Engineering Daily 21 [Home Practice] EV

Functions and Graphs of Functions + Permutations and Combinations

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. What is the maximum value of $f(x) = 3\cos x + 4\sin x$?

Ⓐ 7 Ⓑ 3 Ⓒ 4 Ⓓ 5

2. What is the domain of the function $f(x) = \log_2\left(\log_3(x^2-4)\right)$?

Ⓐ $(-\sqrt{5}, \sqrt{5})$ Ⓑ $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$

Ⓒ $(-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ Ⓓ $(-\infty, -2) \cup (2, \infty)$

3. What is the domain of the function $f(x) = \sqrt{\log_{\frac{1}{2}} \frac{x-1}{x+1}}$?

Ⓐ $(1, \infty)$ Ⓑ $(-\infty, -1)$

Ⓒ $(-\infty, -1) \cup (1, \infty)$ Ⓓ $[1, \infty)$

4. If $f(x) = \frac{1}{2+\sin x+\cos x}$, $x \in \mathbb{R}$, what is the range?

Ⓐ $[2-\sqrt{2}, 2+\sqrt{2}]$ Ⓑ $\left(\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right)$

Ⓒ $\left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right]$ Ⓓ $\left[\frac{1}{4}, 1\right]$

5. If $f : [1, \infty) \to [0, 1)$ and $f(x) = \sqrt{\frac{x-1}{x+1}}$, which of the following is $f^{-1}(x)$?

Ⓐ $\frac{1+x}{1-x}$ Ⓑ $\frac{1-x^2}{1+x^2}$

Ⓒ $\frac{1+x^2}{1-x^2}$ Ⓓ $\frac{x^2-1}{x^2+1}$

6. If $f(x) = \ln x$ and $g(x) = e^{x^2}$, what is the product of all real solutions of $(f \circ g)(x) = (g \circ f)(x)$?

Ⓐ e^4 Ⓑ e^2 Ⓒ 1 Ⓓ e

7. For all $x \in \mathbb{R} \setminus \{0, 1\}$, $f(x) + 2f\left(\frac{1}{1-x}\right) = x$. What is $f(2)$?

Ⓐ $\frac{4}{3}$ Ⓑ $-\frac{2}{3}$ Ⓒ $\frac{2}{3}$ Ⓓ $\frac{1}{3}$

8. For $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$ when $x \le 0$ and $f(x) = \sqrt{x}$ when $x > 0$. What is $f^{-1}(-8) + f^{-1}(3)$?

Ⓐ 1 Ⓑ −5 Ⓒ 11 Ⓓ 7

9. The graphs of $y = |x|$ and $y = ax + 1$ have exactly one common point. Which is the set of all possible values of a?

Ⓐ $(-\infty, -1) \cup (1, \infty)$ Ⓑ $\{-1, 1\}$

Ⓒ $(-\infty, -1] \cup [1, \infty)$ Ⓓ $(-1, 1)$

10. $f(x) = 2x + 4$ for $-2 \le x \le 0$ and $f(x) = 4 - x$ for $0 < x \le 3$. What is the solution set of the inequality $f(2x-1) \ge 2$?

Ⓐ $\left[\frac{1}{2}, \frac{3}{2}\right]$ Ⓑ $\left[-\frac{3}{2}, 2\right]$

Ⓒ $\left[0, \frac{3}{2}\right]$ Ⓓ $[-1, 2]$

11. For all real x, $f(2+x) = f(2-x)$ and $f(5+x) = f(5-x)$. If $f(1) = 7$, what is $f(2025)$?

Ⓐ 0 Ⓑ 14 Ⓒ −7 Ⓓ 7

12. If the domain of f is $[-3, 5]$, what is the domain of $g(x) = f\left(\frac{x-1}{x+1}\right)$?

Ⓐ $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$ Ⓑ $\mathbb{R} \setminus \{-1\}$

Ⓒ $\left[-\frac{3}{2}, -\frac{1}{2}\right]$ Ⓓ $\left(-\infty, -\frac{3}{2}\right] \cup \left[-\frac{1}{2}, \infty\right)$

13. Let $b > 0$ and $f(x) = \frac{e^x+a}{e^x+b}$, $x \in \mathbb{R}$. If the function is odd, which is (a, b)?

Ⓐ $(1, 2)$ Ⓑ $(-1, 1)$ Ⓒ $(-2, 2)$ Ⓓ $(1, 1)$

14. If $f(x) = x^3 + x$, $x \in \mathbb{R}$, and $h = f^{-1}$, what is $h(7) + h(-7) + h(30)$?

Ⓐ 3 Ⓑ 6 Ⓒ 30 Ⓓ 0

15. $f : \mathbb{R} \to \mathbb{R}$ is a one-one and onto function with $f(2) = 3$ and $f(5) = 7$. If $g(x) = f^{-1}(2f(x) + 1)$, what is $g^{-1}(5)$?

Ⓐ 7 Ⓑ 3 Ⓒ 5 Ⓓ 2

16. If $f(x) = x + \frac{1}{x}$, $x > 0$, which is the solution of the equation $f(f(x)) = \frac{5}{2}$?

Ⓐ $x = \pm 1$ Ⓑ $x = 2$ Ⓒ $x = \frac{1}{2}, 2$ Ⓓ $x = 1$

17. Which is the set of all values of k for which $|2^x - 3| = k$ has exactly one real solution?

Ⓐ $[0, 3)$ Ⓑ $\{0\} \cup [3, \infty)$

Ⓒ $[3, \infty)$ Ⓓ $\{0\} \cup (3, \infty)$

18. If $f(x) = \frac{x^2+1}{x^2-1}$, what is the domain of $(f \circ f)(x)$?

Ⓐ $\mathbb{R} \setminus \{0\}$ Ⓑ $\mathbb{R} \setminus \{-1, 0\}$

Ⓒ $\mathbb{R} \setminus \{-1, 1\}$ Ⓓ $\mathbb{R} \setminus \{-1, 0, 1\}$

19. If $f(x) = \frac{|x|}{x} + \frac{|x-1|}{x-1} + \frac{|x-3|}{x-3}$, what is the solution set of the equation $f(x) = 1$?

Ⓐ $(0, 1)$ Ⓑ $(1, 3)$ Ⓒ $(3, \infty)$ Ⓓ $[1, 3]$

20. If $f(x) = \frac{1}{1+e^x}$, what is the value of $\sum_{k=-5}^{5} f(k)$?

Ⓐ $\frac{9}{2}$ Ⓑ 5 Ⓒ 6 Ⓓ $\frac{11}{2}$

21. If $f(x) = x^3$, $x \in \mathbb{R}$, how many distinct intersection points do the graphs of $y = f(x)$ and $y = f^{-1}(x)$ have?

Ⓐ 3 Ⓑ 1 Ⓒ 9 Ⓓ 2

22. $f(x) = x^2$ for $x < 0$ and $f(x) = 2 - x$ for $x \ge 0$. For which values of t does the equation $f(x) = t$ have exactly two distinct real solutions?

Ⓐ $0 < t < 2$ Ⓑ $0 < t \le 2$

Ⓒ $0 \le t \le 2$ Ⓓ $t > 2$

23. Using each of the digits 0,1,2,3,4,5 exactly once,how many numbers are odd and greater than 300000?

Ⓐ 192 Ⓑ 144 Ⓒ 168 Ⓓ 216

24. What is the sum of all five-digit numbers formed by using each of the digits $1, 2, 3, 4, 5$ exactly once?

Ⓐ 3999960 Ⓑ 3333300

Ⓒ 7999920 Ⓓ 1999980

25. In how many ways can 8 distinct persons be seated in a row so that B is to the right of A and C is to the right of B? (They need not be adjacent)

Ⓐ 20160 Ⓑ 3360 Ⓒ 6720 Ⓓ 13440

26. 5 mathematics, 4 physics and 3 chemistry books, all different, are to be arranged on one shelf. Books of the same subject stay together and the mathematics group is not in the middle of the other two groups. How many arrangements are possible?

Ⓐ 69120 Ⓑ 34560

Ⓒ 17280 Ⓓ 103680

27. A circular necklace is to be made from 8 distinct beads so that two specified beads are opposite each other. Necklaces that coincide after rotating or flipping are counted once. How many necklaces are possible?

Ⓐ 1440 Ⓑ 180 Ⓒ 720 Ⓓ 360

28. A committee of 3 men and 3 women is to be formed from 5 men and 5 women. A particular man and a particular woman must either both be on the committee or both be left out. How many committees are possible?

Ⓐ 64 Ⓑ 100 Ⓒ 52 Ⓓ 36

29. In how many ways can exactly 20 taka be made using an unlimited supply of coins of 1, 2 and 5 taka?

Ⓐ 30 Ⓑ 22 Ⓒ 29 Ⓓ 28

30. Arranging the letters A, A, A, A, B, B, B, C, C in a row, how many distinct arrangements have the same first and last letter?

Ⓐ 420 Ⓑ 350 Ⓒ 315 Ⓓ 700

31. Using the digits $0, 1, 2, 3$ (repetition allowed), how many five-digit numbers can be formed in which 3 appears exactly twice?

Ⓐ 324 Ⓑ 162 Ⓒ 270 Ⓓ 216

32. How many subsets of $A = \{1, 2, \dots, 8\}$ have an even number of elements, contain 1, but do not contain 2?

Ⓐ 31 Ⓑ 32 Ⓒ 64 Ⓓ 16

33. In a convex nonagon, no three diagonals meet at the same interior point. How many distinct interior intersection points do the diagonals create in total?

Ⓐ 108 Ⓑ 126 Ⓒ 252 Ⓓ 84

34. In how many ways can four identical rooks be placed on a chessboard of 5 rows and 6 columns so that no two are in the same row or the same column?

Ⓐ 1800 Ⓑ 7200

Ⓒ 900 Ⓓ 360

35. How many rectangles can be formed by choosing four of the vertices of a regular octagon?

Ⓐ 6 Ⓑ 4 Ⓒ 12 Ⓓ 8

36. Using the digits $1, 2, \dots, 9$ without repetition, how many three-digit numbers can be formed whose digits add up to 12?

Ⓐ 36 Ⓑ 54 Ⓒ 48 Ⓓ 42

37. Four balls are chosen from 4 red, 3 blue and 2 green balls and arranged in a row. Every colour must appear at least once. If balls of the same colour are identical, how many different colour sequences are possible?

Ⓐ 24 Ⓑ 36 Ⓒ 18 Ⓓ 72

38. What is the value of $\sum_{r=0}^{8} r^2 \, {}^8C_r$?

Ⓐ 4608 Ⓑ 4096 Ⓒ 3584 Ⓓ 8192

39. What is the domain of the function $f(x) = \sin^{-1}\left(\log_2 \frac{x}{2}\right)$?

Ⓐ $(0, 4]$ Ⓑ $[1, 2]$ Ⓒ $[2, 4]$ Ⓓ $[1, 4]$

40. In how many ways can the letters of the word 'EQUATION' be arranged so that the vowels always stay together?

Ⓐ 2880 Ⓑ 1440 Ⓒ 50400 Ⓓ 4320

41. What is the range of the function $f(x) = \frac{x^2-x+1}{x^2+x+1}$, $x \in \mathbb{R}$?

Ⓐ $\left[\frac{1}{3}, 3\right]$ Ⓑ $\left(-\infty, \frac{1}{3}\right] \cup [3, \infty)$

Ⓒ $[0, 3]$ Ⓓ $[-3, 3]$

42. From 3 identical white balls, 4 identical black balls and 5 balls of different colours, in how many ways can at least one ball be selected?

Ⓐ 638 Ⓑ 639 Ⓒ 119 Ⓓ 640

43. Which of the following functions is an even function?

Ⓐ $f(x) = \frac{e^x - e^{-x}}{2}$ Ⓑ $f(x) = x \cdot \frac{a^x - 1}{a^x + 1}$

Ⓒ $f(x) = \sin x + \cos x$ Ⓓ $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$

44. On a 4×5 grid, how many shortest paths from $(0, 0)$ to $(4, 5)$ must pass through the point $(2, 3)$?

Ⓐ 66 Ⓑ 30 Ⓒ 126 Ⓓ 60

45. For all non-zero real x, $2f(x) + 3f\left(\frac{1}{x}\right) = x^2$. What is the value of $f(2)$?

Ⓐ $\frac{19}{20}$ Ⓑ $-\frac{19}{20}$ Ⓒ $-\frac{29}{20}$ Ⓓ $\frac{29}{20}$

46. If $f(x) = \sqrt{x - 2}$ and $g(x) = x^2 + 1$, what is the domain of $(f \circ g)(x)$?

Ⓐ $(-\infty, -1] \cup [1, \infty)$ Ⓑ $\mathbb{R}$

Ⓒ $[-1, 1]$ Ⓓ $[2, \infty)$

47. In how many ways can 12 players be divided into 3 equal teams?

Ⓐ 11550 Ⓑ 5775 Ⓒ 34000 Ⓓ 15400

48. If $f(x) = \frac{2^x - 2^{-x}}{2}$, which of the following is $f^{-1}(x)$?

Ⓐ $\ln\left(x + \sqrt{x^2 + 1}\right)$ Ⓑ $\log_2\left(x + \sqrt{x^2 - 1}\right)$

Ⓒ $\log_2\left(x + \sqrt{x^2 + 1}\right)$ Ⓓ $\log_2\left(x - \sqrt{x^2 + 1}\right)$

49. How many integer solutions does the equation $x + y + z = 15$ have, where $x \geq 2$, $y \geq 3$ and $z \geq 4$?

Ⓐ 28 Ⓑ 36 Ⓒ 45 Ⓓ 21

50. What is the fundamental period of the function $f(x) = \sin^4 x + \cos^4 x$?

Ⓐ $\frac{\pi}{2}$ Ⓑ $\frac{\pi}{4}$ Ⓒ π Ⓓ 2π