Engineering Daily 25 [Home Practice] EV - Answer Sheet

Physical Optics, Modern Physics

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. **B.** 52.2°
Explanation: $15 = 40\cos^2\theta$
$\cos\theta = \sqrt{0.375} \Rightarrow \theta = 52.2°$

2. **B.** $\frac{4}{3}m_0c$
Explanation: $\gamma = \frac{5}{3}$
$p = \gamma m_0 v = \frac{5}{3} \times 0.8m_0c = \frac{4}{3}m_0c$

3. **A.** 0.3 mm
Explanation: $\beta = \lambda\frac{D}{a}$
$\beta' = \frac{\lambda\frac{D}{2}}{2a} = \frac{\beta}{4} = 0.3\,\text{mm}$

4. **C. 38 years**
Explanation: $v = 0.8c$, $t_0 = 30 \times 0.6 = 18$ years
Age = 20 + 18 = 38 years

5. **C.** 1.39×10^{-14} kg
Explanation: $\Delta U = \frac{3}{2}nR\Delta T = 1.5 \times 2 \times 8.314 \times 50 = 1247\,\text{J}$
$\Delta m = \Delta\frac{U}{c^2} = 1.39 \times 10^{-14}\,\text{kg}$

6. **D.** 0.4 m
Explanation: $\sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0}{m} = 0.4$
$L = 1 \times 0.4 = 0.4\,\text{m}$

7. **D.** 0.344°
Explanation: $\sin\theta_1 = \frac{\lambda}{a} = 3 \times 10^{-3}$
$\theta_1 = 0.172°$
Angular separation $= 2\theta_1 = 0.344°$

8. **B.** 2 mm
Explanation: $x_1 = \frac{\lambda f}{a} = \frac{5\times10^{-7}\times0.4}{10^{-4}} = 2 \times 10^{-3}\,\text{m}$

9. **B.** $0.354I_0$
Explanation: $I = \frac{I_0}{2}(\cos^2 15°)^5$
$= 0.354I_0$

10. **B.** 70.9°
Explanation: $L_x = 2\cos 60° \times 0.6 = 0.6\,\text{m}$
$L_y = 2\sin 60° = 1.732\,\text{m}$ (unchanged)
$\theta' = \tan^{-1}\frac{L_y}{L_x} = 70.9°$

11. **A. 1.7**
Explanation: $(\mu - 1)t = n\lambda$, with t and λ the same
$\frac{\mu_2 - 1}{0.5} = \frac{7}{5} \Rightarrow \mu_2 = 1.7$

12. **C. 7**
Explanation: First minimum of a single slit: $\sin\theta = \frac{\lambda}{a}$
Interference maxima: $\sin\theta = \frac{n\lambda}{d}$
Inside the central maximum, $|n| < \frac{d}{a} = 4$
$n = \pm4$ fall on the minima and are absent; so $n = -3$ to 3, 7 in total

13. **A.** 0.6c
Explanation: $\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$; $\Delta\frac{x}{c} = 3\,\text{s}$
$4\sqrt{1 - \beta^2} = 5 - 3\beta$
$25\beta^2 - 30\beta + 9 = 0 \Rightarrow (5\beta - 3)^2 = 0$
$\beta = 0.6$

14. **B.** 1.78×10^{-7} s
Explanation: Relative speed $= \frac{1.2c}{1.36} = 0.8824c$
Length of the other train $= 100\sqrt{1 - 0.8824^2} = 47.06\,\text{m}$
$t = \frac{47.06}{0.8824c} = 1.78 \times 10^{-7}\,\text{s}$

15. **C.** $\frac{1}{9}$
Explanation: $I = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$
$\frac{I_2}{I_1} = \left(\frac{10}{30}\right)^2 = \frac{1}{9}$

16. **C.** $u = u' + v$
Explanation: $\frac{u'v}{c^2} \to 0$
$u = u' + v$ (Galilean addition)

17. **D.** $\frac{\sqrt{15}}{4}c$
Explanation: $\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{4}$
$v = c\sqrt{1 - \frac{1}{16}} = \frac{\sqrt{15}}{4}c$

18. **D.** 0.675 mm
Explanation: $\beta' = \frac{\beta}{\mu} = \frac{0.9}{\frac{4}{3}} = 0.675\,\text{mm}$

19. **A.** 1.52×10^{-30} kg
Explanation: $m = \frac{m_0}{\sqrt{1-0.64}} = \frac{9.11\times10^{-31}}{0.6}$
$= 1.52 \times 10^{-30}\,\text{kg}$

20. **A.** 5×10^5 s
Explanation: $v \ll c$: $\sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{v^2}{2c^2}$
$\Delta t \approx t\frac{v^2}{2c^2}$

$t = \frac{2c^2\Delta t}{v^2} = \frac{2\times9\times10^{16}\times10^{-6}}{3.6\times10^5} = 5\times10^5\ s$

21. **D.** 2 μm, 1.5×10^{14} Hz
Explanation: $\lambda = \frac{2\pi}{k} = 2\times10^{-6}$ m
$f = \frac{\omega}{2\pi} = 1.5\times10^{14}$ Hz
Check: $f\lambda = 3\times10^8\ ms^{-1}$

22. **A.** 0.2 mm
Explanation: $\beta \propto \lambda \propto \frac{1}{f}$
$\beta_2 = 0.5\times\frac{4.5}{7.5} = 0.3$ mm
Decrease = 0.2 mm

23. **B. 81 : 1**
Explanation: $I_2 = 0.64I_1 \Rightarrow a_2 = 0.8a_1$
$\frac{I_{max}}{I_{min}} = \left(\frac{1+0.8}{1-0.8}\right)^2 = 81$

24. **C. 3522**
Explanation: $N = \frac{\sin\theta}{n\lambda} = \frac{\sin 25°}{2\times600\times10^{-9}}$
$= 3.52\times10^5\ m^{-1} = 3522\ cm^{-1}$

25. **B.** 33.7°
Explanation: $\theta_p = \tan^{-1}1.5 = 56.31°$ (with the normal)
Angle with the surface = $90° - \theta_p = 33.7°$

26. **B.** 5.91×10^{12} km
Explanation: Distance travelled by light in one year: $s = vt = \frac{c}{\mu}t$
$s = \frac{9.46\times10^{12}}{1.6} = 5.91\times10^{12}$ km

27. **B.** 0.4 mm
Explanation: $\beta = \frac{3}{10} = 0.3$ cm
$a = \frac{\lambda D}{\beta} = \frac{5\times10^{-7}\times2.4}{3\times10^{-3}} = 4\times10^{-4}$ m

28. **D.** 600 nm
Explanation: $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$
$\frac{1}{\lambda_2} = \frac{1}{150} - \frac{1}{200} = \frac{1}{600}$
$\lambda_2 = 600$ nm

29. **A.** $1070\ Wm^{-2}$
Explanation: $E_0 = cB_0 = 900\ Vm^{-1}$
$I = \frac{E_0B_0}{2\mu_0} = \frac{900\times3\times10^{-6}}{2\times4\pi\times10^{-7}} = 1074\ Wm^{-2}$

30. **A.** $4.18\times10^{-25}\ kgms^{-1}$
Explanation: $K = 3.6 - 3.0 = 0.6$ eV
Impulse = $\Delta p = \sqrt{2mK}$
$= \sqrt{2\times9.11\times10^{-31}\times0.6\times1.6\times10^{-19}} = 4.18\times10^{-25}\ kgms^{-1}$

31. **B.** 0.085°
Explanation: $d = 1.43\times10^{-6}$ m
$\theta_1 = \sin^{-1}\frac{2\times589\times10^{-9}}{d} = 55.548°$
$\theta_2 = 55.633°$
$\Delta\theta = 0.085°$

32. **A.** 500 nm
Explanation: $x_4 = \frac{4\lambda D}{a}$
$\lambda = \frac{9.6\times10^{-3}\times0.25\times10^{-3}}{4\times1.2} = 5\times10^{-7}$ m

33. **D.** 110.8 keV
Explanation: $\Delta\lambda = \frac{h}{m_0c}(1-\cos60°) = 1.21\times10^{-12}$ m
$\lambda' = 0.011212$ nm
$E' = \frac{hc}{\lambda} = 110.8$ keV

34. **A.** $\frac{\sqrt{24}}{5}c$
Explanation: $E_0 = 0.2E \Rightarrow \gamma = 5$
$v = c\sqrt{1-\frac{1}{25}} = \frac{\sqrt{24}}{5}c$

35. **A.** 128 kV
Explanation: $eV = K = (\gamma-1)E_0 = 0.25\times0.511$ MeV
$V = 127.8$ kV

36. **C. 2 : 3 : 5**
Explanation: At the intercept, $\phi = \frac{hc}{\lambda_0} \propto \frac{1}{\lambda_0}$
$\phi_1 : \phi_2 : \phi_3 = 2 : 3 : 5$

37. **D.** 4.44 ns
Explanation: $L = 1\times0.8 = 0.8$ m
$t = \frac{0.8}{0.6\times3\times10^8} = 4.44\times10^{-9}$ s

38. **C. 8.33 years**
Explanation: Half-life is a time interval, so it is dilated
$T = \frac{5}{0.6} = 8.33$ years

39. **D. Unchanged**
Explanation: In its own frame the clock is at rest, so time runs normally
Time dilation is seen only when measured from another frame

40. **D.** 1.8 mm
Explanation: $\lambda = \frac{c}{f} = 750$ nm, $a = \frac{\lambda D}{\beta} = 1$ mm
$\Delta\beta = \frac{\lambda\Delta D}{a} = \frac{750\times10^{-9}\times0.4}{10^{-3}} = 0.3$ mm
New width = 1.5 + 0.3 = 1.8 mm

41. **D. Tends to infinity**

Explanation: $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$; as $v \to c$, the denominator $\to 0$
So $m \to \infty$

42. **C.** $(1 - 1.49 \times 10^{-7})c$
Explanation: $\sqrt{1 - \frac{v^2}{c^2}} = \frac{m_e}{m_p} = 5.46 \times 10^{-4}$
$v = c\sqrt{1-\varepsilon} \approx c\left(1 - \frac{\varepsilon}{2}\right)$
$\varepsilon = 2.98 \times 10^{-7} \Rightarrow v = (1 - 1.49 \times 10^{-7})c$

43. **A.** 400 nm
Explanation: Condition for darkness in reflection: $2\mu t = m\lambda$
First: $m = 1 \Rightarrow t = 200\,\text{nm}$
Second: $m = 2 \Rightarrow t = \frac{2\times500}{2\times1.25} = 400\,\text{nm}$

44. **C. 5**
Explanation: $\sqrt{1 - 0.96} = \sqrt{0.04} = 0.2$
$\frac{m}{m_0} = \frac{1}{0.2} = 5$

45. **C.** 0.5c**, opposite to the vehicle's motion**
Explanation: $u = \frac{-0.8c+0.5c}{1-0.4} = -0.5c$
Negative sign: opposite to the vehicle's motion

46. **D.** 5.96 mm
Explanation: $\lambda = \frac{hc}{E} = 497\,\text{nm}$
$a = 2 \times 0.2 = 0.4\,\text{mm}$
$x_3 = \frac{3\lambda D}{a} = 5.96\,\text{mm}$

47. **C. 1.49**
Explanation: $(\mu - 1)t = \lambda$
$\mu = 1 + \frac{5.88\times10^{-7}}{1.2\times10^{-6}} = 1.49$

48. **B. The fifth dark fringe**
Explanation: $\frac{\Delta}{\lambda} = \frac{2.25}{0.5} = 4.5 = \left(5 - \frac{1}{2}\right)$
A half-odd multiple, so dark; $n = 5$, i.e. the fifth dark fringe

49. **D.** 4.48×10^{15} Hz
Explanation: Required $K_{max} = 13.6\,\text{eV}$
$f = \frac{c}{\lambda_0} + \frac{13.6e}{h}$
$= 1.2 \times 10^{15} + 3.28 \times 10^{15} = 4.48 \times 10^{15}$ Hz

50. **A.** $238.7\sin^2(kx - \omega t)\,\hat{i}\,\text{Wm}^{-2}$
Explanation: $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$
$\hat{j} \times \hat{k} = \hat{i}$
$|\vec{S}| = \frac{300\times10^{-6}}{4\pi\times10^{-7}}\sin^2(\cdots) = 238.7\sin^2(\cdots)$