Engineering Daily 18 [Set 04] EV

Trigonometric Ratios + Trigonometric Ratios of Associated Angles + Inverse Trigonometric Functions and Trigonometric Equations

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. What is the fundamental period of the function $f(x) = |\sin 2x| + |\cos 2x|$?

(A) $\frac{\pi}{4}$ (B) π (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{2}$

2. If $0 < \theta < \frac{\pi}{2}$ and $\sec\theta + \tan\theta = 3$, what is the value of $\sin 4\theta$?

(A) $-\frac{336}{625}$ (B) $-\frac{168}{625}$ (C) $\frac{24}{25}$ (D) $\frac{336}{625}$

3. Which is the range of the function $f(x) = 3\sin^2 x + 7\cos^2 x + 4\sin x\cos x$, where $x \in \mathbb{R}$?

(A) $[1, 9]$ (B) $\left[5 - 2\sqrt{2}, 5 + 2\sqrt{2}\right]$

(C) $\left[5 - \sqrt{2}, 5 + \sqrt{2}\right]$ (D) $[3, 7]$

4. Which is the range of the function $f(x) = \frac{2+\sin x}{3+\cos x}$, where $x \in \mathbb{R}$?

(A) $\left[\frac{1}{3}, 1\right]$ (B) $\left[\frac{3-\sqrt{3}}{4}, \frac{3+\sqrt{3}}{4}\right]$

(C) $\left[\frac{1}{4}, \frac{3}{2}\right]$ (D) $\left[\frac{3-\sqrt{2}}{4}, \frac{3+\sqrt{2}}{4}\right]$

5. If $0 < x < \frac{\pi}{2}$, what is the minimum value of $\frac{9}{\sin^2 x} + \frac{16}{\cos^2 x}$?

(A) 25 (B) 48 (C) 50 (D) 49

6. Which is the range of the function $f(x) = \sin x + \cos x + \sin x\cos x$?

(A) $\left[-\sqrt{2}, \sqrt{2}\right]$ (B) $\left[-\frac{1}{2}, \frac{3}{2}\right]$

(C) $\left[-1, \frac{1}{2} + \sqrt{2}\right]$ (D) $\left[1 - \sqrt{2}, 1 + \sqrt{2}\right]$

7. If $\sin\theta + \cos\theta = \sqrt{\frac{3}{2}}$, what is the value of $\tan^6\theta + \cot^6\theta$?

(A) 2744 (B) 2702 (C) 196 (D) 2688

8. If $\sin x + \sin y = \frac{3}{2}$ and $\cos x + \cos y = \frac{1}{2}$, what is the value of $\cos 2(x - y)$?

(A) $-\frac{7}{8}$ (B) $\frac{1}{8}$ (C) $-\frac{1}{4}$ (D) $\frac{7}{8}$

9. What is the value of $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$?

(A) $\frac{3}{16}$ (B) $\frac{\sqrt{3}}{16}$ (C) $\frac{3}{8}$ (D) $\frac{1}{16}$

10. What is the value of $\sin^6 10^\circ + \sin^6 50^\circ + \sin^6 70^\circ$?

(A) $\frac{9}{8}$ (B) $\frac{63}{64}$ (C) $\frac{57}{64}$ (D) $\frac{15}{16}$

11. What is the value of $\tan^2 10^\circ + \tan^2 70^\circ + \tan^2 130^\circ$?

(A) 12 (B) 9 (C) 3 (D) 6

12. What is the value of $\sum_{k=0}^{4} 2^k \tan\left(\frac{2^k\pi}{96}\right)$?

(A) $\cot\frac{\pi}{96} + \frac{32}{\sqrt{3}}$ (B) $\cot\frac{\pi}{96} - 32\sqrt{3}$

(C) $\cot\frac{\pi}{96} - \frac{16}{\sqrt{3}}$ (D) $\cot\frac{\pi}{96} - \frac{32}{\sqrt{3}}$

13. What is the value of $\cos^2\frac{\pi}{7} + \cos^2\frac{2\pi}{7} + \cos^2\frac{3\pi}{7}$?

(A) $\frac{5}{4}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{7}{4}$

14. What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

(A) $-\frac{\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

15. If $\frac{\pi}{2} < \alpha < \pi$, $\frac{3\pi}{2} < \beta < 2\pi$, $\sin\alpha = \frac{3}{5}$ and $\cos\beta = \frac{12}{13}$, what is the value of $\sin(\alpha - \beta)$?

(A) $\frac{56}{65}$ (B) $-\frac{16}{65}$

(C) $-\frac{56}{65}$ (D) $\frac{16}{65}$

16. The perimeter of a sector is 20 cm. If the radius and the central angle vary, what are the maximum area of the sector and the central angle in that case, respectively?

(A) 25 cm^2 and 2 radians (B) 50 cm^2 and 1 radian

(C) 50 cm^2 and 2 radians (D) 25 cm^2 and 1 radian

17. If $0 < \beta < \alpha < \frac{\pi}{2}$, $\tan(\alpha + \beta) = 7$ and $\tan(\alpha - \beta) = \frac{1}{2}$, what is the value of $\tan\alpha$?

(A) $\frac{3+\sqrt{10}}{2}$ (B) $\frac{1+\sqrt{10}}{2}$

(C) $\frac{1+\sqrt{10}}{3}$ (D) $\frac{\sqrt{10}-1}{3}$

18. In $\triangle ABC$, if $\sin^2 A + \sin^2 B - \sin^2 C = \sin A \sin B$, what is $\angle C$?

(A) 60° (B) 30° (C) 90° (D) 120°

19. In $\triangle ABC$, if $\sin A : \sin B : \sin C = 5 : 7 : 8$, what is the value of $\cos 2C$?

(A) $-\frac{1}{7}$ (B) $-\frac{47}{49}$ (C) $\frac{47}{49}$ (D) $\frac{1}{49}$

20. In $\triangle ABC$, if $\cot A + \cot B + \cot C = 2$, what is the value of $\cot^2 A + \cot^2 B + \cot^2 C$?

(A) 3 (B) 1 (C) 4 (D) 2

21. The ratio of the inradius r to the circumradius R of a triangle is 1 : 4. What is the value of $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$?

(A) $\frac{5}{8}$ (B) $\frac{3}{4}$ (C) $\frac{9}{8}$ (D) $\frac{7}{8}$

22. If $4\tan^{-1} x + 2\cot^{-1} x = \frac{5\pi}{4}$, what is the value of x?

(A) $\sqrt{2} + 1$ (B) 1 (C) $\sqrt{2} - 1$ (D) $\sqrt{3}$

23. In $\triangle ABC$, the sides opposite to the angles A, B, C are 13, 14, 15 units respectively. What is the value of $\cos(A - B)$?

(A) $\frac{63}{65}$ (B) $\frac{224}{325}$ (C) $\frac{323}{325}$ (D) $\frac{99}{325}$

24. In $\triangle ABC$, if $a : b = 2 : 3$ and $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{1}{5}$, what is the value of $\cos C$?

(A) $\frac{7}{18}$ (B) $\frac{17}{108}$ (C) $-\frac{17}{108}$ (D) $\frac{17}{54}$

25. In $\triangle ABC$, if $A = 2B$ and $b : c = 2 : 3$, what is the value of $\cos A$?

Ⓐ $\frac{3}{4}$ Ⓑ $\frac{1}{2}$ Ⓒ $-\frac{1}{4}$ Ⓓ $\frac{1}{4}$

26. In $\triangle ABC$, if $A - B = 60°$ and $\sin A + \sin B = \sqrt{3}\sin C$, what is the value of C?

Ⓐ $90°$ Ⓑ $45°$ Ⓒ $60°$ Ⓓ $30°$

27. In $\triangle ABC$, the sides a, b, c are in A.P. in this order and $C = 120°$. What is $a : b : c$?

Ⓐ $3 : 5 : 7$ Ⓑ $4 : 5 : 6$ Ⓒ $2 : 3 : 4$ Ⓓ $1 : 2 : 3$

28. A triangle has semi-perimeter $s = 12$, inradius $r = 2$ and circumradius $R = 5$ units. What is the value of $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$?

Ⓐ $\frac{121}{36}$ Ⓑ $\frac{25}{36}$ Ⓒ $\frac{13}{36}$ Ⓓ $\frac{49}{36}$

29. What is the principal value of $\sin^{-1}\left(\sin\frac{17\pi}{12}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$?

Ⓐ $\frac{5\pi}{12}$ Ⓑ $\frac{7\pi}{12}$ Ⓒ $\frac{3\pi}{4}$ Ⓓ $-\frac{5\pi}{12}$

30. Which is the value of $\tan^{-1}2 + \tan^{-1}3 + \tan^{-1}5$?

Ⓐ $\pi - \tan^{-1}\frac{2}{3}$ Ⓑ $\tan^{-1}\frac{2}{3}$

Ⓒ $\pi + \tan^{-1}\frac{2}{3}$ Ⓓ $\frac{\pi}{2} + \tan^{-1}\frac{2}{3}$

31. If $x > 1$, what is the value of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$?

Ⓐ $2\tan^{-1}x$ Ⓑ $\frac{\pi}{2} - 2\tan^{-1}x$

Ⓒ $2\tan^{-1}x - \pi$ Ⓓ $\pi - 2\tan^{-1}x$

32. Which is the real domain of the function $f(x) = \cos^{-1}\left(\frac{x^2-3}{x^2+1}\right)$?

Ⓐ $[-1, 1]$ Ⓑ $(-\infty, -1] \cup [1, \infty)$

Ⓒ $\left(-\infty, -\sqrt{3}\right] \cup \left[\sqrt{3}, \infty\right)$ Ⓓ $\mathbb{R}$

33. Which is the real solution of the equation $\sin^{-1}x + \sin^{-1}(2x) = \frac{\pi}{6}$?

Ⓐ $\frac{1}{2\sqrt{5-2\sqrt{3}}}$ Ⓑ $\frac{1}{2\sqrt{5+2\sqrt{3}}}$

Ⓒ $-\frac{1}{2\sqrt{5+2\sqrt{3}}}$ Ⓓ $\sqrt{5+2\sqrt{3}}$

34. Which is the real solution of the equation $\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\frac{1}{2}$?

Ⓐ 1 Ⓑ $1, -5$ Ⓒ -1 Ⓓ -5

35. What is the value of $\sum_{n=1}^{10}\tan^{-1}\left(\frac{3}{9n^2+3n-1}\right)$?

Ⓐ $\tan^{-1}\frac{3}{13}$ Ⓑ $\tan^{-1}\frac{6}{13}$ Ⓒ $\tan^{-1}\frac{13}{6}$ Ⓓ $\tan^{-1}\frac{12}{13}$

36. If $x > 0$ and $\tan^{-1}x + \tan^{-1}(2x) + \tan^{-1}(4x) = \pi$, what is the value of x?

Ⓐ $\frac{\sqrt{7}}{4}$ Ⓑ $\frac{\sqrt{14}}{4}$ Ⓒ $\frac{\sqrt{14}}{2}$ Ⓓ $\frac{7}{8}$

37. If $x > 0$, what is the maximum value of $\tan^{-1}x + \tan^{-1}\frac{4}{x}$?

Ⓐ $\frac{\pi}{2}$ Ⓑ $\pi - \tan^{-1}\frac{3}{4}$ Ⓒ $2\tan^{-1}2$ Ⓓ $\frac{3\pi}{4}$

38. If $x, y \in [-1, 1]$ and $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{3}$, which of the following relations is always true?

Ⓐ $x^2 + y^2 + xy = \frac{3}{4}$ Ⓑ $x^2 + y^2 + xy = 1$

Ⓒ $x^2 + y^2 = \frac{3}{4}$ Ⓓ $x^2 + y^2 - xy = \frac{3}{4}$

39. In the interval $[0, 2\pi)$, how many distinct real solutions does the equation $\tan 3x \tan 2x = 1$ have?

Ⓐ 10 Ⓑ 12 Ⓒ 6 Ⓓ 8

40. In the interval $[0, 2\pi)$, in which interval must k lie for the equation $\sin^2 x - \cos x = k$ to have exactly 4 distinct real solutions?

Ⓐ $\left[1, \frac{5}{4}\right)$ Ⓑ $(-1, 1)$ Ⓒ $\left(1, \frac{5}{4}\right)$ Ⓓ $\left(1, \frac{5}{4}\right]$

41. In the interval $[0, 2\pi)$, how many distinct solutions does the equation $\cos 4x + 6\sin x \cos x = 2$ have?

Ⓐ 6 Ⓑ 8 Ⓒ 2 Ⓓ 4

42. In the interval $[0, 2\pi)$, how many distinct solutions does the equation $\sin 3x = \cos 2x$ have?

Ⓐ 4 Ⓑ 6 Ⓒ 5 Ⓓ 10

43. In the interval $[0, 2\pi)$, how many solutions does the equation $|\sin x| + |\cos x| = \sqrt{\frac{3}{2}}$ have?

Ⓐ 4 Ⓑ 12 Ⓒ 6 Ⓓ 8

44. In the interval $[0, 2\pi)$, how many solutions does the equation $4^{\sin^2 x} + 4^{\cos^2 x} = \frac{9}{2}$ have?

Ⓐ 0 Ⓑ 8 Ⓒ 2 Ⓓ 4

45. In the interval $[0, 2\pi)$, how many distinct solutions does the equation $\sin x + \sin 2x + \sin 3x = 0$ have?

Ⓐ 4 Ⓑ 6 Ⓒ 8 Ⓓ 5

46. In the interval $[0, 2\pi)$, what is the sum of all distinct solutions of the equation $2\cos^2 x + 3\sin x = 3$?

Ⓐ $\frac{5\pi}{2}$ Ⓑ 2 Ⓒ $\frac{3\pi}{2}$ Ⓓ π

47. In the interval $0 \le \theta \le \pi$, which is the solution of the equation $\sin\theta + \cos\theta = 1$?

Ⓐ $0, \frac{\pi}{4}$ Ⓑ $\frac{\pi}{4}, \frac{\pi}{2}$ Ⓒ $0, \frac{\pi}{2}$ Ⓓ $\frac{\pi}{2}$

48. Which is the general solution of the equation $3\tan^2\theta - 1 = 0$?

Ⓐ $\theta = 2n\pi \pm \frac{\pi}{3}$ Ⓑ $\theta = n\pi \pm \frac{\pi}{3}$

Ⓒ $\theta = 2n\pi \pm \frac{\pi}{6}$ Ⓓ $\theta = n\pi \pm \frac{\pi}{6}$

49. If $\sin\theta + \cos\theta = \frac{1}{2}$, what is the value of $\sin^{10}\theta + \cos^{10}\theta$?

Ⓐ $\frac{1621}{4096}$ Ⓑ $\frac{25}{64}$ Ⓒ $\frac{2029}{4096}$ Ⓓ $\frac{1216}{4096}$

50. An angle θ is divided into two parts α and β such that $\tan\alpha = k\tan\beta$. Then what is the value of $\sin(\alpha - \beta)$?

Ⓐ $\frac{k-1}{k+1}\tan(\alpha+\beta)$ Ⓑ $\frac{k-1}{k+1}\sin(\alpha+\beta)$

Ⓒ $\frac{k-1}{k+1}\cot(\alpha+\beta)$ Ⓓ $\frac{k-1}{k+1}\cos(\alpha+\beta)$