MCQ 50 marks

1. **D.** 6
Explanation: $11 + 6\sqrt{2} = (3 + \sqrt{2})^2$ and $11 - 6\sqrt{2} = (3 - \sqrt{2})^2$
Since $3 - \sqrt{2} > 0$, the expression $= (3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$

2. **B.** $\max S = 7$, $\inf S = 2$ **but** $\min S$ **does not exist**
Explanation: $(x - 2)(x - 4) < 0 \Rightarrow 2 < x < 4$
$\therefore S = (2, 4) \cup \{7\}$
$7 \in S$ and it is the largest element, so $\sup S = \max S = 7$
$\inf S = 2$, but $2 \notin S$, so $\min S$ does not exist

3. **B.** $\min S = 2$, $\sup S = 3 \notin S$
Explanation: $\frac{3n-1}{n} = 3 - \frac{1}{n}$, which increases as n increases
$S = \{2, \frac{5}{2}, \frac{8}{3}, \cdots\}$
2 is attained at $n = 1$, so $\min S = 2$
$\sup S = 3$, but $3 \notin S$

4. **D.** $x \leq -\frac{3}{2}$ **or** $x \geq 3$
Explanation: $4x - 3 \geq 9 \Rightarrow x \geq 3$
or, $4x - 3 \leq -9 \Rightarrow x \leq -\frac{3}{2}$

5. **A.** $x < \frac{5}{3}$ **or** $x > \frac{7}{3}$
Explanation: $x \neq 2$; $|x - 2| > 0$, so $|x - 2| > \frac{1}{3}$
$\Rightarrow x - 2 > \frac{1}{3}$ or $x - 2 < -\frac{1}{3}$
$\Rightarrow x > \frac{7}{3}$ or $x < \frac{5}{3}$ ($x = 2$ is excluded anyway)

6. **B.** $\left(-\frac{5}{3}, -\frac{1}{3}\right) \cup \left(1, \frac{7}{3}\right)$
Explanation: $-2 < |3x - 1| - 4 < 2 \Rightarrow 2 < |3x - 1| < 6$
$\Rightarrow 3x - 1 \in (-6, -2) \cup (2, 6)$
$\Rightarrow 3x \in (-5, -1) \cup (3, 7)$
$\Rightarrow x \in \left(-\frac{5}{3}, -\frac{1}{3}\right) \cup \left(1, \frac{7}{3}\right)$

7. **A.** 12
Explanation: $x < -2$: $-2x + 3 \leq 11 \Rightarrow x \geq -4$
$-2 \leq x \leq 5$: $(x + 2) + (5 - x) = 7 \leq 11$ (always true)
$x > 5$: $2x - 3 \leq 11 \Rightarrow x \leq 7$
Solution set $= [-4, 7]$; integers from -4 to 7 give 12

8. **D.** $k = 5$
Explanation: $2 \leq x \leq 7$: $(x - 2) + (7 - x) = 5$ (constant)
$x < 2$: $9 - 2x$ and $x > 7$: $2x - 9$, which are strictly monotonic
For $k = 5$ the whole interval $[2, 7]$ is a solution (infinitely many)
For $k > 5$ there are exactly two solutions; for $k < 5$ there is none

9. **A.** $x < -7$ **or** $x > -1$
Explanation: For $x \neq -2$: $|2x - 1| < 3|x + 2|$; squaring
$4x^2 - 4x + 1 < 9x^2 + 36x + 36$
$\Rightarrow 5x^2 + 40x + 35 > 0 \Rightarrow x^2 + 8x + 7 > 0$
$\Rightarrow (x + 1)(x + 7) > 0 \Rightarrow x < -7$ or $x > -1$
$x = -2$ lies outside this set, so no separate exclusion is needed

10. **B.** $[-3, 2]$
Explanation: $|x - 2| + |x + 3| = |2 - x| + |x + 3| \geq |(2 - x) + (x + 3)| = 5$
Equality holds when $(2 - x)(x + 3) \geq 0 \Rightarrow -3 \leq x \leq 2$

11. **D.** 10
Explanation: $-11 < 2x - 3 < 11 \Rightarrow -8 < 2x < 14$
$\Rightarrow -4 < x < 7$
Integers: $-3, -2, \ldots, 6 \Rightarrow 10$

12. **B.** $x < -2$ **or** $x > 5$
Explanation: $(x - 5)(x + 2) > 0$
$\therefore x < -2$ or $x > 5$

13. **C.** $|x^2 - 16| < 0.1604$
Explanation: $3.98 < x < 4.02 \Rightarrow 7.98 < x + 4 < 8.02$
$|x^2 - 16| = |x - 4|\,|x + 4| < 0.02 \times 8.02 = 0.1604$
$\therefore |x^2 - 16| < 0.1604$ [the value approaches 0.1604 as $x \to 4.02$, so 0.16 is not guaranteed]

14. **B.** $[-4, 12]$
Explanation: $x^2 + 4x = (x + 2)^2 - 4$
$x + 2 \in [-1, 4] \Rightarrow (x + 2)^2 \in [0, 16]$
$\Rightarrow x^2 + 4x \in [-4, 12]$

15. **A.** $\{-2\} \cup [1, 4]$

Explanation: At $x = -2$ the expression is 0, which satisfies $\leq 0 \Rightarrow x = -2$ is a solution
For $x \neq -2$, $(x+2)^2 > 0$; $(x-4)^3$ has the sign of $(x-4)$
$\therefore (x-1)(x-4) \leq 0 \Rightarrow 1 \leq x \leq 4$
Solution: $\{-2\} \cup [1, 4]$

16. **B.** $[-2, -1) \cup [4, \infty)$
Explanation: $\frac{(x-4)(x+2)}{x+1} \geq 0$; critical points: $-2, -1, 4$
Signs: $(-\infty, -2)$: $-$, $(-2, -1)$: $+$, $(-1, 4)$: $-$, $(4, \infty)$: $+$
$x = -2, 4$ are allowed (zero numerator), $x = -1$ is excluded (zero denominator)
$\therefore$ Solution: $[-2, -1) \cup [4, \infty)$

17. **B.** $(-\infty, -5) \cup (-5, 5)$
Explanation: $\frac{2x}{(x-5)(x+5)} - \frac{1}{x+5} \leq 0$
$\Rightarrow \frac{2x-(x-5)}{(x-5)(x+5)} \leq 0 \Rightarrow \frac{x+5}{(x-5)(x+5)} \leq 0$
For $x \neq -5$: $\frac{1}{x-5} \leq 0 \Rightarrow x < 5$
But $x = -5$ makes the original expression undefined
$\therefore$ Solution: $(-\infty, -5) \cup (-5, 5)$

18. **C.** $[-5, -3] \cup [3, 5]$
Explanation: Since $x^2 = |x|^2$: $(|x| - 3)(|x| - 5) \leq 0$
$\Rightarrow 3 \leq |x| \leq 5$
$\Rightarrow x \in [-5, -3] \cup [3, 5]$

19. **C.** $|x| \leq 1$ **or** $|x| \geq \sqrt{5}$
Explanation: $x^2 - 3 \geq 2 \Rightarrow x^2 \geq 5 \Rightarrow |x| \geq \sqrt{5}$
or, $x^2 - 3 \leq -2 \Rightarrow x^2 \leq 1 \Rightarrow |x| \leq 1$

20. **A.** $x > 6$
Explanation: Conditions: $x + 10 \geq 0$ and $x - 2 > 0 \Rightarrow x > 2$
Squaring: $x + 10 < x^2 - 4x + 4 \Rightarrow x^2 - 5x - 6 > 0$
$\Rightarrow (x-6)(x+1) > 0 \Rightarrow x < -1$ or $x > 6$
Intersection with $x > 2$: $x > 6$

21. **D.** $(-\infty, -3] \cup \left(\frac{13}{4}, \infty\right)$
Explanation: Domain: $x^2 \geq 9 \Rightarrow x \leq -3$ or $x \geq 3$
$x \leq -3$: the right side $x - 2 < 0$, always true
$x \geq 3$: squaring gives $x^2 - 9 > x^2 - 4x + 4 \Rightarrow x > \frac{13}{4}$
$\therefore$ Solution: $(-\infty, -3] \cup \left(\frac{13}{4}, \infty\right)$

22. **D.** $1 \leq m < 4$
Explanation: For $m = 1$ the expression is $3 > 0$ (always true)
For $m \neq 1$: $m - 1 > 0$ and discriminant < 0
$4(m-1)^2 - 12(m-1) < 0 \Rightarrow (m-1)(m-4) < 0 \Rightarrow 1 < m < 4$
Combining: $1 \leq m < 4$

23. **C.** 9
Explanation: $x + \frac{108}{x^2} = \frac{x}{2} + \frac{x}{2} + \frac{108}{x^2}$
By AM $\geq$ GM: $\frac{x}{2} + \frac{x}{2} + \frac{108}{x^2} \geq$
$3\sqrt[3]{\frac{x}{2} \cdot \frac{x}{2} \cdot \frac{108}{x^2}} = 3\sqrt[3]{27} = 9$
Equality when $\frac{x}{2} = \frac{108}{x^2} \Rightarrow x = 6$
$\therefore$ minimum value = 9

24. **A.** $-6 \leq a \leq 6$
Explanation: No solution means $x^2 - ax + 9 \geq 0$ for all x
Condition: discriminant $\leq 0 \Rightarrow a^2 - 36 \leq 0 \Rightarrow -6 \leq a \leq 6$
For $a = \pm 6$ the expression is a perfect square and never negative, so these values are included

25. **D.** 60
Explanation: By AM $\geq$ GM: $4a + 9b \geq 2\sqrt{36ab} = 2\sqrt{900} = 60$
Equality when $4a = 9b$ and $ab = 25 \Rightarrow a = \frac{15}{2}$, $b = \frac{10}{3}$
$\therefore$ minimum value = 60

26. **C.** $7 + 4\sqrt{3}$
Explanation: $x + y = (x + y)\left(\frac{3}{x} + \frac{4}{y}\right) = 7 + \frac{4x}{y} + \frac{3y}{x}$
By AM $\geq$ GM: $\frac{4x}{y} + \frac{3y}{x} \geq 2\sqrt{12} = 4\sqrt{3}$
$\therefore x + y \geq 7 + 4\sqrt{3}$

27. **B.** $|x^2 - 2x + 3| < 18$
Explanation: Triangle inequality: $|x^2 - 2x + 3| \leq |x|^2 + 2|x| + 3 < 9 + 6 + 3 = 18$
As $x \to -3$ the value $\to 9 + 6 + 3 = 18$; so the bounds 6, 9, 12 can be exceeded

28. **C.** 20
Explanation: Axis intercepts: $(4, 0)$ and $(0, 10)$
Area $= \frac{1}{2} \times 4 \times 10 = 20$

29. **D.** 28

Explanation: Large triangle ($x + y \leq 9$): $\frac{1}{2} \times 9 \times 9 = \frac{81}{2}$
Small triangle ($x + y < 5$): $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$
Required area = $\frac{81}{2} - \frac{25}{2} = 28$

30. **B.** 42
Explanation: The area of $|x| + |y| \leq a$ is $2a^2$
Outer rhombus ($a = 5$): 50; inner rhombus ($a = 2$): 8
Required area = $50 - 8 = 42$

31. **D.** $\frac{25}{2}$
Explanation: $y = |x - 1|$ and $y = 5 - |x - 1|$ meet where $|x - 1| = \frac{5}{2}$
The region is a square centred on $x = 1$; vertical diagonal = 5, horizontal diagonal = 5
Area = $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$

32. **A.** 21
Explanation: For each y, $2 \leq x \leq 8 - y$:
$y = 1$: 6 points; $y = 2$: 5; $y = 3$: 4; $y = 4$: 3; $y = 5$: 2; $y = 6$: 1
Total = $6 + 5 + 4 + 3 + 2 + 1 = 21$

33. **C.** 7
Explanation: $z(1, 3) = 10,\ z(4, 1) = 7,\ z(6, 4) = 18,\ z(3, 6) = 21$
$\therefore z_{min} = 7$ [at $(4, 1)$]

34. **A.** 17
Explanation: Intersection: $(2x + y) - (x + y) = 10 - 7 \Rightarrow x = 3,\ y = 4$
Corner points: $(0, 0), (5, 0), (3, 4), (0, 7)$
z: 0, 15, 17, 14
$\therefore z_{max} = 17$

35. **D.** 460
Explanation: Let x bags of A and y bags of B be made
Constraints: $2x + 3y \leq 24$, $3x + 2y \leq 26$, $x, y \geq 0$; profit $z = 50x + 40y$
Intersection: $x = 6,\ y = 4$ [$2(6) + 3(4) = 24$, $3(6) + 2(4) = 26$]
Corner points: $(0, 0), \left(\frac{26}{3}, 0\right), (6, 4), (0, 8)$
z: 0, $\frac{1300}{3}$, 460, 320
$\therefore$ Maximum profit = 460

36. **D.** $1 < k < 3$
Explanation: $z(5, 3) = 15 + 3k,\ z(6, 0) = 18,\ z(0, 8) = 8k$
For a unique maximum the inequalities must be strict:
$15 + 3k > 18 \Rightarrow k > 1$
$15 + 3k > 8k \Rightarrow k < 3$
$\therefore 1 < k < 3$ [for $k = 1$ or 3 there are infinitely many solutions along an edge]

37. **A.** 4
Explanation: The segment lies on $x + y = 7$; the objective line must be parallel to it
$\frac{4}{1} = \frac{p}{1} \Rightarrow p = 4$
Check: $z = 4(x + y)$; $z(4, 3) = z(0, 7) = 28$, $z\left(\frac{11}{2}, 0\right) = 22$, $z(0, 0) = 0 \Rightarrow z_{max} = 28$

38. **C.** 12
Explanation: Intersection: adding gives $2x = 10 \Rightarrow x = 5,\ y = 3$
Corner points: $(0, 0), (2, 0), (5, 3), (0, 8)$
z: 0, 6, 12, −8
$\therefore z_{max} = 12$ [at $(5, 3)$]

39. **A.** 8
Explanation: The feasible region is a small triangle with corner points $(3, 3), (4, 2), (4, 3)$
z: 9, 8, 10
$\therefore z_{min} = 8$ [at $(4, 2)$]

40. **C.** 1020
Explanation: Intersection: $(3x + y) - (x + y) = 540 - 300 \Rightarrow x = 120,\ y = 180$
Corner points:
$(0, 0), (180, 0), (120, 180), (0, 300)$
z: 0, 720, 1020, 900
$\therefore z_{max} = 1020$

41. **A.** 25
Explanation: Adding: $3x + 3y = 21 \Rightarrow x + y = 7$; subtracting: $x - y = 1$; so the intersection is $(4, 3)$
Corner points: $(0, 11), (4, 3), (10, 0)$ [the region is unbounded]
z: 33, 25, 40
The coefficients of z are positive, so $z_{min} = 25$ [at $(4, 3)$]

42. **C.** $\frac{170}{7}$
Explanation: Intersection: subtracting $4x + 3y = 24$ from $4x + 10y = 40$ gives $7y = 16 \Rightarrow y = \frac{16}{7},\ x = \frac{30}{7}$
Corner points: $(0, 0), (6, 0), \left(\frac{30}{7}, \frac{16}{7}\right), (0, 4)$

z: 0, 18, $\frac{90+80}{7} = \frac{170}{7}$, 20
$\therefore z_{max} = \frac{170}{7}$ (≈ 24.29)

43. **A.** 39000
Explanation: Let x acres be used for A and y acres for B
Constraints: $x + y \leq 12$, $3x + y \leq 24$, $x, y \geq 0$; profit $z = 4000x + 2500y$
Intersection: subtracting gives $2x = 12 \Rightarrow x = 6,\ y = 6$
Corner points: $(0, 0), (8, 0), (6, 6), (0, 12)$
z: 0, 32000, 39000, 30000
$\therefore$ Maximum profit = 39000

44. **D.** 160
Explanation: Let x units of P and y units of Q be used
Constraints: $3x + y \geq 10$, $x + 2y \geq 10$, $x, y \geq 0$; cost $z = 20x + 30y$
Intersection: $y = 10 - 3x \Rightarrow x + 20 - 6x = 10 \Rightarrow x = 2,\ y = 4$
Corner points: $(0, 10), (2, 4), (10, 0)$
z: 300, 160, 200
$\therefore$ Minimum cost = 160

45. **C.** 16
Explanation: For each y, $y \leq x \leq 6 - y$:
$y = 0$: $x = 0, \ldots, 6 \Rightarrow 7$ points
$y = 1$: $x = 1, \ldots, 5 \Rightarrow 5$ points
$y = 2$: $x = 2, 3, 4 \Rightarrow 3$ points
$y = 3$: $x = 3 \Rightarrow 1$ point
Total = $7 + 5 + 3 + 1 = 16$

46. **B.** $(3, 4)$
Explanation: At $(3, 4)$: $x + y = 7 \geq 5$
$3x - y = 9 - 4 = 5 \leq 6$
$y = 4 \leq 5$
All three conditions hold, so the point lies inside the region

47. **C.** 23
Explanation: Corner points:
$(0, 0), (5, 0), (5, 4), (0, 4)$
z: 0, 15, 23, 8
$\therefore z_{max} = 23$ [at $(5, 4)$]

48. **A.** $[4, 5]$
Explanation: $|x - 2| \leq 3 \Rightarrow [-1, 5]$ and $|x - 6| \leq 2 \Rightarrow [4, 8]$
Intersection: $[4, 5]$

49. **B.** $(-2, 0) \cup (2, \infty)$
Explanation: $\frac{4}{x} - x < 0 \Rightarrow \frac{4-x^2}{x} < 0 \Rightarrow \frac{(x-2)(x+2)}{x} > 0$
Critical points: $-2, 0, 2$; signs: $(-\infty, -2)$: −, $(-2, 0)$: +, $(0, 2)$: −, $(2, \infty)$: +
$\therefore$ Solution: $(-2, 0) \cup (2, \infty)$

50. **B.** 6
Explanation: $\frac{(x-2)(x+2)(x-6)}{x+1} \leq 0$; critical points: $-2, -1, 2, 6$
Signs: $(-\infty, -2)$: +, $(-2, -1)$: −, $(-1, 2)$: +, $(2, 6)$: −, $(6, \infty)$: +
Solution: $[-2, -1) \cup [2, 6]$ ($x = -1$ excluded)
Integers: $-2,\ 2, 3, 4, 5, 6 \Rightarrow 6$