Engineering Daily 27 [Home Practice] EV - Answer Sheet

Statics + Motion of a Particle in a Plane

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. **A.** $60°$

Explanation: Let each force be P and the angle between them α

Given, $R = 2P\cos\frac{\alpha}{2} = \sqrt{3}P \Rightarrow \cos\frac{\alpha}{2} = \frac{\sqrt{3}}{2}$

$\frac{\alpha}{2} = 30°$

Therefore, $\alpha = 60°$

2. **C.** $15\,N$ **and** $9\,N$

Explanation: Moments about C: $P_A \cdot AC = P_B \cdot CB \Rightarrow 3P_A = 5P_B$

Also, $P_A + P_B = 24$

Therefore, $P_A = 15\,N$ and $P_B = 9\,N$

3. **D.** $90°$

Explanation: Given, $P^2 + Q^2 + 2PQ\cos\alpha = P^2 + Q^2 - 2PQ\cos\alpha$

$\Rightarrow 4PQ\cos\alpha = 0$

Therefore, $\alpha = 90°$

4. **D.** $60°$

Explanation: The resultant of 6 and 10 equals 14: $196 = 36 + 100 + 120\cos\alpha$

$\cos\alpha = \frac{60}{120} = \frac{1}{2}$

Therefore, $\alpha = 60°$

5. **A.** $\cos^{-1}\left(\frac{1}{4}\right)$

Explanation: Let the forces be P and $2P$ and the angle between them α

Given, $R^2 = 3 \cdot P \cdot 2P = 6P^2$

$6P^2 = P^2 + 4P^2 + 4P^2\cos\alpha \Rightarrow \cos\alpha = \frac{1}{4}$

Therefore, $\alpha = \cos^{-1}\left(\frac{1}{4}\right) \approx 75.52°$

6. **C.** $2\sqrt{19}\,N$

Explanation: Taking the directions as $0°$, $60°$, $120°$: $X = 2 + 4\cos 60° + 6\cos 120° = 2 + 2 - 3 = 1$

$Y = 0 + 4\sin 60° + 6\sin 120° = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

Therefore, $R = \sqrt{1 + 75} = \sqrt{76} = 2\sqrt{19}\,N$

7. **A.** $4\,N$

Explanation: Let the other force be Q; given, $37 = 9 + Q^2 + 2 \cdot 3 \cdot Q\cos 60°$

$\Rightarrow Q^2 + 3Q - 28 = 0 \Rightarrow (Q + 7)(Q - 4) = 0$

Therefore, $Q = 4\,N$

8. **A.** $113.16\,N$

Explanation: Three forces act: the weight mg, the horizontal force F and the reaction R (normal to the plane, at $30°$ to the vertical)

Vertical equilibrium: $R\cos 30° = mg \Rightarrow R = \frac{98}{0.866}$

Therefore, $R \approx 113.16\,N$

9. **B.** $30\,kg$

Explanation: Let the father's mass be F and the son's mass S; moments about the fulcrum: $F \times 2 = S \times 5$

Also, $F = S + 45 \Rightarrow 2S + 90 = 5S$

Therefore, $S = 30\,kg$

10. **C.** $\sqrt{3}Q$

Explanation: From the triangle of forces (P, Q, R) with $R = 2Q$: $Q^2 = P^2 + R^2 - 2PR\cos 30°$

$Q^2 = P^2 + 4Q^2 - 2\sqrt{3}PQ \Rightarrow P^2 - 2\sqrt{3}PQ + 3Q^2 = 0$

$\left(P - \sqrt{3}Q\right)^2 = 0$

Therefore, $P = \sqrt{3}Q$

11. **A.** $7\,N$

Explanation: Taking AB as $0°$, the direction of AC is $60°$ and of BC is $120°$

$X = 5 + 3\cos 60° + 2\cos 120° = 5 + 1.5 - 1 = 5.5$

$Y = 3\sin 60° + 2\sin 120° = \frac{5\sqrt{3}}{2}$

Therefore, $R = \sqrt{30.25 + 18.75} = \sqrt{49} = 7\,N$

12. **C.** $40\,kg$

Explanation: At the point of tipping the reaction at A is zero; take moments about C. The plank's centre of gravity is 4 m from A, that is from C $5 - 4 = 1$ m

Let the boy's mass be m: $m \times 3 = 120 \times 1$

Therefore, $m = 40\,kg$

13. **D.** $135°$

Explanation: Let the angle between them be α: $R^2 = 2P^2(1 + \cos\alpha) = P^2\left(2 - \sqrt{2}\right)$

$\Rightarrow 1 + \cos\alpha = 1 - \frac{1}{\sqrt{2}} \Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}}$

Therefore, $\alpha = 135°$

14. **C.** 720 N

Explanation: Let RQ = k and PR = 2k, so PQ = 3k; moments about P : $R_Q \cdot 3k = W \cdot 2k \Rightarrow R_Q = \frac{2}{3}W$

$R_P = W - R_Q = \frac{1}{3}W$

Given, $R_Q - R_P = \frac{1}{3}W = 240$

Therefore, W = 720 N

15. **C.** 40 cm

Explanation: In the like case the resultant is 14 N, inside, at a distance from the 10 N force of $\frac{4\times42}{14} = 12$ cm

In the unlike case the resultant is 6 N, outside, at a distance from the 10 N force of $\frac{4\times42}{6} = 28$ cm

The two positions lie on opposite sides of the 10 N force, so the separation is 12 + 28 = 40 cm

Therefore, the distance is 40 cm

16. **A.** 35 N **and** 25 N

Explanation: The resultant of unlike forces lies outside, on the side of the larger force; it is 5 cm from the larger force P and 7 cm from the smaller force Q

Moments about a point on the resultant: 5P = 7Q, P − Q = 10

$\frac{7}{5}Q - Q = 10 \Rightarrow Q = 25$

Therefore, P = 35 N and Q = 25 N

17. **B.** 150°

Explanation: Taking magnitudes in $\vec{A} + \vec{C} = -\vec{B}$: $A^2 + 3A^2 + 2\sqrt{3}A^2\cos\theta_{AC} = A^2$

$\cos\theta_{AC} = -\frac{3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$

Therefore, $\theta_{AC} = 150°$

18. **B.** 32.64 N **and** 43.97 N

Explanation: Let the smaller force be P and the larger Q; the resultant makes with the larger force 100° − 60° = 40°

Sine rule: $\frac{P}{\sin 40°} = \frac{Q}{\sin 60°} = \frac{R}{\sin(180°-100°)}$

$P = \frac{50\sin 40°}{\sin 80°} \approx 32.64$ N, $Q = \frac{50\sin 60°}{\sin 80°} \approx 43.97$ N

Therefore, the forces are 32.64 N and 43.97 N

19. **C.** $2\sqrt{2} : \sqrt{7} : \sqrt{3}$

Explanation: Let the angle between the forces be α: (i) $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

(ii) $4R^2 = P^2 + 4Q^2 + 4PQ\cos\alpha$

(iii) $9R^2 = P^2 + Q^2 - 2PQ\cos\alpha$

Adding (i) and (iii): $10R^2 = 2\left(P^2 + Q^2\right) \Rightarrow R^2 = \frac{P^2+Q^2}{5}$

Subtracting 2×(i) from (ii): $2R^2 = 2Q^2 - P^2$

Substituting, $2P^2 + 2Q^2 = 10Q^2 - 5P^2 \Rightarrow 7P^2 = 8Q^2$

$P^2 : Q^2 : R^2 = 8 : 7 : 3$

Therefore, $P : Q : R = 2\sqrt{2} : \sqrt{7} : \sqrt{3}$

20. **C.** 3.04 m

Explanation: Relation: $\frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON}$

Here, $R = \sqrt{36 + 64 + 2\cdot6\cdot8\cos60°} = \sqrt{148} \approx 12.17$ N

$\frac{6}{3} + \frac{8}{4} = 4 = \frac{12.17}{ON}$

Therefore, ON ≈ 3.04 m

21. **D.** $\left(2\sqrt{3} - 2\right)$ N**, along the negative y-axis**

Explanation: $X = 2\sqrt{2}\cos45° - 4 + 4\cos60° = 2 - 4 + 2 = 0$

$Y = 2\sqrt{2}\sin45° + 0 - 4\sin60° = 2 - 2\sqrt{3}$

Therefore, the resultant = $2\sqrt{3} - 2 \approx 1.46$ N, along the negative y-axis

22. **B.** 808.5 N **and** 955.5 N

Explanation: Moments about the left leg:

$F_2 \times 12 = (150 \times 6 + 30 \times 9)g = 1170g$

$F_2 = 97.5 \times 9.8 = 955.5$ N

$F_1 = 180 \times 9.8 - 955.5 = 808.5$ N

Therefore, the reactions are 808.5 N and 955.5 N respectively

23. **D.** $\frac{8}{3}$ N

Explanation: The resultant is 20 − 12 = 8 N, outside the larger force, at $x = \frac{12\times30}{8} = 45$ cm

After adding F the forces are 20 + F and 12 + F ; the resultant is still 8 N: $x' = \frac{(12+F)\times30}{8}$

Shift $x' - x = \frac{30F}{8} = 10$

Therefore, $F = \frac{8}{3}$ N

24. **B.** 6 kg

Explanation: Let the mass of the rod be W and its half-length l: 10 × 0.5 = W (l − 0.5), 4 × 0.8 = W (l − 0.8)

Subtracting: 5 − 3.2 = W (0.3)

Therefore, W = 6 kg

25. **D.** 5 kg-wt

Explanation: Let the string make an angle θ with the vertical: $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

Vertical equilibrium: $T\cos\theta = 12 \Rightarrow T = 13$ kg-wt
Therefore, $F = T\sin\theta = 5$ kg-wt

26. **B.** 19.6 m
Explanation: Taking upward as positive: $-h = 14.7 \times 4 - \frac{1}{2}(9.8)(16) = 58.8 - 78.4$
Therefore, $h = 19.6$ m

27. **C.** 400π m^2
Explanation: Maximum range (at 45°): $R_{max} = \frac{u^2}{g} = \frac{196}{9.8} = 20$ m
Area $= \pi R_{max}^2 = 400\pi$ m^2
Therefore, the area is 400π m^2

28. **A.** $\cos^{-1}\left(\frac{14}{3\sqrt{145}}\right)$
Explanation: $\overrightarrow{AB} = 6\hat{i} + 8\hat{j} + 4\hat{k}$, $\overrightarrow{AC} = 4\hat{i} - 2\hat{j} + 5\hat{k}$
$\overrightarrow{AB}\cdot\overrightarrow{AC} = 24 - 16 + 20 = 28$, $\left|\overrightarrow{AB}\right| = \sqrt{116}$, $\left|\overrightarrow{AC}\right| = \sqrt{45}$
$\cos\theta = \frac{28}{\sqrt{5220}} = \frac{28}{6\sqrt{145}}$
Therefore, $\theta = \cos^{-1}\left(\frac{14}{3\sqrt{145}}\right) \approx 67.2°$

29. **C.** 3 : 2
Explanation: Let the width of the river be d: $t_1 = \frac{d}{\sqrt{13^2-5^2}} = \frac{d}{12}$, $t_2 = \frac{d}{13+5} = \frac{d}{18}$
$t_1 : t_2 = 18 : 12$
Therefore, $t_1 : t_2 = 3 : 2$

30. **D.** 42 kmh^{-1}
Explanation: Each half being d, the total time $t = \frac{d}{30} + \frac{d}{70} = \frac{10d}{210}$
Average speed $= \frac{2d}{t} = 2d \times \frac{210}{10d}$
Therefore, the average speed = 42 kmh^{-1}

31. **B.** 5.10 ms^{-1}
Explanation: East component: $6 - 5\cos 45° = 6 - 3.536 = 2.464$
North component: $8 - 5\sin 45° = 8 - 3.536 = 4.464$
Therefore, $R = \sqrt{2.464^2 + 4.464^2} = \sqrt{26.0} \approx 5.10$ ms^{-1}

32. **A.** $2\left(\sqrt{3} - \sqrt{2}\right)$ s
Explanation: Let AB = h; the times to cover h, 2h, 3h from A are proportional to $\sqrt{1} : \sqrt{2} : \sqrt{3}$
$t_1 : t_2 : t_3 = 1 : \left(\sqrt{2} - 1\right) : \left(\sqrt{3} - \sqrt{2}\right)$
Therefore, $t_3 = 2\left(\sqrt{3} - \sqrt{2}\right) \approx 0.636$ s

33. **A.** 14.64 ms^{-1}
Explanation: Sine rule: $\frac{u}{\sin 45°} = \frac{v}{\sin 30°} = \frac{w}{\sin 75°}$
Therefore, $u = \frac{20\times 0.7071}{0.9659} \approx 14.64$ ms^{-1}

34. **A.** 176.4 m
Explanation: Let the aeroplane's speed be u and its height at release h; for the bomb $-h = 6u - \frac{1}{2}g(36)$
$\Rightarrow h + 6u = \frac{1}{2}g(36) = 176.4$ m
The aeroplane's height at the moment of impact is $h + 6u = 176.4$ m, which does not depend on u

35. **D.** 32 m
Explanation: Let the speed of the first stone after falling $x = 2$ m be $v = \sqrt{2gx}$ and $y = 14$ m; the common time is t: $h - x = vt + \frac{1}{2}gt^2$, $h - y = \frac{1}{2}gt^2$
Subtracting, $y - x = vt \Rightarrow t = \frac{y-x}{\sqrt{2gx}}$
$h = y + \frac{1}{2}gt^2 = y + \frac{(y-x)^2}{4x} = \frac{(x+y)^2}{4x}$
Therefore, $h = \frac{(2+14)^2}{8} = 32$ m

36. **B.** 193.6 m
Explanation: At the end of the free fall: $v_1^2 = 2(9.8)(40) = 784 \Rightarrow v_1 = 28$ ms^{-1}
In the retardation stage: $4^2 = 784 - 2(2.5)h_2 \Rightarrow h_2 = \frac{768}{5} = 153.6$ m
Therefore, the total height = 40 + 153.6 = 193.6 m

37. **B.** 3 s
Explanation: $-h = ut_1 - \frac{1}{2}gt_1^2$, $-h = -ut_2 - \frac{1}{2}gt_2^2$
Multiplying the first by t_2, the second by t_1 and adding: $-h(t_1 + t_2) = -\frac{1}{2}gt_1t_2(t_1 + t_2) \Rightarrow h = \frac{1}{2}gt_1t_2$
For free fall $h = \frac{1}{2}gt^2$, so $t = \sqrt{t_1t_2} = \sqrt{9}$
Therefore, t = 3 s

38. **B.** 4 m
Explanation: Range R = 12 + 6 = 18 m
Path: $y = x\tan 45°\left(1 - \frac{x}{R}\right)$
Therefore, $h = 12\left(1 - \frac{12}{18}\right) = 4$ m

39. **B.** 28.17 ms^{-1}

Explanation: Path: $y = x\tan 45° - \frac{gx^2}{2u^2\cos^2 45°} = x - \frac{gx^2}{u^2}$
At $x = 45$, $y \geq 20$: $45 - \frac{9.8\times 2025}{u^2} \geq 20 \Rightarrow u^2 \geq \frac{19845}{25} = 793.8$
Therefore, $u_{min} \approx 28.17\,ms^{-1}$

40. **D.** 811.4 m
Explanation: $\frac{u^2}{g} = 640 \Rightarrow u = \sqrt{6272} \approx 79.20\,ms^{-1}$
From the car the vertical velocity $u\sin 45°$ is unchanged, so $T = \frac{2u\sin 45°}{g} \approx 11.43$ s
The horizontal speed is $u\cos 45° + 15$; the range $= 640 + 15 \times 11.43$
Therefore, the range ≈ 811.4 m

41. **A.** $4.67\,ms^{-1}$
Explanation: Time of fall: $3.6 = \frac{1}{2}(9.8)t^2 \Rightarrow t = \sqrt{0.7347} \approx 0.857$ s
$ut \geq 4$
Therefore, $u_{min} = \frac{4}{0.857} \approx 4.67\,ms^{-1}$

42. **A.** 50.21 m
Explanation: Displacement in the first 4 s = 20 m
In the next 3 s: $\Delta x = 5 \times 3 = 15$, $\Delta y = 12 \times 3 = 36$
The position is $(35, 36)$; the distance $= \sqrt{35^2 + 36^2} = \sqrt{2521}$
Therefore, the distance ≈ 50.21 m

43. **C.** $20\,kmh^{-1}$**, from** $\tan^{-1}\left(\frac{3}{4}\right)$ **west of north**
Explanation: $\vec{v}_{wind} = \vec{v}_{rel} + \vec{v}_{ship} = (0, -16) + (12, 0) = (12, -16)$
Magnitude $= \sqrt{144 + 256} = 20\,kmh^{-1}$
The wind blows towards the south-east, that is, it comes from $\tan^{-1}\frac{12}{16} = \tan^{-1}\frac{3}{4} \approx 36.87°$ west of north

44. **A.** $\tan^{-1}\left(\frac{3}{4}\right)$
Explanation: Path: $y = x\tan\alpha\left(1 - \frac{x}{R}\right) \Rightarrow 5 = 10\tan\alpha\left(1 - \frac{10}{30}\right)$
$\tan\alpha = \frac{5}{10\times\frac{2}{3}} = \frac{3}{4}$
Therefore, $\alpha = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.87°$

45. **D.** 5514.5 m
Explanation: Here, $u = 288\,kmh^{-1} = 80\,ms^{-1}$, $4900 = 4.9t^2 \Rightarrow t = \sqrt{1000} \approx 31.62$ s
Horizontal distance $= 80 \times 31.62 \approx 2529.8$ m
Therefore, the straight-line distance $= \sqrt{4900^2 + 2529.8^2} \approx 5514.5$ m

46. **D.** $\tan^{-1}(1.47)$
Explanation: Vertical velocity $v_y = \sqrt{2gh} = \sqrt{864.36} = 29.4\,ms^{-1}$
$\tan\theta = \frac{29.4}{20} = 1.47$
Therefore, $\theta = \tan^{-1}(1.47) \approx 55.78°$

47. **B.** 1.77 s
Explanation: $v_x = 30\cos 60° = 15$, $v_y = 30\sin 60° - 9.8t = 25.98 - 9.8t$
While rising, $\tan 30° = \frac{v_y}{v_x} \Rightarrow 25.98 - 9.8t = 8.66$
Therefore, $t = \frac{17.32}{9.8} \approx 1.77$ s

48. **B.** $10\sqrt{13}\,kmh^{-1}$
Explanation: $|\vec{v}_B - \vec{v}_A|^2 = 40^2 + 30^2 - 2 \cdot 40 \cdot 30\cos 60° = 1600 + 900 - 1200 = 1300$
Therefore, $v_{BA} = \sqrt{1300} = 10\sqrt{13} \approx 36.06\,kmh^{-1}$

49. **D.** 80 m
Explanation: $R_{max} = \frac{u^2}{g} = 160$
Vertically upward: $H = \frac{u^2}{2g} = \frac{160}{2}$
Therefore, $H = 80$ m

50. **C.** $\frac{5}{9}$
Explanation: $h \propto t^2$ ($u = 0$); in the first 3 seconds $h \propto 9$
In the 3rd second $\propto 9 - 4 = 5$
Therefore, the fraction $= \frac{5}{9}$