MCQ 50 marks

1. **D.** $20°$

Explanation: Formula for the angle between clock hands: $\theta = \left|30H - \frac{11M}{2}\right|$

Here, $H = 3$ and $M = 20$

$\theta = \left|30 \times 3 - \frac{11 \times 20}{2}\right|$

$= |90 - 110| = 20°$

Therefore, the smaller angle is $20°$

2. **B.** $\frac{\pi}{2}$

Explanation: The period of $f(x) = A \sin(bx + c) + d$ is $T = \frac{2\pi}{|b|}$

Here, $b = 4$

So, $T = \frac{2\pi}{4} = \frac{\pi}{2}$

Therefore, the fundamental period is $\frac{\pi}{2}$

3. **A.** $\frac{x^2-1}{x^2+1}$

Explanation: Given, $\sec\theta + \tan\theta = x$ ... (i)

We know, $\sec^2\theta - \tan^2\theta = 1$

So, $\sec\theta - \tan\theta = \frac{1}{x}$ ... (ii)

Adding (i) and (ii), $2\sec\theta = x + \frac{1}{x} = \frac{x^2+1}{x}$, so $\cos\theta = \frac{2x}{x^2+1}$

Subtracting (ii) from (i), $2\tan\theta = x - \frac{1}{x} = \frac{x^2-1}{x}$, so $\tan\theta = \frac{x^2-1}{2x}$

Again, $\sin\theta = \tan\theta\cos\theta = \frac{x^2-1}{2x} \times \frac{2x}{x^2+1}$

Therefore, $\sin\theta = \frac{x^2-1}{x^2+1}$

4. **A.** $168.75\ \text{cm}^2$

Explanation: Area of a sector $A = \frac{1}{2}r^2\theta$ (with θ in radians)

Here, $r = 15$ cm and $\theta = 1.5$

$A = \frac{1}{2} \times 15^2 \times 1.5 = \frac{1}{2} \times 225 \times 1.5$

Therefore, the area is $168.75\ \text{cm}^2$

5. **C.** $\frac{1}{\sqrt{2}}$

Explanation: We know, $\cos(-\theta) = \cos\theta$

So, $\cos(-1125°) = \cos 1125°$

Here, $1125° = 3 \times 360° + 45°$

$\cos 1125° = \cos 45°$

Therefore, the value is $\frac{1}{\sqrt{2}}$

6. **B.** 2

Explanation: Let $\sin A = s$

Given, $s + \frac{1}{s} = 2$

$s^2 - 2s + 1 = 0$

$(s-1)^2 = 0$

So, $s = 1$, i.e. $\sin A = 1$ and $\text{cosec}\ A = 1$

$\sin^{10} A + \text{cosec}^{10} A = 1^{10} + 1^{10}$

Therefore, the value is 2

7. **C.** $[2, 8]$

Explanation: We know, $-1 \le \cos 2x \le 1$

So, $-3 \le -3\cos 2x \le 3$

$5 - 3 \le 5 - 3\cos 2x \le 5 + 3$

$2 \le y \le 8$

Therefore, the range is $[2, 8]$

8. **A.** 4

Explanation: $f(x) = 3\left(\sin^2 x + \cos^2 x\right) + \sin^2 x = 3 + \sin^2 x$

We know, $0 \le \sin^2 x \le 1$

So, the maximum value is $3 + 1 = 4$

Therefore, the maximum value is 4

9. **B.** $10\pi\ \text{rads}^{-1}$

Explanation: One rotation covers an angle of 2π radians

Here, in 60 seconds the wheel rotates 300 times

So, in 1 second it rotates $\frac{300}{60} = 5$ times

$\omega = 5 \times 2\pi = 10\pi\ \text{rads}^{-1}$

Therefore, the angular velocity is $10\pi\ \text{rads}^{-1}$

10. **A.** 2

Explanation: Here, $\cos\frac{7\pi}{8} = \cos\left(\pi - \frac{\pi}{8}\right) = -\cos\frac{\pi}{8}$

$\cos\frac{5\pi}{8} = \cos\left(\pi - \frac{3\pi}{8}\right) = -\cos\frac{3\pi}{8}$

So, the expression $= 2\cos^2\frac{\pi}{8} + 2\cos^2\frac{3\pi}{8}$

Again, $\cos\frac{3\pi}{8} = \cos\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \sin\frac{\pi}{8}$

$= 2\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right) = 2 \times 1$

Therefore, the value is 2

11. **B.** $4\sin A\sin B\sin C$

Explanation: $\sin 2A + \sin 2B + \sin 2C = 2\sin(A+B)\cos(A-B) + 2\sin C\cos C$

Here, $A + B = \pi - C$, so $\sin(A+B) = \sin C$ and $\cos C = -\cos(A+B)$

So, the expression =

$2 \sin C [\cos(A - B) - \cos(A + B)]$
$= 2 \sin C \times 2 \sin A \sin B$
Therefore, the value is $4 \sin A \sin B \sin C$

12. **C.** $\frac{c^2}{ab}$
Explanation: Here, $\angle C = 90°$, so $a^2 + b^2 = c^2$
In the right-angled triangle, $\tan A = \frac{a}{b}$ and $\tan B = \frac{b}{a}$
$\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab}$
Therefore, the value is $\frac{c^2}{ab}$

13. **D.** -5
Explanation: The range of $a \sin x + b \cos x$ is $\left[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}\right]$
Here, $a = 3$, $b = -4$
$\sqrt{a^2 + b^2} = \sqrt{9 + 16} = 5$
Therefore, the minimum value is -5

14. **D.** $\frac{2x}{1+x^2}$
Explanation: Let $\tan^{-1} x = \theta$, so $\tan \theta = x$
$\sin 2\theta = \frac{2 \tan \theta}{1+\tan^2 \theta}$
So, $\sin\left(2 \tan^{-1} x\right) = \frac{2x}{1+x^2}$
Therefore, the value is $\frac{2x}{1+x^2}$

15. **A.** $\frac{3}{2}$
Explanation: We know, $\sin \frac{7\pi}{8} = \sin\left(\pi - \frac{\pi}{8}\right) = \sin \frac{\pi}{8}$
and $\sin \frac{5\pi}{8} = \sin\left(\pi - \frac{3\pi}{8}\right) = \sin \frac{3\pi}{8} = \cos \frac{\pi}{8}$
So, the expression $= 2\left(\sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8}\right)$
$= 2\left[1 - 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8}\right]$
$= 2\left[1 - \frac{1}{2} \sin^2 \frac{\pi}{4}\right]$
$= 2\left[1 - \frac{1}{2} \times \frac{1}{2}\right] = 2 \times \frac{3}{4}$
Therefore, the value is $\frac{3}{2}$

16. **D.** $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$
Explanation: Let $g(x) = \cos x - \sin x$
The range of $g(x)$ is $\left[-\sqrt{2}, \sqrt{2}\right]$, but the denominator cannot be zero, so $g(x) \not\equiv 0$
When $0 < g(x) \leq \sqrt{2}$, $f(x) = \frac{1}{g(x)} \geq \frac{1}{\sqrt{2}}$
When $-\sqrt{2} \leq g(x) < 0$, $f(x) \leq -\frac{1}{\sqrt{2}}$
Therefore, the range is $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$

17. **B.** $\frac{3\sqrt{3}}{8}$
Explanation: For the positive maximum take $x + y < \pi$, and let $z = \pi - (x + y)$
Here, $x + y + z = \pi$ and $\sin(x + y) = \sin(\pi - z) = \sin z$
So, the given expression $= \sin x \sin y \sin z$
For angles x, y, z of a triangle, this product is maximum when $x = y = z = \frac{\pi}{3}$
$\left(\sin \frac{\pi}{3}\right)^3 = \left(\frac{\sqrt{3}}{2}\right)^3$
Therefore, the maximum value is $\frac{3\sqrt{3}}{8}$

18. **D.** $\frac{\pi}{4}$
Explanation: Here, $2n = (n^2 + n + 1) - (n^2 - n + 1)$ and $n^4 + n^2 + 2 = 1 + (n^2 + n + 1)(n^2 - n + 1)$
So, $T_n = \tan^{-1} \frac{(n^2+n+1)-(n^2-n+1)}{1+(n^2+n+1)(n^2-n+1)}$
$= \tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)$
Again, since $n^2 + n + 1 = (n + 1)^2 - (n + 1) + 1$, the terms cancel in succession
$S_n = \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1$
So, $S_\infty = \frac{\pi}{2} - \frac{\pi}{4}$
Therefore, the value is $\frac{\pi}{4}$

19. **B.** 0
Explanation: Here, $10 \sin x \cos x = 5 \sin 2x$ and $\cos 4x = 1 - 2 \sin^2 2x$
Let $\sin 2x = s$
Given, $1 - 2s^2 + 5s = 5$
$2s^2 - 5s + 4 = 0$
Discriminant, $D = (-5)^2 - 4 \times 2 \times 4 = 25 - 32 = -7 < 0$
So, s has no real value
Therefore, there are 0 real solutions

20. **C.** 1
Explanation: Let $\cos^{-1} x = A$, $\cos^{-1} y = B$, $\cos^{-1} z = C$
Given, $A + B + C = \pi$, so $A + B = \pi - C$
So, $\cos(A + B) = -\cos C$
$\cos A \cos B - \sin A \sin B = -z$
$xy + z = \sqrt{1 - x^2}\sqrt{1 - y^2}$
Squaring, $x^2y^2 + 2xyz + z^2 = 1 - x^2 - y^2 + x^2y^2$
Therefore, $x^2 + y^2 + z^2 + 2xyz = 1$

21. **A.** $\frac{\pi}{3}$
Explanation: The range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\sin \frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin \frac{\pi}{3}$
Since $\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Therefore, the principal value is $\frac{\pi}{3}$

22. **D.** 10

Explanation: Let the number of sides be n.
The sum of interior angles is $(n-2)\times 180°$
Given, $(n-2)\times 180° = 1440°$
$n-2 = 8$
Therefore, $n = 10$

23. **A. $\frac{\pi}{12}$ to the right**
Explanation: $f(x) = 2\sin\left(3x - \frac{\pi}{4}\right) = 2\sin\left[3\left(x - \frac{\pi}{12}\right)\right]$
This is of the form $A\sin(b(x-h))$ with $h = \frac{\pi}{12}$
Therefore, the graph is shifted $\frac{\pi}{12}$ to the right

24. **B.** 1
Explanation: We know, $a\sin x + b\cos x$ lies in $\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$
So, $3\sin x - 4\cos x \in [-5, 5]$
The denominator $3\sin x - 4\cos x + 6 \in [1, 11]$, which is always positive
The fraction is maximum when the denominator is minimum, i.e. 1
Therefore, the maximum value is $\frac{1}{1} = 1$

25. **C.** $\sec^2\theta\,\mathrm{cosec}^2\theta$
Explanation: $\sec^2\theta + \mathrm{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$
$= \frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}$
$= \frac{1}{\sin^2\theta\cos^2\theta}$
Therefore, the expression equals $\sec^2\theta\,\mathrm{cosec}^2\theta$

26. **C.** 4
Explanation: Here, the number of terms is $\frac{80-10}{10} + 1 = 8$
Again, $\cos 80° = \sin 10°$, $\cos 70° = \sin 20°$, $\cos 60° = \sin 30°$, $\cos 50° = \sin 40°$
So, $\cos^2 10° + \cos^2 80° = \cos^2 10° + \sin^2 10° = 1$
Similarly, each of the other three pairs sums to 1
Therefore, the total is $4\times 1 = 4$

27. **D.** 2
Explanation: $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$
Here, $A + B = 45°$, so $\tan 45° = 1$
So, $\tan A + \tan B + \tan A\tan B = 1$
$(1+\tan A)(1+\tan B) = 1 + \tan A + \tan B + \tan A\tan B = 1 + 1$
Therefore, the value is 2

28. **B. Isosceles or right-angled**
Explanation: From the sine rule, $a = 2R\sin A$ and $b = 2R\sin B$
Given, $2R\sin A\cos A = 2R\sin B\cos B$
$\sin 2A = \sin 2B$
Either $2A = 2B$, i.e. $A = B$ (isosceles)
or $2A = 180° - 2B$, i.e. $A + B = 90°$ and $C = 90°$ (right-angled)
Therefore, the triangle is isosceles or right-angled

29. **B.** 90°
Explanation: By the sine rule, $a : b : c = \sin A : \sin B : \sin C = 3 : 4 : 5$
Here, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$
So, $a^2 + b^2 = c^2$, the triangle is right-angled with hypotenuse c
Therefore, $\angle C = 90°$

30. **A.** 15
Explanation: We know, $\sec^2\theta = 1 + \tan^2\theta$ and $\mathrm{cosec}^2\phi = 1 + \cot^2\phi$
So, $\sec^2\left(\tan^{-1}2\right) = 1 + 2^2 = 5$
Again, $\mathrm{cosec}^2\left(\cot^{-1}3\right) = 1 + 3^2 = 10$
Therefore, the value is $5 + 10 = 15$

31. **D.** $\sqrt{\frac{x^2+1}{x^2+2}}$
Explanation: Let $\tan^{-1}x = \theta$, so $\tan\theta = x$
So, $\cos\theta = \frac{1}{\sqrt{1+x^2}}$
Again, let $\cot^{-1}(\cos\theta) = \phi$, so $\cot\phi = \frac{1}{\sqrt{1+x^2}}$
$\mathrm{cosec}^2\phi = 1 + \cot^2\phi = 1 + \frac{1}{1+x^2} = \frac{x^2+2}{x^2+1}$
Since $\phi \in (0, \pi)$, $\sin\phi > 0$
Therefore, $\sin\phi = \sqrt{\frac{x^2+1}{x^2+2}}$

32. **C.** $x + y + z = xyz$
Explanation: Let $\tan^{-1}x = A$, $\tan^{-1}y = B$, $\tan^{-1}z = C$
Given, $A + B = \pi - C$, so $\tan(A+B) = -\tan C$
$\frac{x+y}{1-xy} = -z$
$x + y = -z + xyz$
Therefore, $x + y + z = xyz$

33. **D.** $2n\pi + \frac{\pi}{4}$
Explanation: $\sin x + \cos x = \sqrt{2}$
$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = 1$
$\sin\left(x + \frac{\pi}{4}\right) = 1$
$x + \frac{\pi}{4} = 2n\pi + \frac{\pi}{2}$
Therefore, $x = 2n\pi + \frac{\pi}{4}$

34. **B.** $n\pi \pm \frac{\pi}{6}$

Explanation: $\tan 2x \tan x = 1$

$\frac{2\tan x}{1-\tan^2 x} \times \tan x = 1$

$2\tan^2 x = 1 - \tan^2 x$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}} = \tan\left(\pm\frac{\pi}{6}\right)$

Therefore, $x = n\pi \pm \frac{\pi}{6}$

35. **D.** 2

Explanation: $\sin^2 x - \cos x = \frac{1}{4}$

$1 - \cos^2 x - \cos x = \frac{1}{4}$

$4\cos^2 x + 4\cos x - 3 = 0$

$(2\cos x + 3)(2\cos x - 1) = 0$

$\cos x = -\frac{3}{2}$ is impossible, since $-1 \le \cos x \le 1$

So, $\cos x = \frac{1}{2}$, i.e. $x = \frac{\pi}{3}, \frac{5\pi}{3}$ in $[0, 2\pi]$

Therefore, there are 2 solutions

36. **B.** $2n\pi + \frac{\pi}{6}$ **and** $2n\pi - \frac{\pi}{2}$

Explanation: Let $\sqrt{3} = r\cos\alpha$ and $1 = r\sin\alpha$

So, $r = \sqrt{3+1} = 2$ and $\tan\alpha = \frac{1}{\sqrt{3}}$, i.e. $\alpha = \frac{\pi}{6}$

Given, $2\cos\left(x + \frac{\pi}{6}\right) = 1$

$\cos\left(x + \frac{\pi}{6}\right) = \cos\frac{\pi}{3}$

$x + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

Taking (+), $x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{6} = 2n\pi + \frac{\pi}{6}$

Taking (-), $x = 2n\pi - \frac{\pi}{3} - \frac{\pi}{6} = 2n\pi - \frac{\pi}{2}$

Therefore, $x = 2n\pi + \frac{\pi}{6}$ and $x = 2n\pi - \frac{\pi}{2}$

37. **D.** $\sqrt{2} : 1 : \sqrt{5}$

Explanation: Given, $\sin A = \frac{1}{\sqrt{5}}$, so $\tan A = \frac{1}{2}$

Again, $\cos B = \frac{3}{\sqrt{10}}$, so $\sin B = \frac{1}{\sqrt{10}}$ and $\tan B = \frac{1}{3}$

$\tan(A+B) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = 1$

Since $A, B < 45°$, $A + B = 45°$ and $C = 135°$

So, $\sin C = \frac{1}{\sqrt{2}}$

By the sine rule, $a : b : c = \frac{1}{\sqrt{5}} : \frac{1}{\sqrt{10}} : \frac{1}{\sqrt{2}}$

Multiplying each by $\sqrt{10}$, $a : b : c = \sqrt{2} : 1 : \sqrt{5}$

Therefore, the ratio is $\sqrt{2} : 1 : \sqrt{5}$

38. **A.** 4

Explanation: Here, $\cos^2 x = 1 - \sin^2 x$, so $2^{\sin^2 x} + 2^{1-\sin^2 x} = 2\sqrt{2}$

Let $2^{\sin^2 x} = y$

So, $y + \frac{2}{y} = 2\sqrt{2}$

$y^2 - 2\sqrt{2}y + 2 = 0$

$\left(y - \sqrt{2}\right)^2 = 0$

So, $2^{\sin^2 x} = \sqrt{2}$, i.e. $\sin^2 x = \frac{1}{2}$

For $\sin x = \pm\frac{1}{\sqrt{2}}$, $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi]$

Therefore, there are 4 solutions

39. **C.** a, c, b **are in A.P.**

Explanation: By the half-angle formulas, $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ and $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

So, $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{s-c}{s}$

Given, $\frac{s-c}{s} = \frac{1}{3}$

$3s - 3c = s$

$2s = 3c$

Here, $2s = a + b + c$, so $a + b + c = 3c$

$a + b = 2c$

Therefore, a, c, b are in A.P.

40. **C.** $-\frac{1}{12}$

Explanation: Since the sum of the two $\sin^{-1}$ terms is negative, $x < 0$. Let $x = -y$ (where $y > 0$)

Given, $\sin^{-1}(6y) + \sin^{-1}\left(6\sqrt{3}y\right) = \frac{\pi}{2}$

$\sin^{-1}\left(6\sqrt{3}y\right) = \frac{\pi}{2} - \sin^{-1}(6y) = \cos^{-1}(6y)$

$6\sqrt{3}y = \sqrt{1 - 36y^2}$

$108y^2 = 1 - 36y^2$

$y^2 = \frac{1}{144}$

Since $y > 0$, $y = \frac{1}{12}$

Therefore, $x = -\frac{1}{12}$

41. **B.** $\frac{\pi}{3}$

Explanation: Let $y = \cos A \sin\left(A - \frac{\pi}{6}\right)$

Here, $y = \frac{1}{2}\left\{2\sin\left(A - \frac{\pi}{6}\right)\cos A\right\}$

$= \frac{1}{2}\left\{\sin\left(2A - \frac{\pi}{6}\right) - \sin\frac{\pi}{6}\right\}$

y is maximum when $\sin\left(2A - \frac{\pi}{6}\right)$ is maximum, i.e. $\sin\left(2A - \frac{\pi}{6}\right) = 1 = \sin\frac{\pi}{2}$

So, $2A - \frac{\pi}{6} = \frac{\pi}{2}$

or, $2A = \frac{2\pi}{3}$

Therefore, $A = \frac{\pi}{3}$

42. **D.** 0

Explanation: Given, $\sin\theta + \sin\phi = \sqrt{3}(\cos\phi - \cos\theta)$

or, $2\sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = 2\sqrt{3}\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

or, $\cos\frac{\theta-\phi}{2} = \sqrt{3}\sin\frac{\theta-\phi}{2}$

or, $\cot\frac{\theta-\phi}{2} = \sqrt{3}$

or, $\frac{\theta-\phi}{2} = 30°$

So, $\theta - \phi = 60°$

Now, $\sin 3\theta + \sin 3\phi = 2 \sin \frac{3\theta+3\phi}{2} \cos \frac{3\theta-3\phi}{2}$
$= 2 \sin \frac{3\theta+3\phi}{2} \cos 90°$
Therefore, $\sin 3\theta + \sin 3\phi = 0$

43. D. Equilateral
Explanation: Let the triangle be right-angled isosceles, with $C = 90°$ and $A = B = 45°$
Then, $\cot A + \cot B + \cot C = \cot 45° + \cot 45° + \cot 90°$
$= 1 + 1 + 0 = 2$
But $2 \neq \sqrt{3}$, so the condition is not satisfied
So, the right-angled and isosceles options are eliminated
Now, for the equilateral case put $A = B = C = 60°$
$\cot A + \cot B + \cot C = 3 \cot 60° = 3 \times \frac{1}{\sqrt{3}} = \sqrt{3}$
The condition is satisfied, therefore the triangle is equilateral

44. B. $n\pi,\ n\pi \pm \frac{\pi}{3}$
Explanation: Let $\tan x = u$
We know, $\sec 2x = \frac{1+\tan^2 x}{1-\tan^2 x}$
Given, $u^2 + \frac{1+u^2}{1-u^2} = 1$
or, $\frac{u^2-u^4+1+u^2}{1-u^2} = 1$
or, $2u^2 + 1 - u^4 = 1 - u^2$
or, $3u^2 - u^4 = 0$
or, $u^2(3 - u^2) = 0$
So, $u = 0$ or $u^2 = 3$, i.e. $\tan x = 0$ or $\tan x = \pm\sqrt{3}$
If $\tan x = 0$, then $x = n\pi$
If $\tan x = \pm\sqrt{3}$, then $x = n\pi \pm \frac{\pi}{3}$
Therefore, the solutions are $n\pi$ and $n\pi \pm \frac{\pi}{3}$

45. C. $\frac{1}{4}$
Explanation: Given, $\sin\theta + \cos\theta = \sqrt{2}$
Squaring, $1 + 2 \sin\theta \cos\theta = 2$
So, $\sin\theta \cos\theta = \frac{1}{2}$
Now, $(\sin\theta - \cos\theta)^2 = 1 - 2 \sin\theta \cos\theta = 1 - 1 = 0$
So, $\sin\theta = \cos\theta$
Again, $2 \sin\theta = \sqrt{2}$, i.e. $\sin\theta = \frac{1}{\sqrt{2}}$
Therefore, $\theta = 45°$
Putting in the question, $\sin^6 45° + \cos^6 45° = \left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6$
$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

46. A. $2n\pi + 2\alpha,\ 2n\pi$
Explanation: Given, $\cos\theta + 2 \sin\theta = 1$
Let $1 = R \cos\alpha$ and $2 = R \sin\alpha$
Here, $R = \sqrt{1^2 + 2^2} = \sqrt{5}$ and $\tan\alpha = 2$, i.e. $\cos\alpha = \frac{1}{\sqrt{5}}$
Then, $R \cos(\theta - \alpha) = 1$
or, $\sqrt{5} \cos(\theta - \alpha) = 1$
or, $\cos(\theta - \alpha) = \frac{1}{\sqrt{5}} = \cos\alpha$
So, $\theta - \alpha = 2n\pi \pm \alpha$
Therefore, $\theta = 2n\pi + 2\alpha$ or $\theta = 2n\pi$.

47. C. 1
Explanation: Given, $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$
or, $\cos^{-1} x + \cos^{-1} y = \pi - \cos^{-1} z$
Taking cos on both sides, $xy - \sqrt{(1 - x^2)(1 - y^2)} = -z$
or, $xy + z = \sqrt{(1 - x^2)(1 - y^2)}$
Squaring, $x^2y^2 + 2xyz + z^2 = 1 - x^2 - y^2 + x^2y^2$
or, $x^2 + y^2 + z^2 + 2xyz = 1$

48. B. $\frac{n\pi}{3}$
Explanation: Here, $\tan 3x = \tan(x + 2x) = \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x}$
or, $\tan x + \tan 2x = \tan 3x - \tan x \tan 2x \tan 3x$
Given, $\tan x + \tan 2x = \tan x \tan 2x \tan 3x - \tan 3x$
So, $\tan x + \tan 2x = -(\tan x + \tan 2x)$
or, $\tan x + \tan 2x = 0$
or, $\tan 2x = \tan(-x)$
or, $2x = n\pi - x$
Therefore, $x = \frac{n\pi}{3}$

49. D. $\frac{13}{2}$
Explanation: Here, $\theta = \frac{\pi}{36} = 5°$
So, the expression $= \sin^2 15° + \sin^2 20° + \cdots + \sin^2 75°$
Pairing terms from both ends, $\sin^2 15° + \sin^2 75° = \sin^2 15° + \cos^2 15° = 1$
Similarly, there are 6 such pairs and the middle term is $\sin^2 45° = \frac{1}{2}$
Therefore, the value $= 6 + \frac{1}{2} = \frac{13}{2}$

50. D. $\frac{1}{2}\sqrt{\frac{3}{7}}$
Explanation: Let $\sin^{-1} x = \alpha$, then $x = \sin\alpha$
Given, $\sin^{-1} 2x = \frac{\pi}{3} - \alpha$
or, $2x = \sin\left(\frac{\pi}{3} - \alpha\right) = \frac{\sqrt{3}}{2}\sqrt{1 - x^2} - \frac{x}{2}$

or, $5x = \sqrt{3}\sqrt{1 - x^2}$
or, $25x^2 = 3 - 3x^2$
or, $x^2 = \frac{3}{28}$
Since $5x > 0$, we have $x > 0$
Therefore, $x = \frac{1}{2}\sqrt{\frac{3}{7}}$
Using the calculator's table mode with the left side as the function and substituting the options would be faster.