Engineering Daily 21 [Set 03] EV

Functions and Graphs of Functions + Permutations and Combinations

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. What is the domain of the function $f(x) = \sqrt{\frac{x-1}{x+2}} + \log_2(4-x)$?

(A) $[1,4)$ (B) $(-\infty,-2) \cup [1,4)$

(C) $(-2,1] \cup (1,4)$ (D) $(-\infty,-2] \cup [1,4)$

2. What is the domain of the function $f(x) = \log_{x-1}(5-x)$?

(A) $(1,2) \cup (2,5]$ (B) $(2,5)$

(C) $(1,2) \cup (2,5)$ (D) $(1,5)$

3. If $f(x) = \frac{x^2-4x+7}{x^2-4x+5}$, $x \in \mathbb{R}$, what is the range of the function?

(A) $(1,3]$ (B) $[1,3]$ (C) $[3,\infty)$ (D) $(1,2]$

4. What is the range of the function $f(x) = \sqrt{x-1} + \sqrt{5-x}$?

(A) $[2,4]$ (B) $[2,2\sqrt{2}]$ (C) $[\sqrt{2},2]$ (D) $[0,2\sqrt{2}]$

5. If $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R} \setminus \{2\}$ and $f(x) = \frac{2x-3}{x+1}$, which of the following is $f^{-1}(x)$?

(A) $\frac{3-x}{x+2}$ (B) $\frac{x+3}{x-2}$ (C) $\frac{x-3}{x+2}$ (D) $\frac{x+3}{2-x}$

6. If $f(x) = \sqrt{3-x}$ and $g(x) = \frac{1}{x-2}$, what is the domain of $(f \circ g)(x)$?

(A) $(2, \frac{7}{3}]$ (B) $(-\infty,2] \cup [\frac{7}{3},\infty)$

(C) $(-\infty,2) \cup (\frac{7}{3},\infty)$ (D) $(-\infty,2) \cup [\frac{7}{3},\infty)$

7. If $f(x) + 2f(1-x) = 3x^2$ for all real x, which of the following is $f(x)$?

(A) $x^2 - 4x + 2$ (B) $x^2 + 4x - 2$

(C) $3x^2 - 2x + 1$ (D) $2x^2 - 4x + 1$

8. For $x \neq -1$, if $f\left(\frac{x-1}{x+1}\right) = \frac{x^2+1}{(x+1)^2}$, what is $f(2)$?

(A) 5 (B) $\frac{5}{4}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$

9. $f(x) = \frac{ax+3}{2x-1}$ is a non-constant function. Wherever the composite function is defined, $f(f(x)) = x$. What is a?

(A) 3 (B) -1 (C) 1 (D) -6

10. If $f : [2,\infty) \to [3,\infty)$ and $f(x) = x^2 - 4x + 7$, which of the following is $f^{-1}(x)$?

(A) $-2 + \sqrt{x-3}$ (B) $2 - \sqrt{x-3}$

(C) $2 + \sqrt{x+3}$ (D) $2 + \sqrt{x-3}$

11. The point $(2,-1)$ on the graph of $y = f(x)$ is transformed to which point on the graph of $y = -2f(3-x) + 4$?

(A) $(-1,6)$ (B) $(5,6)$

(C) $(1,2)$ (D) $(1,6)$

12. The line $y = k$ meets the graph of $y = |x^2 - 4x + 3|$ at four distinct points. In which interval does k lie?

(A) $0 < k < 1$ (B) $0 \leq k \leq 1$

(C) $0 < k \leq 1$ (D) $k > 1$

13. If the equation $\big||x| - 2\big| = a$ has exactly three distinct real solutions, what is a?

(A) 1 (B) 2 (C) 0 (D) 3

14. What is the sum of the squares of the x-coordinates of the intersection points of the graphs $y = x^2$ and $y = 2|x| + 3$?

(A) 18 (B) 9 (C) 6 (D) 10

15. If $f(x) = x^2 - 2ax + 3$ is one-one on the interval $[2,\infty)$, which is the set of all possible values of a?

(A) $\{2\}$ (B) $(-\infty,2]$

(C) $[2,\infty)$ (D) $(-\infty,2)$

16. If $f(x) = \frac{x}{1+|x|}$, $x \in \mathbb{R}$, what is the range of the function?

(A) $(-1,1)$ (B) $[0,1)$

(C) $[-1,1]$ (D) $\mathbb{R} \setminus \{-1,1\}$

17. If $f : \mathbb{R} \to (-1,1)$ and $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$, which of the following is $f^{-1}(x)$?

(A) $\frac{1}{2}\log_2\left(\frac{1-x}{1+x}\right)$ (B) $\frac{1}{2}\log_2\left(\frac{1+x}{1-x}\right)$

(C) $\log_2\left(\frac{1+x}{1-x}\right)$ (D) $\frac{1}{2}\log_2\left(\frac{x+1}{x-1}\right)$

18. What is the range of the function $f(x) = \log_2 x + \log_x 2$?

(A) $[2,\infty)$ (B) $(-\infty,-2) \cup (2,\infty)$

(C) $(-\infty,-2] \cup [2,\infty)$ (D) $\mathbb{R} \setminus \{0\}$

19. If $f(x) = x^2 - 1$ and $g(x) = 2x + 1$, what is the product of the real roots of the equation $(f \circ g)(x) = (g \circ f)(x)$?

(A) 2 (B) -2 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

20. For all real x, $f(x) + f(-x) = 2x^2$ and $f(x) - f(-x) = 4x$. What is the range of f?

(A) $[-1,\infty)$ (B) $(-1,\infty)$

(C) $[0,\infty)$ (D) $\mathbb{R}$

21. The line $y = k$ meets the graph of $y = x^4 - 4x^2$ at exactly three distinct points. What is k?

(A) -2 (B) -4 (C) 4 (D) 0

22. Let $f(x) = \sqrt{4-(x-1)^2}$ and $g(x) = |f(x) - 1|$. How many intersection points are there between the graph of $y = g(x)$ and the line $y = \frac{1}{2}$?

(A) 6 (B) 3 (C) 4 (D) 2

23. If $f(x) = |x^2 - 1| + |x^2 - 4|$, $x \in \mathbb{R}$, what is the range of the function?
Ⓐ $(3, \infty)$ Ⓑ $[0, \infty)$ Ⓒ $[3, \infty)$ Ⓓ $[3, 5]$

24. What is the range of $f(x) = \frac{x^2-1}{x-1}$ over its natural domain?
Ⓐ $\mathbb{R}$ Ⓑ $\mathbb{R} \setminus \{2\}$ Ⓒ $\mathbb{R} \setminus \{1\}$ Ⓓ $\mathbb{R} \setminus \{-1\}$

25. The parabola $y = x^2$ and the line $y = mx - 1$ have exactly one common point. What are the possible values of m?
Ⓐ $m = \pm 2$ Ⓑ $m = \pm 1$ Ⓒ $m = 2$ Ⓓ $m = 0$

26. In how many distinct ways can the letters of the word BANANA be arranged so that no two A's are adjacent?
Ⓐ 36 Ⓑ 12 Ⓒ 60 Ⓓ 24

27. In how many distinct ways can the letters of the word MISSISSIPPI be arranged keeping all the vowels together?
Ⓐ 1680 Ⓑ 420 Ⓒ 5040 Ⓓ 840

28. In how many distinct ways can the letters of the word MATHEMATICS be arranged so that no two vowels are adjacent?
Ⓐ 529200 Ⓑ 2116800
Ⓒ 1058400 Ⓓ 151200

29. Using the digits $0, 1, 2, 3, 4, 5$ without repetition, how many five-digit numbers divisible by 5 can be formed?
Ⓐ 240 Ⓑ 192 Ⓒ 216 Ⓓ 120

30. Among the six-digit numbers formed by using each of the digits $1, 2, 3, 4, 5, 6$ exactly once, how many are divisible by 4?
Ⓐ 144 Ⓑ 360 Ⓒ 240 Ⓓ 192

31. In how many ways can 5 boys and 4 girls be seated in a row so that no two girls sit together? (All are distinct persons)
Ⓐ 86400 Ⓑ 8640 Ⓒ 43200 Ⓓ 2880

32. In how many ways can 6 distinct persons be seated around a round table so that two specified persons do not sit next to each other? (Arrangements that differ only by rotation count as one)
Ⓐ 72 Ⓑ 48 Ⓒ 96 Ⓓ 120

33. From 6 men and 5 women, how many committees of 5 members can be formed with at least 2 men and at least 2 women?
Ⓐ 450 Ⓑ 350 Ⓒ 462 Ⓓ 300

34. Among 8 persons are A and B. A committee of 4 is formed containing exactly one of them, and then a president and a secretary are chosen from that committee. In how many ways can this be done?
Ⓐ 480 Ⓑ 960
Ⓒ 40 Ⓓ 240

35. Among 10 persons, four are A, B, C, D. How many committees of 5 members can be formed such that A, B are not together and C, D are also not together?
Ⓐ 152 Ⓑ 146 Ⓒ 196 Ⓓ 140

36. In how many ways can 12 distinct books be divided into three unnamed groups of 4 each?
Ⓐ 11550 Ⓑ 5775 Ⓒ 34650 Ⓓ 17325

37. In how many ways can four numbers be chosen from $\{1, 2, \ldots, 10\}$ so that no two of the chosen numbers are consecutive integers?
Ⓐ 35 Ⓑ 70 Ⓒ 126 Ⓓ 15

38. In how many ways can three distinct numbers be chosen from $\{1, 2, \ldots, 12\}$ so that their sum is even?
Ⓐ 120 Ⓑ 110 Ⓒ 100 Ⓓ 220

39. 8 different letters are to be placed one each in their 8 correspondingly addressed envelopes. In how many ways can exactly 2 letters be placed in the correct envelopes?
Ⓐ 14840 Ⓑ 20160 Ⓒ 7420 Ⓓ 1855

40. Four dice of different colours are thrown once. In how many different outcomes is the sum of the top faces 10?
Ⓐ 80 Ⓑ 72 Ⓒ 96 Ⓓ 84

41. On the integer-coordinate grid, going from $(0, 0)$ to $(4, 5)$, each step adds only $(1, 0)$ or $(0, 1)$. How many paths do not pass through the point $(2, 2)$?
Ⓐ 126 Ⓑ 66 Ⓒ 56 Ⓓ 60

42. From $(0, 0)$ to $(5, 5)$, each step moves one unit right or one unit up. How many paths never reach a point with $y > x$?
Ⓐ 252 Ⓑ 42 Ⓒ 84 Ⓓ 126

43. Among 10 distinct points in a plane, 4 lie on one straight line and another 3 lie on a different straight line. Apart from these two groups, no three points are collinear. How many triangles can be formed?
Ⓐ 116 Ⓑ 114 Ⓒ 120 Ⓓ 115

44. From 4 red, 5 blue and 6 green balls, in how many ways can 5 balls be selected so that every colour has at least one ball?
Ⓐ 2170 Ⓑ 2296 Ⓒ 2163 Ⓓ 3003

45. What is the value of $\sum_{r=1}^{5} r \, {}^{8}C_r \, {}^{6}C_{5-r}$?
Ⓐ 10010 Ⓑ 2002 Ⓒ 5720 Ⓓ 715

46. Among 7 distinct books, three specified books are A, B, C. In how many ways can the books be arranged in a row so that exactly two of these three are adjacent and the third is not next to either of them?
Ⓐ 1440 Ⓑ 4320
Ⓒ 2880 Ⓓ 2160

47. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{a, b, c\}$. How many onto functions $f : A \to B$ satisfy $f(1) \neq f(2)$?

(A) 150 (B) 486 (C) 390 (D) 540

48. If ${}^{n}P_{r} = 360$ and ${}^{n}C_{r} = 15$, what is $n + r$? (Here n, r are positive integers and $r \leq n$)

(A) 10 (B) 12 (C) 11 (D) 9

49. What is the range of the function $f(x) = e^{\ln\left(\frac{5x^3-2x^2}{x-4}\right)}$?

(A) $\mathbb{R}$ (B) $[0, \infty)$

(C) $(-\infty, 0)$ (D) $(0, \infty)$

50. How many 6-digit numbers can be formed?

(A) 900000 (B) 999999

(C) 810000 (D) 600000