MCQ 50 marks

1. **C.** 7

Explanation: $x < -3$: $(1 - x) + (-x - 3) = -2x - 2 < 8 \Rightarrow x > -5$

$-3 \le x \le 1$: $(1 - x) + (x + 3) = 4 < 8$ (always true)

$x > 1$: $(x - 1) + (x + 3) = 2x + 2 < 8 \Rightarrow x < 3$

Solution set $= (-5, 3)$

Integers: $-4, -3, \ldots, 2 \Rightarrow 7$

2. **A.** $\sup S = 2$, $\inf S = -3$**; but** max**,** min **do not exist**

Explanation: $0 < \frac{2}{n} \le 2$ and $0 < \frac{3}{m} \le 3$, so $-3 < \frac{2}{n} - \frac{3}{m} < 2$

For $n = 1$, $m \to \infty$ the value $\to 2$; so $\sup S = 2$

For $m = 1$, $n \to \infty$ the value $\to -3$; so $\inf S = -3$

Since $\frac{3}{m} > 0$ and $\frac{2}{n} > 0$, neither 2 nor -3 is attained, so max and min do not exist

3. **C.** $\frac{7}{3}$, -3

Explanation: $-8 < 3x + 1 < 8 \Rightarrow -9 < 3x < 7 \Rightarrow -3 < x < \frac{7}{3}$

$S = \left(-3, \frac{7}{3}\right)$; hence $\sup S = \frac{7}{3}$, $\inf S = -3$ (neither is in S)

4. **C.** $\left[-\frac{5}{3}, 5\right]$

Explanation: $-10 \le 5 - 3x \le 10$

$\Rightarrow -15 \le -3x \le 5$

$\Rightarrow -\frac{5}{3} \le x \le 5$ [the signs reverse on dividing by -3]

5. **B.** $\frac{5}{9} \le x \le \frac{7}{9}$**,** $x \ne \frac{2}{3}$

Explanation: For $x \ne \frac{2}{3}$ we have $|3x - 2| > 0$, so $|3x - 2| \le \frac{1}{3}$

$\Rightarrow -\frac{1}{3} \le 3x - 2 \le \frac{1}{3}$

$\Rightarrow \frac{5}{3} \le 3x \le \frac{7}{3}$

$\Rightarrow \frac{5}{9} \le x \le \frac{7}{9}$

Solution: $\frac{5}{9} \le x \le \frac{7}{9}$, $x \ne \frac{2}{3}$

6. **C.** $[-4, 0] \cup [4, 8]$

Explanation: $-2 \le |x - 2| - 4 \le 2 \Rightarrow 2 \le |x - 2| \le 6$

$\Rightarrow x - 2 \in [-6, -2] \cup [2, 6]$

$\Rightarrow x \in [-4, 0] \cup [4, 8]$

7. **D.** $\left(-\frac{10}{3}, \frac{10}{3}\right)$

Explanation: The expression is an even function, so solve for $x \ge 0$ and reflect

$0 \le x \le 3$: $x + (3 - x) + (x + 3) = x + 6 < 10$ (always true)

$x > 3$: $x + (x - 3) + (x + 3) = 3x < 10 \Rightarrow x < \frac{10}{3}$

$\therefore 0 \le x < \frac{10}{3}$; reflecting gives the solution $\left(-\frac{10}{3}, \frac{10}{3}\right)$

8. **C.** $-2 < a < 6$

Explanation: 1st inequality: $x \in (a - 3, a + 3)$; 2nd inequality: $x \in (1, 3)$

Two open intervals intersect if $a - 3 < 3$ and $a + 3 > 1$

$\Rightarrow a < 6$ and $a > -2$

$\therefore -2 < a < 6$

9. **C.** $-5 \le x \le -1$**,** $x \ne -2$

Explanation: For $x \ne -2$: $|x - 1| \ge 2|x + 2|$; squaring

$x^2 - 2x + 1 \ge 4x^2 + 16x + 16$

$\Rightarrow 3x^2 + 18x + 15 \le 0 \Rightarrow x^2 + 6x + 5 \le 0$

$\Rightarrow (x + 1)(x + 5) \le 0 \Rightarrow -5 \le x \le -1$

Solution: $-5 \le x \le -1$, $x \ne -2$

10. **D.** $[-3, \infty)$

Explanation: $(x^2 - 9) + (x - 3) = x^2 + x - 12$; so the equation has the form $|a| + |b| = |a + b|$

Condition: $ab \ge 0 \Rightarrow (x^2 - 9)(x - 3) \ge 0$

$\Rightarrow (x - 3)^2(x + 3) \ge 0$

$(x - 3)^2 \ge 0$ always, so $x + 3 \ge 0$ or $x = 3 \Rightarrow x \ge -3$

11. **D.** $(-4, -3) \cup (-1, 6)$

Explanation: $|x - 1| < 5 \Rightarrow -4 < x < 6$

$|x + 2| > 1 \Rightarrow x < -3$ or $x > -1$

Intersection: $(-4, -3) \cup (-1, 6)$

12. **B.** $x > -2$

Explanation: Both sides are non-negative, so squaring:

$x^2 - 6x + 9 < x^2 + 14x + 49$

$\Rightarrow -40 < 20x \Rightarrow x > -2$

13. **A.** 7**;** $1 \le x \le 8$

Explanation: $|x-1|+|8-x| \geq |(x-1)+(8-x)| = 7$
Equality when $(x-1)(8-x) \geq 0 \Rightarrow 1 \leq x \leq 8$
$\therefore$ the minimum value is 7, attained on $[1, 8]$

14. **D.** $|3x-2y-8| < 1.8$
Explanation: $3x-2y-8 = 3(x-2)-2(y+1)$
$\Rightarrow |3x-2y-8| \leq 3|x-2|+2|y+1|$
$< 3(0.4)+2(0.3) = 1.8$
$\therefore |3x-2y-8| < 1.8$ [the value approaches 1.8 as $x \to 2.4,\ y \to -1.3$]

15. **D.** $\left(\frac{1}{5}, 1\right)$
Explanation: $-1 < x < 3 \Rightarrow 1 < x+2 < 5$
All values are positive, so taking reciprocals reverses the inequalities: $\frac{1}{5} < \frac{1}{x+2} < 1$

16. **B.** $[-4, 21]$
Explanation: $x^2-4x = (x-2)^2-4$
$x \in [-3, 2] \Rightarrow x-2 \in [-5, 0] \Rightarrow (x-2)^2 \in [0, 25]$
$\Rightarrow (x-2)^2-4 \in [-4, 21]$

17. **D.** $(-2, 1) \cup (1, 3)$
Explanation: $(x-1)^2 \geq 0$; at $x = 1$ the expression is 0, not < 0, so $x \neq 1$
For $x \neq 1$, $(x-1)^2 > 0$, so $(x+2)(x-3) < 0 \Rightarrow -2 < x < 3$
$\therefore$ Solution: $(-2, 1) \cup (1, 3)$

18. **B.** $(-\infty, -1] \cup [2, 4) \cup (4, \infty)$
Explanation: $x \neq 4$ (zero denominator) and $(x-4)^2 > 0$
$(x-2)^3$ has the same sign as $(x-2)$
$\therefore (x-2)(x+1) \geq 0 \Rightarrow x \leq -1$ or $x \geq 2$
Excluding $x = 4$: $(-\infty, -1] \cup [2, 4) \cup (4, \infty)$

19. **D.** $0 \leq x < 1$
Explanation: $\frac{3x-2}{x-1} - 2 \leq 0$
$\Rightarrow \frac{3x-2-2x+2}{x-1} \leq 0$
$\Rightarrow \frac{x}{x-1} \leq 0$
$\therefore 0 \leq x < 1$ [undefined at $x = 1$]

20. **A.** $2 < x < 5$
Explanation: Denominator: $x^2+x+2 = \left(x+\frac{1}{2}\right)^2+\frac{7}{4} > 0$ for all x
$\therefore$ the numerator must be negative: $(x-2)(x-5) < 0$
$\Rightarrow 2 < x < 5$

21. **B.** $x < -5$ **or** $x > 2$
Explanation: The numerator and denominator must have the same sign
Critical points: $-5,\ 2$; positive for $x > 2$, negative on $(-5, 2)$, positive for $x < -5$
$\therefore x < -5$ or $x > 2$

22. **A.** $(-1, 2)$
Explanation: $(x+5)(x-2) < 0 \Rightarrow -5 < x < 2$
Intersection with $x > -1$: $(-1, 2)$

23. **C.** 10
Explanation: $(|x|-1)(|x|-5) \leq 0 \Rightarrow 1 \leq |x| \leq 5$
$\Rightarrow x \in [-5, -1] \cup [1, 5]$
Integers: $\pm1, \pm2, \pm3, \pm4, \pm5 \Rightarrow 10$ (0 excluded)

24. **C.** 8
Explanation: Put $t = x^2 \geq 0$: $(t-1)(t-36) < 0 \Rightarrow 1 < t < 36$
$\Rightarrow 1 < x^2 < 36 \Rightarrow 1 < |x| < 6$
Integers: $\pm2, \pm3, \pm4, \pm5 \Rightarrow 8$

25. **A.** $[2, 4]$
Explanation: $|f|+f = 0 \Leftrightarrow |f| = -f \Leftrightarrow f \leq 0$
$\therefore x^2-6x+8 \leq 0$
$\Rightarrow (x-2)(x-4) \leq 0$
$\Rightarrow 2 \leq x \leq 4$

26. **D.** $[-5, -1)$
Explanation: Domain: $x \geq -5$
Case 1: for $x < -3$ the right side is negative; always true: $-5 \leq x < -3$
Case 2: for $x \geq -3$, squaring gives $x+5 > x^2+6x+9 \Rightarrow x^2+5x+4 < 0 \Rightarrow -4 < x < -1$, so $-3 \leq x < -1$
Combining: $[-5, -1)$

27. **C.** $\left[-3, \frac{3}{\sqrt{2}}\right]$
Explanation: Domain: $9-x^2 \geq 0 \Rightarrow -3 \leq x \leq 3$
$-3 \leq x < 0$: the right side is negative, always true
$0 \leq x \leq 3$: squaring gives $9-x^2 \geq x^2 \Rightarrow x^2 \leq \frac{9}{2} \Rightarrow 0 \leq x \leq \frac{3}{\sqrt{2}}$
Combining: $\left[-3, \frac{3}{\sqrt{2}}\right]$

28. **C.** $k < -2$
Explanation: Condition for always negative: $k < 0$ and discriminant < 0

$16 - 4k^2 < 0 \Rightarrow k^2 > 4 \Rightarrow k < -2$ or $k > 2$
Combining with $k < 0$: $k < -2$

29. **B.** $-4 < a < 3$
Explanation: Condition: discriminant < 0
$4(a+1)^2 - 4(a+13) < 0$
$\Rightarrow a^2 + 2a + 1 - a - 13 < 0 \Rightarrow a^2 + a - 12 < 0$
$\Rightarrow (a+4)(a-3) < 0 \Rightarrow -4 < a < 3$

30. **D.** 25
Explanation: $\frac{4}{x} + \frac{9}{y} = \left(\frac{4}{x} + \frac{9}{y}\right)(x+y) = 13 + \frac{4y}{x} + \frac{9x}{y}$
By AM $\geq$ GM: $\frac{4y}{x} + \frac{9x}{y} \geq 2\sqrt{36} = 12$
$\therefore \frac{4}{x} + \frac{9}{y} \geq 25$
Equality when $2y = 3x$, i.e. $x = \frac{2}{5},\ y = \frac{3}{5}$
$\therefore$ minimum value = 25

31. **B.** 13
Explanation: $\frac{x^2+5x+16}{x} = x + \frac{16}{x} + 5$
By AM $\geq$ GM: $x + \frac{16}{x} \geq 2\sqrt{16} = 8$
$\therefore$ minimum value $= 8 + 5 = 13$ (at $x = 4$)

32. **B.** 64
Explanation: By AM $\geq$ GM: $\frac{a+b+c}{3} \geq \sqrt[3]{ab}c$
$\Rightarrow 4 \geq \sqrt[3]{abc}$
$\Rightarrow abc \leq 64$
Equality when $a = b = c = 4$
$\therefore$ maximum value = 64

33. **A.** 30
Explanation: Axis intercepts: $(10, 0)$ and $(0, 6)$
Area $= \frac{1}{2} \times 10 \times 6 = 30$

34. **A.** 27
Explanation: $x + y = 9$ and $y = 2x$ meet where $3x = 9 \Rightarrow (3, 6)$
Vertices: $(0, 0), (9, 0), (3, 6)$
Area $= \frac{1}{2} \times 9 \times 6 = 27$

35. **D.** 24
Explanation: The region is a rectangle; distances between parallel lines are $\frac{6}{\sqrt{2}} = 3\sqrt{2}$ and $\frac{8}{\sqrt{2}} = 4\sqrt{2}$
Area $= 3\sqrt{2} \times 4\sqrt{2} = 24$

36. **B.** 18
Explanation: $y = |x|$ and $y = 6 - |x|$ meet where $|x| = 3 \Rightarrow (\pm 3, 3)$
The region is a square with vertices $(0, 0), (3, 3), (0, 6), (-3, 3)$ and diagonals 6 and 6
Area $= \frac{1}{2} \times 6 \times 6 = 18$

37. **A.** 33
Explanation: For each y, $0 \leq x \leq \frac{24-4y}{3}$:
$y = 0$: $x \leq 8 \Rightarrow 9$ points; $y = 1$: $x \leq \frac{20}{3} \Rightarrow 7$ points
$y = 2$: $x \leq \frac{16}{3} \Rightarrow 6$ points; $y = 3$: $x \leq 4 \Rightarrow 5$ points
$y = 4$: $x \leq \frac{8}{3} \Rightarrow 3$ points; $y = 5$: $x \leq \frac{4}{3} \Rightarrow 2$ points; $y = 6$: $x = 0 \Rightarrow 1$ point
Total $= 9 + 7 + 6 + 5 + 3 + 2 + 1 = 33$

38. **D.** 10
Explanation: The region is a rhombus centred at $(2, 0)$; corner points:
$(5, 0), (-1, 0), (2, 3), (2, -3)$
z: 10, -2, 1, 7
$\therefore z_{max} = 10$ [at $(5, 0)$]

39. **A.** -7
Explanation: $z(2, 1) = 0,\ z(6, 2) = 2,\ z(5, 5) = -5,\ z(1, 4) = 1 - 8 = -7$
$\therefore z_{min} = -7$ [at $(1, 4)$]

40. **A.** 20
Explanation: Intersection: $(x + 2y) - (x + y) = 12 - 8 \Rightarrow y = 4,\ x = 4$
Corner points: $(0, 0), (8, 0), (4, 4), (0, 6)$
z: 0, 16, 20, 18
$\therefore z_{max} = 20$

41. **B.** 12
Explanation: Intersection: $(3x + y) - (x + y) = 10 - 6 \Rightarrow x = 2,\ y = 4$
Corner points: $(0, 10), (2, 4), (6, 0)$ [the region is unbounded]
z: 30, 16, 12
The coefficients of z are positive, so $z_{min} = 12$ [at $(6, 0)$]

42. **B.** 180
Explanation: Let x units of A and y units of B be made
Constraints: $3x + y \leq 15$, $2x + 2y \leq 14 \Rightarrow x + y \leq 7$, $x, y \geq 0$; profit $z = 30x + 20y$
Intersection: $(3x + y) - (x + y) = 15 - 7 \Rightarrow x = 4,\ y = 3$
Corner points: $(0, 0), (5, 0), (4, 3), (0, 7)$

z: 0, 150, 180, 140
∴ Maximum profit = 180

43. **D.** $\frac{1}{3} \le a \le 1$

Explanation: Corner points:
$(0,0), (8,0), (3,5), (0,6)$
$z(3,5) = 3a + 5,\ z(8,0) = 8a,\ z(0,6) = 6$
$3a + 5 \ge 8a \Rightarrow a \le 1$
$3a + 5 \ge 6 \Rightarrow a \ge \frac{1}{3}$
$\therefore \frac{1}{3} \le a \le 1$

44. **C.** 2

Explanation: The segment lies on the boundary line $2x + y = 14$
The objective line must be parallel to it: $\frac{4}{2} = \frac{k}{1} \Rightarrow k = 2$
Check: $z = 2(2x + y)$; $z(5,4) = z(7,0) = 28$, $z(0,9) = 18$, $z(0,0) = 0 \Rightarrow z_{max} = 28$

45. **B.** 10

Explanation: Corner points:
$(3,0), (5,0), (5,4), (0,4), (0,3)$
z: 6, 10, 6, −4, −3
$\therefore z_{max} = 10$ [at $(5,0)$]

46. **A.** $z_{min} = 11$**; no finite** z_{max}

Explanation: $x - y = 3$ and $x + y = 5$ meet at $(4,1)$
On the x-axis ($y = 0$): $x \ge 5$ and $x \le 3$ together are impossible, so there is no corner point there
Corner points: $(0,5), (4,1)$; z: 15, 11
The region is unbounded upwards, so there is no finite maximum; $z_{min} = 11$

47. **B. No feasible solution exists**

Explanation: Corner points of the region $x + y \le 4$, $x, y \ge 0$: $(0,0), (4,0), (0,4)$
Values of $3x + y$: 0, 12, 4; the maximum is $12 < 14$
But the condition requires $3x + y \ge 14$, which is impossible
$\therefore$ the feasible region is empty (no feasible solution)

48. **C.** $\frac{102}{7}$

Explanation: Intersection: $x = 12 - 3y \Rightarrow 36 - 9y + 2y = 14 \Rightarrow y = \frac{22}{7},\ x = \frac{18}{7}$
Corner points: $(0,0), \left(\frac{14}{3}, 0\right), \left(\frac{18}{7}, \frac{22}{7}\right), (0,4)$
z: 0, $\frac{28}{3}$, $\frac{36+66}{7} = \frac{102}{7}$, 12
$\therefore z_{max} = \frac{102}{7}$

49. **A.** 200

Explanation: Let x kg of X and y kg of Y be used
Constraints: $3x + y \ge 15$, $2x + 4y \ge 20 \Rightarrow x + 2y \ge 10$, $x, y \ge 0$; cost $z = 35x + 20y$
Intersection: $y = 15 - 3x \Rightarrow x + 30 - 6x = 10 \Rightarrow x = 4,\ y = 3$
Corner points: $(0,15), (4,3), (10,0)$
z: 300, 200, 350
∴ Minimum cost = 200

50. **A.** 220

Explanation: Intersection: $(2x + y) - (x + y) = 80 - 60 \Rightarrow x = 20,\ y = 40$
Corner points: $(0,0), (40,0), (20,40), (0,60)$
z: 0, 200, 220, 180
$\therefore z_{max} = 220$