac{2\tan\alpha}{1 + \tan^2\alpha} = \frac{4}{5} \Rightarrow 2\tan^2\alpha - 5\tan\alpha + 2 = 0$
$\tan\alpha = \frac{1}{2},\ 2$
Therefore, the smaller angle $\alpha = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.57°$