MCQ 50 marks

1. **B.** 17 N

Explanation: Let the forces be P and Q ($P > Q$)

Here, $P + Q = 23$, $P - Q = 7$

So, $P = 15$ N, $Q = 8$ N

When they act at right angles, $R = \sqrt{P^2 + Q^2} = \sqrt{225 + 64} = \sqrt{289}$

Therefore, R = 17 N

2. **D.** $\cos^{-1}\left(-\frac{3}{8}\right)$

Explanation: Let the angle between the forces be α

Here, $R^2 = 4P^2 + P^2 + 4P^2\cos\alpha = P^2(5 + 4\cos\alpha)$

Given, $4R^2 = 16P^2 + P^2 + 8P^2\cos\alpha = P^2(17 + 8\cos\alpha)$

So, $4(5 + 4\cos\alpha) = 17 + 8\cos\alpha$

$\Rightarrow 8\cos\alpha = -3$

Therefore, $\alpha = \cos^{-1}\left(-\frac{3}{8}\right)$

3. **B.** $\cos^{-1}\left(-\frac{5}{13}\right)$

Explanation: The resultant is perpendicular to P, so $P^2 + R^2 = Q^2$

Here, $P = \sqrt{26^2 - 24^2} = 10$ N

Also, $P + Q\cos\alpha = 0 \Rightarrow \cos\alpha = -\frac{P}{Q} = -\frac{10}{26} = -\frac{5}{13}$

Therefore, $\alpha = \cos^{-1}\left(-\frac{5}{13}\right)$

4. **A.** 3 **units**

Explanation: If the direction of the resultant is unchanged, the ratio of the two forces is unchanged.

Here, $\frac{9P}{2P+12} = \frac{3P}{2P} = \frac{3}{2}$

$\Rightarrow 18P = 6P + 36 \Rightarrow 12P = 36$

Therefore, P = 3 units

5. **C.** 71.74 N

Explanation: Let the tensions in the strings inclined at 30° and 45° to the vertical be T_1 and T_2

Here, $W = 10 \times 9.8 = 98$ N; the angle between the strings = 75°

Lami's theorem: $\frac{T_1}{\sin 135°} = \frac{T_2}{\sin 150°} = \frac{W}{\sin 75°}$

$T_1 = \frac{98 \times 0.7071}{0.9659} = 71.74$ N, $T_2 = \frac{98 \times 0.5}{0.9659} = 50.73$ N

Therefore, the greater tension = 71.74 N

6. **B.** sin A : sin B : sin C

Explanation: Here, $\angle BHC = 180° - A$, $\angle CHA = 180° - B$, $\angle AHB = 180° - C$

Lami's theorem: $\frac{P}{\sin\angle BHC} = \frac{Q}{\sin\angle CHA} = \frac{R}{\sin\angle AHB}$

$\Rightarrow \frac{P}{\sin A} = \frac{Q}{\sin B} = \frac{R}{\sin C}$

Therefore, P : Q : R = sin A : sin B : sin C

7. **A.** 15 N

Explanation: Let BC = a, CA = b, AB = c

The resultant passes through the incentre, so

$F_A : F_B : F_C = a : b : c = 6 : 8 : 10$

Here, $F_A = 9 \Rightarrow$ multiplying factor $= \frac{9}{6} = 1.5$

Check: $F_B = 8 \times 1.5 = 12$ N

Therefore, $F = 10 \times 1.5 = 15$ N

8. **C.** 12 cm

Explanation: Initially the resultant acts at a distance from the 11 N force of $x_1 = \frac{5 \times 32}{16} = 10$ cm

After interchange, from the same end, $x_2 = \frac{11 \times 32}{16} = 22$ cm

Therefore, the shift = 22 − 10 = 12 cm

9. **C.** 0.75 m

Explanation: Let the reactions at A and at the second support be R_1 and R_2

Here, $R_1 + R_2 = 90 + 30 = 120$, $R_2 = 2R_1$

So, $R_1 = 40$ N, $R_2 = 80$ N

Moments about A: $R_2(12 - x) = 90 \times 6 + 30 \times 12 = 900$

$\Rightarrow 12 - x = 11.25$

Therefore, x = 0.75 m

10. **A.** 12 kg

Explanation: Let the table edge be O and the maximum mass m; at the point of tipping the moments about O are equal

Distance of the centre of gravity from O = 1.5 − 0.6 = 0.9 m

Here, $m \times 0.6 = 8 \times 0.9$

Therefore, m = 12 kg

11. **D.** 200 N

Explanation: Let the force at the hand be F; moments about the shoulder: $F \times 0.3 = 50 \times 0.9 \Rightarrow F = 150\,\text{N}$
Pressure on the shoulder = 50 + 150
Therefore, the pressure = 200 N

12. **D.** 7.5 N
Explanation: Let the weight be W and the inclination α; parallel to the plane: $W \sin\alpha = 6$
With the horizontal force: $10\cos\alpha = W\sin\alpha = 6 \Rightarrow \cos\alpha = 0.6 \Rightarrow \sin\alpha = 0.8$
So, $W = \frac{6}{0.8}$
Therefore, $W = 7.5\,\text{N}$

13. **B.** 4.54 m
Explanation: Let the man be at distance x from the foot A; floor reaction N, wall reaction S, friction F
Here, $N = (20 + 60)g = 80g$
At limiting equilibrium $F = \mu N = 0.4 \times 80g = 32g = S$
Moments about A: $S \cdot 6\sin 60° = 20g \cdot 3\cos 60° + 60g \cdot x\cos 60°$
$\Rightarrow 32 \times 5.196 = 30 + 30x \Rightarrow 166.28 = 30 + 30x$
Therefore, $x = 4.54$ m

14. **C.** 45.83 N
Explanation: Let the wall point be A, the centre O, and the string inclined at θ to the vertical
Here, $AO = 1.5r + r = 2.5r$ and the perpendicular distance of O from the wall = r
$\sin\theta = \frac{r}{2.5r} = 0.4 \Rightarrow \cos\theta = \sqrt{0.84} = 0.9165$
Vertical equilibrium: $T\cos\theta = W \Rightarrow T = \frac{42}{0.9165}$
Therefore, $T = 45.83\,\text{N}$

15. **B.** $\sqrt{19}$ N
Explanation: Here, the forces are inclined at 120° to each other
$R^2 = a^2 + b^2 + c^2 - ab - bc - ca$
$= 16 + 49 + 81 - 28 - 63 - 36 = 19$
Therefore, $R = \sqrt{19}\,\text{N}$

16. **C.** $\sqrt{82}$ N
Explanation: Take A as the origin and AB as the x-axis
$\sum F_x = 6 - 2 + 5\sqrt{2}\cos 45° = 4 + 5 = 9$
$\sum F_y = 3 - 7 + 5\sqrt{2}\sin 45° = -4 + 5 = 1$
Therefore, $R = \sqrt{81 + 1} = \sqrt{82}\,\text{N}$

17. **A.** $2\sqrt{34}$ **units**
Explanation: Let $\vec{F} = (F_x, F_y)$; the moment about P is $(\vec{O} - \vec{P}) \times \vec{F}$
About A: $(-4)F_y - 0 \cdot F_x = -24 \Rightarrow F_y = 6$
About B: $0 \cdot F_y - (-3)F_x = 30 \Rightarrow F_x = 10$
Therefore, $|\vec{F}| = \sqrt{100 + 36} = \sqrt{136} = 2\sqrt{34}$

18. **C.** $5\sqrt{2}$ N
Explanation: Let each force be P; the resultant of two equal perpendicular forces is $P\sqrt{2}$, equal and opposite to 10 N
$P\sqrt{2} = 10$
Therefore, $P = 5\sqrt{2}\,\text{N}$

19. **D.** 36 cm
Explanation: The resultant is $9 - 5 = 4$ N, in the direction of the larger force and lying outside it; if its distance from the larger force is x, $9x = 5(x + 16) \Rightarrow x = 20$ cm
The balancing force must act at the point of application of the resultant, so the rod must extend at least to that point
Therefore, the minimum length = 16 + 20 = 36 cm

20. **A.** $\cos^{-1}\left(-\frac{11}{16}\right)$
Explanation: Let the forces be P and Q ($P > Q$)
Given, $P + Q = 3(P - Q) \Rightarrow P = 2Q$
Resultant $R = \frac{P+Q}{2} = \frac{3Q}{2}$
$R^2 = P^2 + Q^2 + 2PQ\cos\alpha \Rightarrow \frac{9Q^2}{4} = 4Q^2 + Q^2 + 4Q^2\cos\alpha$
$\Rightarrow 4\cos\alpha = \frac{9}{4} - 5 = -\frac{11}{4}$
Therefore, $\cos\alpha = -\frac{11}{16} \Rightarrow \alpha = \cos^{-1}\left(-\frac{11}{16}\right)$

21. **A.** 4758.4 N
Explanation: y-components: $T_1\sin 25° = T_2\sin 50° \Rightarrow T_2 = \frac{T_1\sin 25°}{\sin 50°}$
x-components: $T_1\cos 25° + T_2\cos 50° = 6000$
Substituting, $T_1 \cdot \frac{\sin(25°+50°)}{\sin 50°} = 6000$
$T_1 = \frac{6000\sin 50°}{\sin 75°} = \frac{6000 \times 0.7660}{0.9659}$
Therefore, $T_1 = 4758.4$ N

22. **A.** 5 N
Explanation: By Lami's theorem each force is proportional to the sine of the angle between

the other two
$F_1 : F_2 : F_3 = \sin 150° : \sin 120° : \sin 90° = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$
Let the smallest be k; the largest $2k = 10 \Rightarrow k = 5$
Therefore, the smallest force = 5 N

23. **C.** $2\sqrt{3}$ N**; on** BC **produced,** 30 cm **beyond** C
Explanation: Take B = (0, 0), C = (30, 0), A = $(15, 15\sqrt{3})$
$\vec{F}_{BC} = (6, 0)$, $\vec{F}_{CA} = (-2, 2\sqrt{3})$, $\vec{F}_{AB} = (-1, -\sqrt{3})$
$\vec{R} = (3, \sqrt{3}) \Rightarrow R = \sqrt{12} = 2\sqrt{3}$ N
Moment about B: only the force along CA contributes, $30 \times 2\sqrt{3} = 60\sqrt{3}$
If the line of action meets BC at distance x from B, $x \cdot \sqrt{3} = 60\sqrt{3} \Rightarrow x = 60$ cm
Therefore, 30 cm beyond C on BC produced

24. **B.** 28.8 kg-wt **and** 43.2 kg-wt
Explanation: Positions from one end: A at 1.5 m, centre of gravity at 6 m, B at 12 − 3 = 9 m
Moments about B: $P_A(9 - 1.5) = 72(9 - 6) \Rightarrow P_A = \frac{216}{7.5} = 28.8$ kg-wt
$P_B = 72 - 28.8 = 43.2$ kg-wt
Therefore, the pressures are 28.8 kg-wt and 43.2 kg-wt respectively

25. **C.** $\frac{W}{2\sqrt{3}}$
Explanation: Let the rod have length 2a, the wall reaction S and the applied force P ; the floor reaction is W
Horizontal equilibrium: P = S
Moments about the lower end: $S \cdot 2a \sin 60° = W \cdot a \cos 60°$
$\Rightarrow S = \frac{W \cot 60°}{2}$
Therefore, $P = \frac{W}{2\sqrt{3}}$

26. **B.** 7 ms^{-1}
Explanation: Let the velocity imparted by the bat be v; the resultant w = 25 and the original velocity u = 24 are perpendicular
$w^2 = u^2 + v^2 \Rightarrow v = \sqrt{25^2 - 24^2} = \sqrt{49}$
Therefore, v = 7 ms^{-1}

27. **B.** $\cos^{-1}\left(-\frac{3}{5}\right)$
Explanation: Let the angle of throw be α, the train's speed u = 15 and the relative speed v = 25
The resultant is perpendicular to the train's velocity, so $u + v\cos\alpha = 0$
$\cos\alpha = -\frac{u}{v} = -\frac{15}{25} = -\frac{3}{5}$
Therefore, $\alpha = \cos^{-1}\left(-\frac{3}{5}\right)$

28. **D.** $\cos^{-1}\left(\frac{1}{3}\right)$
Explanation: The particle is at rest, so the resultant of 5 and 6 equals 9 in magnitude and is opposite
$9^2 = 5^2 + 6^2 + 2 \cdot 5 \cdot 6\cos\alpha \Rightarrow 81 = 61 + 60\cos\alpha$
$\cos\alpha = \frac{20}{60} = \frac{1}{3}$
Therefore, $\alpha = \cos^{-1}\left(\frac{1}{3}\right)$

29. **D.** $20\sqrt{19}$ m
Explanation: Distances covered in 10 s: 40 m, 60 m
If the distance between them is d, $d^2 = 40^2 + 60^2 - 2 \cdot 40 \cdot 60\cos 120°$
$= 1600 + 3600 + 2400 = 7600$
Therefore, $d = \sqrt{7600} = 20\sqrt{19}$ m

30. **B.** 24 km
Explanation: After t hours, A = (12t, 0), B = (0, 26 − 5t)
$d^2 = 144t^2 + (26 - 5t)^2 = 169t^2 - 260t + 676$
The minimum occurs when $t = \frac{260}{2 \times 169} = \frac{10}{13}$ h
$d^2_{min} = 676 - \frac{260^2}{4 \times 169} = 676 - 100 = 576$
Therefore, $d_{min} = 24$ km

31. **B.** 48 kmh^{-1}
Explanation: Let AB = d; total time = $\frac{d}{40} + \frac{d}{60} = \frac{d}{24}$
Average speed = $2d \times \frac{24}{d} = 48$ kmh^{-1}
Therefore, the average speed = 48 kmh^{-1}

32. **B.** 30 min
Explanation: Crossing by the shortest path (straight across), the resultant speed = $\sqrt{13^2 - 5^2} = \sqrt{144} = 12$ kmh^{-1}
Time = $\frac{6}{12} = 0.5$ h
Therefore, the time = 30 minutes

33. **B.** 3 min
Explanation: Shortest path: $t_1 = \frac{2}{\sqrt{100-36}} = \frac{2}{8} = 0.25$ h

Shortest time (heading perpendicular to the current): $t_2 = \frac{2}{10} = 0.2\,\text{h}$
Difference = 0.05 h
Therefore, the difference = 3 minutes

34. **A.** $4\,\text{ms}^{-1}$
Explanation: Let the current speed be u = $5\,\text{ms}^{-1}$ and the resultant velocity make an angle ϕ with the current
The displacement to the target is (60, 80), so $\sin\phi = \frac{80}{100} = 0.8$
The boat's speed is least when its velocity is perpendicular to the resultant: $v_{min} = u\sin\phi = 5 \times 0.8$
Therefore, $v_{min} = 4\,\text{ms}^{-1}$

35. **D.** 80 s
Explanation: Relative speed = 54 − 36 = $18\,\text{kmh}^{-1} = 5\,\text{ms}^{-1}$
Relative distance to be covered = 150 + 250 = 400 m
Time = $\frac{400}{5}$
Therefore, the time = 80 s

36. **A.** 10.5 s
Explanation: Relative to B, the initial speed of A is $10 + 8 = 18\,\text{ms}^{-1}$, the relative acceleration is $0.2\,\text{ms}^{-2}$ and the relative distance is 120 + 80 = 200 m
$200 = 18t + \frac{1}{2}(0.2)t^2 \Rightarrow t^2 + 180t - 2000 = 0$
$t = \frac{-180+\sqrt{32400+8000}}{2} = \frac{-180+201.0}{2}$
Therefore, $t \approx 10.50$ s

37. **D.** $\tan^{-1}\left(\frac{5}{12}\right)$
Explanation: Relative to the cyclist the rain has 12 downward and 5 backward
If the angle with the vertical is θ, $\tan\theta = \frac{5}{12}$
Therefore, $\theta = \tan^{-1}\left(\frac{5}{12}\right) \approx 22.62°$, tilted forward

38. **A.** $2\sqrt{13}\,\text{kmh}^{-1}$
Explanation: Let the horizontal and downward components of the actual velocity of the rain be v_x and v_y
At $4\,\text{kmh}^{-1}$ the apparent velocity is vertical, so $v_x = 4$
At $10\,\text{kmh}^{-1}$ the relative horizontal component is $4 - 10 = -6$ (from the front): $\tan 45° = \frac{6}{v_y} \Rightarrow v_y = 6$
Therefore, $v = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}\,\text{kmh}^{-1}$

39. **A.** 4 cm
Explanation: Let the initial speed be u and the retardation a; after 5 cm the speed is $\frac{2u}{3}$:
$\left(\frac{2u}{3}\right)^2 = u^2 - 2a \cdot 5 \Rightarrow 2a = \frac{u^2}{9}$
To stop after a further s, $0 = \frac{4u^2}{9} - 2as \Rightarrow s = \frac{\frac{4u^2}{9}}{\frac{u^2}{9}}$
Therefore, s = 4 cm

40. **C.** 6.77 s
Explanation: Condition for catching: displacement of dog = 24+ displacement of cat
$\frac{1}{2}(4)t^2 = 24 + 10t \Rightarrow t^2 - 5t - 12 = 0$
$t = \frac{5+\sqrt{25+48}}{2} = \frac{5+8.544}{2}$
Therefore, $t \approx 6.77$ s

41. **D.** 8.16 s
Explanation: They meet when $20t = 30 + \frac{1}{2}(4)t^2 \Rightarrow t^2 - 10t + 15 = 0 \Rightarrow t = 5 \pm \sqrt{10}$
At $t_1 = 1.84$ s the cycle overtakes the bus; later, once the bus speed exceeds $20\,\text{ms}^{-1}$, the bus catches up again
Therefore, $t_2 = 5 + \sqrt{10} \approx 8.16$ s

42. **D.** $1.04\,\text{ms}^{-2}$
Explanation: Let the retardation be a; stopping distances $s_1 = \frac{30^2}{2a}$, $s_2 = \frac{20^2}{2a}$
To avoid collision $s_1 + s_2 \le 625 \Rightarrow \frac{1300}{2a} \le 625 \Rightarrow a \ge \frac{1300}{1250}$
Therefore, $a_{min} = 1.04\,\text{ms}^{-2}$

43. **C.** $48.99\,\text{ms}^{-1}$
Explanation: Let the maximum speed be V ; distance while accelerating $\frac{V^2}{2\times1.5} = \frac{V^2}{3}$, while retarding $\frac{V^2}{2\times3} = \frac{V^2}{6}$
$\frac{V^2}{3} + \frac{V^2}{6} = 1200 \Rightarrow \frac{V^2}{2} = 1200 \Rightarrow V^2 = 2400$
Therefore, $V = \sqrt{2400} \approx 48.99\,\text{ms}^{-1}$

44. **C.** 196 m
Explanation: The stone's initial velocity equals the balloon's: $14.7\,\text{ms}^{-1}$ upward
Taking upward as positive, $-h = 14.7 \times 8 - \frac{1}{2}(9.8)(8)^2 = 117.6 - 313.6 = -196$
Therefore, h = 196 m

45. **C.** 176.4 m

Explanation: Let the total time be t and the height $h = \frac{1}{2}gt^2$
Distance in the last second $= \frac{1}{2}g(2t - 1)$
$\frac{2t-1}{t^2} = \frac{11}{36} \Rightarrow 11t^2 - 72t + 36 = 0$
$t = \frac{72 \pm 60}{22} \Rightarrow t = 6$ or $t = \frac{6}{11}$
$t < 1$ is not acceptable, so $t = 6$ s. Therefore, $h = \frac{1}{2}(9.8)(36) = 176.4$ m

46. **B.** 95.48 m
Explanation: Taking upward as positive: $-44.1 = (24.5 \sin 30°)t - \frac{1}{2}(9.8)t^2 \Rightarrow 4.9t^2 - 12.25t - 44.1 = 0$
$t = \frac{12.25 + \sqrt{150.06 + 864.36}}{9.8} = \frac{12.25 + 31.85}{9.8} = 4.5$ s
Horizontal distance $= 24.5 \cos 30° \times 4.5 = 21.22 \times 4.5$
Therefore, the distance ≈ 95.48 m

47. **D.** 60.42 ms^{-1}
Explanation: Gravity acts equally on both, so the bullet effectively travels in a straight line towards the object's initial position
Initial distance $= \sqrt{245^2 + 350^2} = 427.2$ m
Fall time of the object $= \sqrt{\frac{2 \times 245}{9.8}} = \sqrt{50} = 7.071$ s
Condition for a hit: $\frac{427.2}{u} \leq 7.071$
Therefore, $u_{min} = 60.42$ ms^{-1}

48. **A.** 39.9 m
Explanation: Here, $\tan \alpha = \frac{4}{3} \Rightarrow \cos \alpha = \frac{3}{5}$
$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$
$= 63 \times \frac{4}{3} - \frac{9.8 \times 3969}{2 \times 1225 \times 0.36} = 84 - 44.1$
Therefore, $y = 39.9$ m

49. **A.** $40\sqrt{3}$ ms^{-1}**,** 1 s
Explanation: Let the second body's initial speed be v; to meet, their horizontal velocities must be equal: $40 \cos 30° = v \cos 60° \Rightarrow v = 40\sqrt{3}$ ms^{-1}
Both accelerate at g vertically, so the relative vertical velocity is constant: $40 \sin 30° + v \sin 60° = 20 + 60 = 80$ ms^{-1}
The relative vertical distance is 80 m, so $t = \frac{80}{80} = 1$ s
Therefore, $v = 40\sqrt{3}$ ms^{-1} and $t = 1$ s

50. **D.** 51.24 ms^{-1}
Explanation: Time of fall: $122.5 = \frac{1}{2}(9.8)t^2 \Rightarrow t = 5$ s
Horizontal speed $u = \frac{75}{5} = 15$ ms^{-1}
Vertical speed $v_y = gt = 49$ ms^{-1}
Therefore, $v = \sqrt{15^2 + 49^2} = \sqrt{2626} \approx 51.24$ ms^{-1}