MCQ 50 marks

1. **B.** 4

Explanation: For even n the terms are $2 - \frac{1}{n}$, which approach 2 but never equal 2

For odd n the terms are $-\left(2 - \frac{1}{n}\right)$, which approach -2 but never equal -2

So, $\sup S = 2$ and $\inf S = -2$

$\sup S - \inf S = 2 - (-2) = 4$

2. **C.** 3

Explanation: $S = \left\{1, 2, \frac{7}{3}, \frac{5}{2}, \cdots\right\}$

As n increases, $3 - \frac{2}{n}$ keeps increasing towards 3 but never equals 3

So, $\sup S = 3$ (although $3 \notin S$)

3. **A.** $\sup S = 5,\ \inf S = -1$

Explanation: $x^2 - 4x - 5 < 0$

$\Rightarrow (x - 5)(x + 1) < 0$

$\Rightarrow -1 < x < 5$, so $S = (-1, 5)$

Therefore, $\sup S = 5$ and $\inf S = -1$ (neither belongs to S)

4. **B.** $(-7, 7)$

Explanation: $|x| < a \Leftrightarrow -a < x < a$

So, $|x| < 7 \Rightarrow -7 < x < 7$

Solution set = $(-7, 7)$

5. **D.** $(-2, 0) \cup (4, 6)$

Explanation: $-1 < |x - 2| - 3 < 1$

$\Rightarrow 2 < |x - 2| < 4$

$|x - 2| > 2 \Rightarrow x < 0$ or $x > 4$

$|x - 2| < 4 \Rightarrow -2 < x < 6$

Intersection: $(-2, 0) \cup (4, 6)$

6. **D.** $a \leq 7$

Explanation: Triangle inequality: $|x - 5| + |x + 2| \geq |(x + 2) - (x - 5)| = 7$

The left side has minimum value 7, so it is never less than 7

So, for $a \leq 7$ the condition (left side) $< a$ is impossible

7. **A.** 5

Explanation: Critical points: $x = -\frac{3}{2},\ 1$

$x < -\frac{3}{2}$: $(1 - x) - (2x + 3) \leq 7 \Rightarrow -2 - 3x \leq 7 \Rightarrow x \geq -3$

$-\frac{3}{2} \leq x \leq 1$: $(1 - x) + (2x + 3) = x + 4 \leq 7$ (always true)

$x > 1$: $(x - 1) + (2x + 3) = 3x + 2 \leq 7 \Rightarrow x \leq \frac{5}{3}$

Solution set = $\left[-3, \frac{5}{3}\right]$

Integers: $-3, -2, -1, 0, 1 \Rightarrow 5$

8. **D.** $(-5, 2)$

Explanation: $|2x + 3| < 7$

$\Rightarrow -7 < 2x + 3 < 7$

$\Rightarrow -10 < 2x < 4$

$\Rightarrow -5 < x < 2$

$\therefore$ Solution set = $(-5, 2)$

9. **C.** $x < -1$ **or** $x > \frac{1}{2}$

Explanation: Case 1: $x < -1$; the denominator is negative and the numerator is ≥ 0, so the left side is $\leq 0 < 1$ (always true)

Case 2: $x > -1$; multiplying by $(x + 1) > 0$ gives $|x - 2| < x + 1$

$\Rightarrow -(x + 1) < x - 2 < x + 1$

The right part is always true; left part: $-x - 1 < x - 2 \Rightarrow x > \frac{1}{2}$

So, $x < -1$ or $x > \frac{1}{2}$

10. **D.** $-\frac{7}{2} < x < -\frac{5}{2}, x \neq -3$

Explanation: $x \neq -3$ (denominator cannot be zero), so $|x + 3| > 0$

$\frac{1}{|x+3|} > 2 \Rightarrow |x + 3| < \frac{1}{2}$

$\Rightarrow -\frac{1}{2} < x + 3 < \frac{1}{2}$

$\Rightarrow -\frac{7}{2} < x < -\frac{5}{2}$

Solution: $-\frac{7}{2} < x < -\frac{5}{2}, x \neq -3$

11. **A.** $x > \frac{4}{3}, x \neq 2$

Explanation: For $x \neq 2$: $|3x - 2| > 3|x - 2|$

Both sides are non-negative, so squaring:

$9x^2 - 12x + 4 > 9x^2 - 36x + 36$

$\Rightarrow 24x > 32 \Rightarrow x > \frac{4}{3}$

Solution: $x > \frac{4}{3}, x \neq 2$

12. **B.** $[0, 10]$

Explanation: $-2 \leq x \leq 4 \Rightarrow -6 \leq 3x \leq 12$

$\Rightarrow -8 \leq 3x - 2 \leq 10$

The interval contains 0 (at $x = \frac{2}{3}$), so the minimum is 0

Maximum value = $\max(|-8|, |10|) = 10$

$\therefore$ Range = $[0, 10]$

13. **A.** $[1, 7]$

Explanation: Critical points: $x = 2, 6$

$x < 2$: $(2 - x) + (6 - x) = 8 - 2x \le 6 \Rightarrow x \ge 1$

$2 \le x \le 6$: $(x - 2) + (6 - x) = 4 \le 6$ (always true)

$x > 6$: $(x - 2) + (x - 6) = 2x - 8 \le 6 \Rightarrow x \le 7$

$\therefore$ Solution set $= [1, 7]$

14. **D.** 5

Explanation: The minimum occurs when x is the median of $1, 4, 6$

Here, the median is 4

$f(4) = |4 - 1| + |4 - 4| + |4 - 6| = 3 + 0 + 2 = 5$

$\therefore$ Minimum value $= 5$

15. **D.** $x > 1$

Explanation: Since $|x - 3| \ge 0$, we need $x > 0$

$|x - 3| < 2x \Rightarrow -2x < x - 3 < 2x$

Right part: $x - 3 < 2x \Rightarrow x > -3$ (always true for $x > 0$)

Left part: $-2x < x - 3 \Rightarrow 3x > 3 \Rightarrow x > 1$

$\therefore$ Solution: $x > 1$

16. **B.** $(-\infty, 2] \cup [5, \infty)$

Explanation: Let $a = x - 2$, $b = x - 5$; then $a + b = 2x - 7$

The equation has the form $|a| + |b| = |a + b|$, which holds $\Leftrightarrow ab \ge 0$

$\Rightarrow (x - 2)(x - 5) \ge 0$

$\therefore x \le 2$ or $x \ge 5$

17. **A.** $|x + y - 6| < 5$

Explanation: Triangle inequality: $|a + b| \le |a| + |b|$

$|x + y - 6| = |(x - 5) + (y - 1)|$

$\le |x - 5| + |y - 1| < 2 + 3 = 5$

$\therefore |x + y - 6| < 5$

18. **B.** 8

Explanation: $x < -1$: $(2 - x) + (-x - 1) = 1 - 2x < 9 \Rightarrow x > -4$

$-1 \le x \le 2$: $(2 - x) + (x + 1) = 3 < 9$ (always true)

$x > 2$: $(x - 2) + (x + 1) = 2x - 1 < 9 \Rightarrow x < 5$

Solution set $= (-4, 5)$

Integers: $-3, -2, \ldots, 4 \Rightarrow 8$

19. **C.** $x < -1$ **or** $2 < x < 4$

Explanation: Critical points: $-1, 2, 4$

For $x > 4$ the expression is positive; the sign changes at each critical point

$\therefore$ the expression is negative when $x < -1$ or $2 < x < 4$

20. **D.** 4

Explanation: Since $x^2 = |x|^2$: $|x|^2 - 7|x| + 10 < 0$

$\Rightarrow (|x| - 2)(|x| - 5) < 0$

$\Rightarrow 2 < |x| < 5$

Integers: $\pm 3, \pm 4 \Rightarrow 4$

21. **A.** $x < 2$ **or** $x \ge 5$

Explanation: $\frac{x+4}{x-2} - 3 \le 0$

$\Rightarrow \frac{x+4-3x+6}{x-2} \le 0$

$\Rightarrow \frac{10-2x}{x-2} \le 0$

$\Rightarrow \frac{x-5}{x-2} \ge 0$

$\therefore x < 2$ or $x \ge 5$ [the expression is undefined at $x = 2$]

22. **D.** $[-4, 4]$

Explanation: Since $x^2 = |x|^2$: $|x|^2 - 3|x| - 4 \le 0$

$\Rightarrow (|x| - 4)(|x| + 1) \le 0$

$|x| + 1 > 0$ always, so $|x| - 4 \le 0$

$\Rightarrow -4 \le x \le 4$

23. **C.** 4

Explanation: $(x - 1)(x - 9) \le 0 \Rightarrow 1 \le x \le 9$

$|x - 5| > 2 \Rightarrow x < 3$ or $x > 7$

Common part: $[1, 3) \cup (7, 9]$

Integers: $1, 2, 8, 9 \Rightarrow 4$

24. **A.** $(-\infty, 2] \cup [3, \infty)$

Explanation: Domain: $(x - 2)(x - 3) \ge 0 \Rightarrow x \le 2$ or $x \ge 3$

Case 1: for $x < 4$ the right side is negative, so it holds on the whole domain: $x \le 2$ or $3 \le x < 4$

Case 2: for $x \ge 4$, squaring gives $x^2 - 5x + 6 > x^2 - 8x + 16 \Rightarrow x > \frac{10}{3}$, so all $x \ge 4$

$\therefore$ Solution: $(-\infty, 2] \cup [3, \infty)$

25. **B.** $(2 - \sqrt{7},\ 1) \cup (3,\ 2 + \sqrt{7})$

Explanation: $-3 < x^2 - 4x < 3$

(i) $x^2 - 4x + 3 > 0 \Rightarrow (x - 1)(x - 3) > 0 \Rightarrow x < 1$ or $x > 3$

(ii) $x^2 - 4x - 3 < 0 \Rightarrow 2 - \sqrt{7} < x < 2 + \sqrt{7}$

Intersection of (i) and (ii): $(2 - \sqrt{7},\ 1) \cup (3,\ 2 + \sqrt{7})$

26. **C.** $(-3, -2] \cup [2, 3)$

Explanation: $\frac{(x-2)(x+2)}{(x-3)(x+3)} \le 0$
Critical points: $-3, -2, 2, 3$; the expression is positive for $x > 3$
Signs: $(-\infty, -3)$: +, $(-3, -2)$: −, $(-2, 2)$: +, $(2, 3)$: −, $(3, \infty)$: +
Zero numerator ($x = \pm 2$) is allowed, zero denominator ($x = \pm 3$) is excluded
$\therefore$ Solution: $(-3, -2] \cup [2, 3)$

27. **B.** $-6 < k < 6$
Explanation: $ax^2 + bx + c > 0$ for all x requires $a > 0$ and $b^2 - 4ac < 0$
Here, $a = 1 > 0$
$k^2 - 4(1)(9) < 0 \Rightarrow k^2 < 36$
$\Rightarrow -6 < k < 6$

28. **D.** $[-3, -1] \cup [1, 3]$
Explanation: $-4 \le x^2 - 5 \le 4 \Rightarrow 1 \le x^2 \le 9$
$x^2 \ge 1 \Rightarrow |x| \ge 1$ and $x^2 \le 9 \Rightarrow |x| \le 3$
$\therefore 1 \le |x| \le 3$, i.e. $x \in [-3, -1] \cup [1, 3]$

29. **C.** $(-2, 0) \cup (2, \infty)$
Explanation: $\frac{x+2}{x-2} - \frac{x-2}{x+2} > 0$
$\Rightarrow \frac{(x+2)^2-(x-2)^2}{(x-2)(x+2)} > 0$
$\Rightarrow \frac{8x}{(x-2)(x+2)} > 0$
Critical points: $-2, 0, 2$; signs: $(-\infty, -2)$: −, $(-2, 0)$: +, $(0, 2)$: −, $(2, \infty)$: +
$\therefore$ Solution: $(-2, 0) \cup (2, \infty)$

30. **C.** 10
Explanation: By AM $\ge$ GM: $\frac{x+\frac{25}{x}}{2} \ge \sqrt{x \cdot \frac{25}{x}} = 5$
$\Rightarrow x + \frac{25}{x} \ge 10$
Equality when $x = \frac{25}{x} \Rightarrow x = 5$
$\therefore$ Minimum value = 10

31. **C.** 24
Explanation: By AM $\ge$ GM: $\frac{3x+2y}{2} \ge \sqrt{3x \cdot 2y}$
$\Rightarrow 12 \ge \sqrt{6xy}$
$\Rightarrow 144 \ge 6xy$
$\Rightarrow xy \le 24$
Equality when $3x = 2y = 12 \Rightarrow x = 4,\ y = 6$
$\therefore$ maximum value = 24

32. **A.** $(1, 2)$
Explanation: At $(1, 2)$: $3(1) + 2 = 5 \le 6$
Also $x = 1 \ge 0$ and $y = 2 \ge 0$
All three conditions hold, so the point lies inside the region

33. **D.** 32
Explanation: The region is a right triangle with vertices $(0, 0), (8, 0), (0, 8)$
Area $= \frac{1}{2} \times 8 \times 8 = 32$

34. **B.** $\frac{9}{2}$
Explanation: Vertices: $(2, 2), (5, 2), (2, 5)$
It is a right triangle with base = 3 and height = 3
Area $= \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$

35. **A.** 18
Explanation: The region is a rhombus with vertices $(\pm 3, 0), (0, \pm 3)$
Its diagonals are $d_1 = d_2 = 6$
Area $= \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 6 \times 6 = 18$

36. **B.** 25
Explanation: $y = |x|$ and $y = 5$ meet at $(-5, 5), (5, 5)$
The region is a triangle with vertices $(0, 0), (5, 5), (-5, 5)$; base = 10, height = 5
Area $= \frac{1}{2} \times 10 \times 5 = 25$

37. **B.** 680
Explanation: Let there be x tables and y chairs
Constraints: $x + y \le 20$, $300x + 100y \le 3600 \Rightarrow 3x + y \le 36$, $x, y \ge 0$
Objective: maximise $z = 40x + 30y$
Intersection: $(3x + y) - (x + y) = 36 - 20 \Rightarrow x = 8,\ y = 12$
Corner points: $(0, 0), (12, 0), (8, 12), (0, 20)$
z: 0, 480, 680, 600
$\therefore$ Maximum profit = 680

38. **A.** $\frac{1}{2} \le k \le 2$
Explanation: Corner points: $(0, 0), \left(\frac{11}{2}, 0\right), (4, 3), (0, 5)$
$z(4, 3) = 4 + 3k,\ z\left(\frac{11}{2}, 0\right) = \frac{11}{2},\ z(0, 5) = 5k$
$z(4, 3) \ge z\left(\frac{11}{2}, 0\right)$: $4 + 3k \ge \frac{11}{2} \Rightarrow k \ge \frac{1}{2}$
$z(4, 3) \ge z(0, 5)$: $4 + 3k \ge 5k \Rightarrow k \le 2$
$\therefore \frac{1}{2} \le k \le 2$

39. **B.** 10
Explanation: Corner points of the region: $(2, 0), (0, 2), (-2, 0), (0, -2)$
$z(2, 0) = 4,\ z(0, 2) = 10,\ z(-2, 0) =$

$-4,\ z(0,-2) = -10$
$\therefore z_{max} = 10$ [at $(0, 2)$]

40. **B.** 22
Explanation: For each y, $0 \le x \le 9 - 3y$:
$y = 0$: $x = 0, \ldots, 9 \Rightarrow 10$ points
$y = 1$: $x = 0, \ldots, 6 \Rightarrow 7$ points
$y = 2$: $x = 0, \ldots, 3 \Rightarrow 4$ points
$y = 3$: $x = 0 \Rightarrow 1$ point
Total $= 10 + 7 + 4 + 1 = 22$

41. **C.** 13
Explanation: Intersection: $(x + 2y) - (x + y) = 8 - 5 \Rightarrow y = 3,\ x = 2$
Corner points: $(0, 0), (5, 0), (2, 3), (0, 4)$
z: $0,\ 10,\ 13,\ 12$
$\therefore z_{max} = 13$ [at $(2, 3)$]

42. **A.** 6
Explanation: Intersection: $x = 8 - 2y \Rightarrow 16 - 4y + y = 10 \Rightarrow y = 2,\ x = 4$
Corner points: $(0, 0), (5, 0), (4, 2), (0, 4)$
z: $0,\ 5,\ 6,\ 4$
$\therefore z_{max} = 6$

43. **B.** 14
Explanation: Intersection: $(x + 3y) - (x + y) = 10 - 6 \Rightarrow y = 2,\ x = 4$
Corner points: $(0, 6), (4, 2), (10, 0)$ [the region is unbounded]
z: $18,\ 14,\ 20$
The coefficients of z are positive, so $z_{min} = 14$ [at $(4, 2)$]

44. **D.** 9
Explanation: $x + \frac{16}{x-1} = (x - 1) + \frac{16}{x-1} + 1$
Since $x - 1 > 0$, by AM $\ge$ GM: $(x - 1) + \frac{16}{x-1} \ge 2\sqrt{16} = 8$
$\therefore x + \frac{16}{x-1} \ge 8 + 1 = 9$
Equality when $(x - 1)^2 = 16 \Rightarrow x = 5$
$\therefore$ minimum value $= 9$

45. **D.** 22
Explanation: $x = 4$ and $x + y = 6$ meet at $(4, 2)$
$y = 5$ and $x + y = 6$ meet at $(1, 5)$
Corner points: $(0, 0), (4, 0), (4, 2), (1, 5), (0, 5)$
z: $0,\ 16,\ 22,\ 19,\ 15$
$\therefore z_{max} = 22$ [at $(4, 2)$]

46. **A.** $\frac{17}{3}$
Explanation: Adding: $3x + 3y = 17 \Rightarrow x + y = \frac{17}{3}$
Subtracting: $(2x + y) - (x + 2y) = 1 \Rightarrow x - y = 1$, so $x = \frac{10}{3},\ y = \frac{7}{3}$
Corner points: $(0, 0), \left(\frac{9}{2}, 0\right), \left(\frac{10}{3}, \frac{7}{3}\right), (0, 4)$
z: $0,\ \frac{9}{2},\ \frac{17}{3},\ 4$
$\therefore z_{max} = \frac{17}{3}$

47. **C. Infinitely many solutions,** $z_{max} = 30$
Explanation: Intersection: $(3x + 2y) - (x + 2y) = 18 - 10 \Rightarrow x = 4,\ y = 3$
Corner points: $(0, 0), (6, 0), (4, 3), (0, 5)$
z: $0,\ 18,\ 30,\ 30$
$z = 3(x + 2y)$, so the objective line is parallel to $x + 2y = 10$
$\therefore$ every point of the segment joining $(0, 5)$ and $(4, 3)$ is optimal; $z_{max} = 30$

48. **A. z has no finite maximum value**
Explanation: $x - 2y \le 4 \Rightarrow y \ge \frac{x-4}{2}$
There is no upper bound on y, so the feasible region is unbounded
For example, $(0, k)$ is feasible for every $k \ge 0$ and $z = k$
As $k \to \infty$, $z \to \infty$; hence there is no finite maximum

49. **C.** 20
Explanation: The region is a rectangle formed by the lines $x - y = \pm 2$ and $x + y = \pm 5$
Distances between the parallel lines: $\frac{4}{\sqrt{2}} = 2\sqrt{2}$ and $\frac{10}{\sqrt{2}} = 5\sqrt{2}$
Area $= 2\sqrt{2} \times 5\sqrt{2} = 20$

50. **C.** 155
Explanation: Let x packets of A and y packets of B be used
Constraints: $2x + y \ge 10,\ x + 3y \ge 15,\ x, y \ge 0$; cost $z = 25x + 20y$
Intersection: $y = 10 - 2x \Rightarrow x + 30 - 6x = 15 \Rightarrow x = 3,\ y = 4$
Corner points: $(0, 10), (3, 4), (15, 0)$
z: $200,\ 155,\ 375$
$\therefore$ Minimum cost $= 155$