Engineering Daily 21 [Set 04] EV

Functions and Graphs of Functions + Permutations and Combinations

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. What is the domain of $f(x) = \sqrt{\log_{x^2-1}\left(\frac{x^2-4}{x^2-9}\right)}$?

(A) $(-\infty,-3) \cup (-1,1) \cup (3,\infty)$

(B) $(-\sqrt{2},-1) \cup (1,\sqrt{2})$

(C) $(-\infty,-3] \cup (-\sqrt{2},-1) \cup (1,\sqrt{2}) \cup [3,\infty)$

(D) $(-\infty,-3) \cup (-\sqrt{2},-1) \cup (1,\sqrt{2}) \cup (3,\infty)$

2. If $f(x) = \frac{x^2-1}{x^2+1} + \frac{x^2+1}{x^2-1}$, $x \in \mathbb{R} \setminus \{-1,1\}$, what is the range?

(A) $(-\infty,-2] \cup [2,\infty)$ (B) $(-\infty,-2] \cup (2,\infty)$

(C) $(-\infty,-2) \cup (2,\infty)$ (D) $(-\infty,-2) \cup [2,\infty)$

3. If $f(x) = \sqrt{x^2-4x+5} + \sqrt{x^2+4x+5}$, $x \in \mathbb{R}$, what is the range?

(A) $(2\sqrt{5},\infty)$ (B) $[2\sqrt{5},\infty)$ (C) $[2,\infty)$ (D) $[4,\infty)$

4. For $x \in \left[\frac{1}{4},8\right]$, the equation $(\log_2 x)^2 - 4|\log_2 x| + 3 = k$ has exactly three distinct solutions. Which gives all values of k?

(A) $-1 < k < 3$ (B) $-1 \le k \le 0$

(C) $0 < k < 3$ (D) $-1 < k \le 0$

5. For all real x, y, $f(x+y) + f(x-y) = 2f(x)f(y)$, where $f : \mathbb{R} \to \mathbb{R}$. If $f(0) = 1$ and $f(1) = 3$, what is $f\left(\frac{1}{4}\right)$?

(A) $\sqrt{\frac{1+\sqrt{2}}{2}}$ (B) $-\sqrt{\frac{1+\sqrt{2}}{2}}$ (C) $\sqrt{1+\sqrt{2}}$ (D) $\sqrt{\frac{\sqrt{2}-1}{2}}$

6. Let $f : \mathbb{Z}_{\ge 0} \to \mathbb{R}$ with $f(0) = 2$ and $f(n+1) = -\frac{1}{1+f(n)}$. What is $\sum_{n=0}^{2026} f(n)$?

(A) $\frac{679}{6}$ (B) $\frac{685}{6}$ (C) $\frac{675}{6}$ (D) $\frac{2027}{6}$

7. A polynomial P with real coefficients satisfies $P(x^2) = P(x)^2$ and $P(2) = 16$. What is the degree of P?

(A) 4 (B) 16 (C) 8 (D) 2

8. f is a polynomial of degree at most 3. For all real x, $f(x) + f(1-x) = 2$ and $f(x) - f(-x) = 2x^3 + 4x$. What is $f(2)$?

(A) $\frac{27}{4}$ (B) $\frac{21}{4}$ (C) $\frac{23}{4}$ (D) $\frac{25}{4}$

9. For how many continuous functions $f : \mathbb{R} \to \mathbb{R}$ does $f(f(x)) = -x$ hold for all real x?

(A) 0 (B) 1 (C) ∞ (D) 2

10. f is a real quadratic polynomial and $f(f(x)) = 8x^4 - 24x^3 + 20x^2 - 3x$. What is the minimum value of $f(x)$?

(A) $\frac{1}{8}$ (B) $-\frac{1}{8}$ (C) -1 (D) $-\frac{1}{4}$

11. For which values of a do both distinct real roots of $x^2 - 2ax + a + 1 = 0$ lie in the interval $(0,3)$?

(A) $\frac{1+\sqrt{5}}{2} \le a < 2$ (B) $1 < a < 3$

(C) $\frac{1+\sqrt{5}}{2} < a \le 2$ (D) $\frac{1+\sqrt{5}}{2} < a < 2$

12. Let $f_a(x) = x^2 - 2ax$ on $-1 \le x \le 2$. If $L(a)$ is the difference between the maximum and minimum values of f_a on this interval, what is $\min_{a\in\mathbb{R}} L(a)$?

(A) $\frac{3}{2}$ (B) $\frac{9}{4}$ (C) 3 (D) $\frac{9}{2}$

13. For how many continuous functions $f : \mathbb{R} \to \mathbb{R}$ do we have $f(x)^2 = (x^2-1)^2$ and $f(0) = 1$?

(A) 8 (B) 2 (C) 4 (D) 16

14. For which values of k does $|x^3 - 3x| = k$ have exactly six distinct real solutions?

(A) $0 \le k < 2$ (B) $0 < k < 2$

(C) $k > 2$ (D) $0 < k \le 2$

15. Let $f(x) = \frac{x^2-3x+2}{x^2-5x+6}$. Keeping the natural domain of the original expression, what is the range of $f(f(x))$?

(A) $\mathbb{R} \setminus \left\{-1, 0, \frac{1}{2}, 1\right\}$ (B) $\mathbb{R} \setminus \{0\}$

(C) $\mathbb{R} \setminus \left\{0, \frac{1}{2}, 1\right\}$ (D) $\mathbb{R} \setminus \{-1, 0, 1\}$

16. Let $a > 1$. If the graphs of $y = a^x$ and $y = \log_a x$ have exactly one common point, what is a?

(A) $e^{\frac{1}{2}}$ (B) $e^{\frac{1}{e}}$ (C) $\sqrt{2}$ (D) e

17. Let $f(x) = x^3 + x$, $x \in \mathbb{R}$. What is the product of all real solutions of $f^{-1}(t) + f^{-1}(8-t) = 2$?

(A) $\frac{16}{3}$ (B) $\frac{40}{27}$ (C) $\frac{74}{27}$ (D) $\frac{20}{9}$

18. Let $f(x) = \sqrt{x + \sqrt{x^2+1}}$. What is the product of all real solutions of $f(x) + f(-x) = 3$?

(A) $-\frac{15}{4}$ (B) $-\frac{5}{4}$ (C) $-\frac{45}{4}$ (D) $\frac{45}{4}$

19. Let $f : [-1,1] \to [0,1]$, $f(x) = \sqrt{1-x^2}$. What is the sum of all real solutions of $f^{\circ 2026}(x) = \frac{x}{2} + \frac{1}{4}$? (Here $f^{\circ 2026}$ means f composed with itself 2026 times)

(A) $\frac{1}{6}$ (B) 0 (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

20. Let $f(x) = \frac{\ln x}{1+(\ln x)^2}$, $x > 0$. What is the product of all real solutions of $f(x) = f\left(\frac{e^3}{x}\right)$?

(A) $e^{\frac{9}{2}}$ (B) e^3 (C) e^6 (D) $e^{\frac{3}{2}}$

21. Let $a > 0$. For which values of a does $|x-1| + |x-2| + |x-4| = ax$ have exactly two distinct real solutions?

(A) $a > \frac{5}{4}$ (B) $\frac{5}{4} < a \le 3$

(C) $\frac{5}{4} < a < 3$ (D) $\frac{5}{4} \le a < 3$

22. $f(x) = x^2$ when x is rational and $f(x) = 2 - x$ when x is irrational. What is the sum of the real points at which the function is continuous?

Ⓐ 1 Ⓑ -2 Ⓒ 3 Ⓓ -1

23. f is an even function and $f(x+3) = -f(x)$ for all real x. If $f(1) = 2$, what is $\sum_{k=0}^{2026} f(k)$?

Ⓐ 0 Ⓑ 4 Ⓒ 2 Ⓓ -2

24. For how many polynomials P with real coefficients does $P(P(x)) = (P(x))^2$ hold for all real x?

Ⓐ 3 Ⓑ 2 Ⓒ 4 Ⓓ 1

25. Using each of the digits $0, 0, 1, 1, 2, 2, 3, 3$ exactly once, how many distinct eight-digit numbers divisible by 11 can be formed?

Ⓐ 540 Ⓑ 504 Ⓒ 432 Ⓓ 486

26. Using the digits $0, 1, 2, 3, 4, 5, 6$ without repetition, how many six-digit numbers divisible by 15 can be formed?

Ⓐ 648 Ⓑ 432 Ⓒ 552 Ⓓ 576

27. 10 distinct persons are to sit around a round table. A, B must be adjacent, but no two of C, D, E may be adjacent. Counting arrangements that differ only by rotation as the same, how many arrangements are possible?

Ⓐ 43200 Ⓑ 28800 Ⓒ 14400 Ⓓ 57600

28. 5 couples are to sit around a round table with men and women alternating. No person may sit next to their own spouse. Counting arrangements that differ only by rotation as the same, how many arrangements are there?

Ⓐ 240 Ⓑ 480 Ⓒ 312 Ⓓ 624

29. The parabola $y = 2x^2$ and the line $y = mx - 2$ have exactly one common point. What are the possible values of m?

Ⓐ $m = 0$ Ⓑ $m = 4$ Ⓒ $m = \pm 2$ Ⓓ $m = \pm 4$

30. In how many distinct ways can the letters of the word CABANA be arranged so that no two A's are adjacent?

Ⓐ 36 Ⓑ 24 Ⓒ 48 Ⓓ 12

31. In how many distinct ways can the letters of the word ASSESSMENT be arranged keeping all the vowels together?

Ⓐ 1680 Ⓑ 10080 Ⓒ 2520 Ⓓ 5040

32. In how many distinct ways can the letters of the word STATISTICS be arranged so that no two vowels are adjacent?

Ⓐ 23520 Ⓑ 47040 Ⓒ 11760 Ⓓ 17640

33. Using the digits $0, 1, 2, 3, 4, 5, 6$ without repetition, how many six-digit numbers divisible by 5 can be formed?

Ⓐ 1440 Ⓑ 1200 Ⓒ 1320 Ⓓ 1260

34. In how many distinct ways can the letters of the word ENGINEERING be arranged so that no two vowels are adjacent?

Ⓐ 12600 Ⓑ 15120 Ⓒ 6300 Ⓓ 25200

35. How many one-one onto functions $f : A \to A$, where $A = \{1, 2, \ldots, 8\}$, satisfy $f(x) \neq x$ for every x and have exactly two unordered pairs $\{a, b\}$ with $f(a) = b$ and $f(b) = a$?

Ⓐ 1260 Ⓑ 840 Ⓒ 1680 Ⓓ 2520

36. 12 distinct students are to be divided into three teams named X, Y, Z with 4 students each. A, B must be in the same team and C, D in different teams. In how many ways can this be done?

Ⓐ 2520 Ⓑ 9450 Ⓒ 6930 Ⓓ 6720

37. 14 identical pens are to be distributed among four distinct students. Each gets at least 1 and at most 5, and the first student gets an odd number. What is the total number of distributions?

Ⓐ 45 Ⓑ 50 Ⓒ 36 Ⓓ 40

38. How many positive integer solutions does $x + y + z = 30$ have with $x < y < z$?

Ⓐ 61 Ⓑ 75 Ⓒ 60 Ⓓ 56

39. Five numbers are to be chosen from $\{1, 2, \ldots, 15\}$ such that no two are consecutive integers and the sum is divisible by 3. How many selections are there?

Ⓐ 162 Ⓑ 155 Ⓒ 153 Ⓓ 154

40. Moving one unit right or up at each step from $(0, 0)$ to $(6, 6)$, how many paths never enter the region $y > x$ and return to the line $y = x$ at exactly two points other than the start and the end?

Ⓐ 28 Ⓑ 42 Ⓒ 25 Ⓓ 35

41. How many binary $(0, 1)$ sequences of length 12 have exactly six 1's, with the 1's forming exactly three blocks, each block containing at least two 1's, and with no three consecutive 0's anywhere?

Ⓐ 16 Ⓑ 12 Ⓒ 8 Ⓓ 35

42. How many different circular necklaces can be made from four identical red and four identical blue beads? Necklaces that coincide after rotation or flipping are counted once.

Ⓐ 10 Ⓑ 12 Ⓒ 8 Ⓓ 16

43. By choosing three vertices of a regular dodecagon, how many acute-angled triangles can be formed?

Ⓐ 40 Ⓑ 60 Ⓒ 36 Ⓓ 44

44. Among 10 distinct gifts, four are A, B, C, D. Exactly two of four distinct children receive gifts, and the numbers they receive are unequal. If A, B go to different children and C, D go to the same child, what is the total number of distributions?

Ⓐ 882 Ⓑ 2352 Ⓒ 1176 Ⓓ 1536

45. How many functions $f : A \to A$, where $A = \{1, 2, \ldots, 9\}$, satisfy $f(f(f(x))) = x$ and have $f(x) = x$ for exactly three elements?

Ⓐ 6720 Ⓑ 2240 Ⓒ 1680 Ⓓ 3360

46. The letters A, B, C are to be written in a 3×3 grid so that each row contains each letter exactly once and no column has all three letters the same. How many arrangements are possible?

Ⓐ 144 Ⓑ 156 Ⓒ 162 Ⓓ 180

47. How many six-digit numbers read the same from the front and from the back and are divisible by 36?

Ⓐ 20 Ⓑ 18 Ⓒ 22 Ⓓ 24

48. 8 distinct persons are to be divided into four unnamed groups of sizes $1, 2, 2, 3$. C, D must be in the same group and A, B in different groups. How many divisions are possible?

Ⓐ 122 Ⓑ 132 Ⓒ 126 Ⓓ 120

49. What is the value of $\sum_{r=0}^{10}(-1)^r \left({}^{10}C_r\right)^2$?

Ⓐ 252 Ⓑ -252 Ⓒ -126 Ⓓ 0

50. Let $A = \{1, 2, \ldots, 8\}$ and $B = \{a, b, c, d\}$. For how many onto functions $f : A \to B$ do we have $f(1) = f(2)$ and every element of B has an odd number of elements in its pre-image?

Ⓐ 1440 Ⓑ 1920 Ⓒ 8064 Ⓓ 2400