Engineering Daily 18 [Set 03] EV

Trigonometric Ratios + Trigonometric Ratios of Associated Angles + Inverse Trigonometric Functions and Trigonometric Equations

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. In a clock, when the time is 3:20, what is the smaller angle between the hour hand and the minute hand?
   (A) 10° (B) 30° (C) 40° (D) 20°

2. What is the fundamental period of the function $f(x) = 3\sin\left(4x - \frac{\pi}{3}\right) + 2$?
   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

3. If $\tan\theta + \sec\theta = x$, what is the value of $\sin\theta$?
   (A) $\frac{x^2-1}{x^2+1}$ (B) $\frac{2x}{x^2+1}$ (C) $\frac{x^2+1}{x^2-1}$ (D) $\frac{2x}{x^2-1}$

4. A circle has radius 15 cm. If an arc subtends an angle of 1.5 radians at the centre, what is the area of the sector formed by the arc?
   (A) $168.75\ \text{cm}^2$ (B) $337.5\ \text{cm}^2$
   (C) $225\ \text{cm}^2$ (D) $112.5\ \text{cm}^2$

5. What is the value of $\cos(-1125°)$?
   (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

6. If $\sin A + \text{cosec}\ A = 2$, what is the value of $\sin^{10} A + \text{cosec}^{10} A$?
   (A) 10 (B) 2 (C) 1 (D) 2^{10}

7. Which is the range of the function $y = 5 - 3\cos(2x)$?
   (A) $[-3, 3]$ (B) $[2, 5]$
   (C) $[2, 8]$ (D) $[5, 8]$

8. What is the maximum value of $f(x) = 4\sin^2 x + 3\cos^2 x$?
   (A) 4 (B) 5 (C) 7 (D) 3

9. A wheel rotates 300 times per minute. What is the angular velocity of any point on the wheel?
   (A) $30\pi\ \text{rads}^{-1}$ (B) $10\pi\ \text{rads}^{-1}$
   (C) $20\pi\ \text{rads}^{-1}$ (D) $40\pi\ \text{rads}^{-1}$

10. What is the value of $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8}$?
    (A) 2 (B) 4 (C) 1 (D) 3

11. If $A + B + C = \pi$, which of the following is the value of $\sin 2A + \sin 2B + \sin 2C$?
    (A) $4\cos A\cos B\cos C$ (B) $4\sin A\sin B\sin C$
    (C) $2\cos A\cos B\cos C$ (D) $2\sin A\sin B\sin C$

12. In $\triangle ABC$, if $\angle C = 90°$, what is the value of $\tan A + \tan B$?
    (A) $\frac{2ab}{a^2+b^2}$ (B) $\frac{ab}{c^2}$
    (C) $\frac{c^2}{ab}$ (D) $\frac{a^2+b^2}{2ab}$

13. What is the minimum value of $3\sin x - 4\cos x$?
    (A) -7 (B) 5 (C) 7 (D) -5

14. What is $\sin\left(2\tan^{-1} x\right)$ when expressed in terms of x?
    (A) $\frac{2x}{1-x^2}$ (B) $\frac{1-x^2}{1+x^2}$
    (C) $\frac{1+x^2}{2x}$ (D) $\frac{2x}{1+x^2}$

15. What is the value of $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$?
    (A) $\frac{3}{2}$ (B) 2 (C) $\frac{5}{2}$ (D) 1

16. Which is the range of the function $f(x) = \frac{1}{\cos x - \sin x}$?
    (A) $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$ (B) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$
    (C) $[-\sqrt{2}, \sqrt{2}]$ (D) $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$

17. For $x, y \in (0, \pi)$, what is the maximum value of $\sin x \sin y \sin(x + y)$?
    (A) $\frac{3}{4}$ (B) $\frac{3\sqrt{3}}{8}$ (C) $\frac{1}{8}$ (D) $\frac{\sqrt{3}}{4}$

18. What is the value of $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2n}{n^4+n^2+2}\right)$?
    (A) π (B) $\frac{\pi}{8}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

19. In the interval $[0, 2\pi]$, how many real solutions does the equation $\cos 4x + 10\sin x\cos x = 5$ have?
    (A) 8 (B) 0 (C) 2 (D) 4

20. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, what is the value of $x^2 + y^2 + z^2 + 2xyz$?
    (A) 0 (B) -1 (C) 1 (D) $4xyz$

21. What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?
    (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{4\pi}{3}$

22. The sum of the interior angles of a regular polygon is 1440°. How many sides does the polygon have?
    (A) 14 (B) 8 (C) 12 (D) 10

23. What is the phase shift of the function $f(x) = 2\sin\left(3x - \frac{\pi}{4}\right)$?
    (A) $\frac{\pi}{12}$ to the right (B) $\frac{\pi}{4}$ to the right
    (C) $\frac{\pi}{12}$ to the left (D) $\frac{\pi}{4}$ to the left

24. What is the maximum value of $\frac{1}{3\sin x - 4\cos x + 6}$?
    (A) $\frac{1}{5}$ (B) 1
    (C) Infinity (D) $\frac{1}{11}$

25. Which of the following is equal to $\sec^2\theta + \text{cosec}^2\theta$?
    (A) 1 (B) $\tan^2\theta - \cot^2\theta$
    (C) $\sec^2\theta\ \text{cosec}^2\theta$ (D) $\sec^2\theta - \text{cosec}^2\theta$

26. What is the value of $\cos^2 10° + \cos^2 20° + \cdots + \cos^2 80°$?

Ⓐ 5 Ⓑ 8 Ⓒ 4 Ⓓ 4.5

27. If $A + B = 45°$, what is the value of $(1 + \tan A)(1 + \tan B)$?

Ⓐ 0 Ⓑ 1 Ⓒ 3 Ⓓ 2

28. In $\triangle ABC$, if $a \cos A = b \cos B$, what type of triangle is it?

Ⓐ Acute-angled Ⓑ Isosceles or right-angled

Ⓒ Scalene Ⓓ Equilateral

29. In $\triangle ABC$, if $\sin A : \sin B : \sin C = 3 : 4 : 5$, what is the value of $\angle C$?

Ⓐ 45° Ⓑ 90°

Ⓒ 36.87° Ⓓ 53.13°

30. What is the value of $\sec^2\left(\tan^{-1} 2\right) + \operatorname{cosec}^2\left(\cot^{-1} 3\right)$?

Ⓐ 15 Ⓑ 25 Ⓒ 13 Ⓓ 11

31. What is $\sin\left(\cot^{-1}\left(\cos\left(\tan^{-1} x\right)\right)\right)$ when expressed in terms of x?

Ⓐ $\frac{x}{\sqrt{x^2+2}}$ Ⓑ $\frac{x^2+1}{x^2+2}$ Ⓒ $\sqrt{\frac{x^2+2}{x^2+1}}$ Ⓓ $\sqrt{\frac{x^2+1}{x^2+2}}$

32. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, which of the following is correct?

Ⓐ $x + y + z = 0$ Ⓑ $xy + yz + zx = 1$

Ⓒ $x + y + z = xyz$ Ⓓ $x^2 + y^2 + z^2 = 1$

33. Which is the general solution of $\sin x + \cos x = \sqrt{2}$?

Ⓐ $2n\pi \pm \frac{\pi}{4}$ Ⓑ $n\pi + \frac{\pi}{4}$

Ⓒ $n\pi \pm \frac{\pi}{4}$ Ⓓ $2n\pi + \frac{\pi}{4}$

34. What is the general solution of $\tan 2x \tan x = 1$?

Ⓐ $(2n + 1)\frac{\pi}{3}$ Ⓑ $n\pi \pm \frac{\pi}{6}$

Ⓒ $(2n + 1)\frac{\pi}{6}$ Ⓓ $n\pi + \frac{\pi}{6}$

35. In the interval $[0, 2\pi]$, how many solutions does the equation $\sin^2 x - \cos x = \frac{1}{4}$ have?

Ⓐ 3 Ⓑ 1 Ⓒ 4 Ⓓ 2

36. What is the general solution of $\sqrt{3}\cos x - \sin x = 1$?

Ⓐ $2n\pi \pm \frac{\pi}{6}$ Ⓑ $2n\pi + \frac{\pi}{6}$ and $2n\pi - \frac{\pi}{2}$

Ⓒ $n\pi + \frac{\pi}{3}$ and $n\pi - \frac{\pi}{6}$ Ⓓ $2n\pi + \frac{\pi}{3}$ and $2n\pi - \frac{\pi}{3}$

37. In a triangle ABC, $A = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $B = \cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$. What is the ratio of the sides $a : b : c$?

Ⓐ $1 : 1 : \sqrt{2}$ Ⓑ $1 : \sqrt{2} : \sqrt{5}$

Ⓒ $2 : 1 : \sqrt{5}$ Ⓓ $\sqrt{2} : 1 : \sqrt{5}$

38. In the interval $[0, 2\pi]$, how many solutions does the equation $2^{\sin^2 x} + 2^{\cos^2 x} = 2\sqrt{2}$ have?

Ⓐ 4 Ⓑ 8 Ⓒ 0 Ⓓ 2

39. In $\triangle ABC$, if $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{1}{3}$, what is the relation among the sides a, b, c?

Ⓐ a, c, b are in G.P. Ⓑ a, b, c are in G.P.

Ⓒ a, c, b are in A.P. Ⓓ a, b, c are in A.P.

40. Which is the solution of the equation $\sin^{-1}(6x) + \sin^{-1}\left(6\sqrt{3}x\right) = -\frac{\pi}{2}$?

Ⓐ $\frac{1}{12}$ Ⓑ $\pm\frac{1}{12}$ Ⓒ $-\frac{1}{12}$ Ⓓ $-\frac{1}{6}$

41. For which value of A will $\cos A \sin\left(A - \frac{\pi}{6}\right)$ attain its maximum value?

Ⓐ $\frac{\pi}{6}$ Ⓑ $\frac{\pi}{3}$

Ⓒ $\frac{\pi}{4}$ Ⓓ None of these

42. If $(\theta - \phi)$ is an acute angle and $\sin\theta + \sin\phi = \sqrt{3}(\cos\phi - \cos\theta)$, what is the value of $\sin 3\theta + \sin 3\phi$?

Ⓐ -1 Ⓑ $\frac{1}{2}$ Ⓒ 1 Ⓓ 0

43. In $\triangle ABC$, if $\cot A + \cot B + \cot C = \sqrt{3}$, what type of triangle is it?

Ⓐ Right-angled Ⓑ Scalene

Ⓒ Isosceles Ⓓ Equilateral

44. Which of the following is the general solution of the equation $\tan^2 x + \sec 2x = 1$?

Ⓐ $2n\pi,\ 2n\pi \pm \frac{\pi}{3}$ Ⓑ $n\pi,\ n\pi \pm \frac{\pi}{3}$

Ⓒ $n\pi \pm \frac{\pi}{4}$ Ⓓ $n\pi \pm \frac{\pi}{6}$

45. If $\sin\theta + \cos\theta = \sqrt{2}$, what is the value of $\sin^6\theta + \cos^6\theta$?

Ⓐ 1 Ⓑ $\frac{1}{8}$ Ⓒ $\frac{1}{4}$ Ⓓ $\frac{1}{2}$

46. Which of the following is the general solution of the equation $1 - 2\sin\theta = \cos\theta$, where $\alpha = \tan^{-1} 2$ and n is an integer?

Ⓐ $2n\pi + 2\alpha,\ 2n\pi$ Ⓑ $\frac{n\pi}{2} + \alpha,\ 2n\pi$

Ⓒ $\frac{n\pi}{2} + \alpha,\ n\pi$ Ⓓ $n\pi + 2\alpha,\ n\pi$

47. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, what is the value of $x^2 + y^2 + z^2 + 2xyz$?

Ⓐ 0 Ⓑ 2 Ⓒ 1 Ⓓ -1

48. What is the value of x in the equation $\tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x$, where n is an integer?

Ⓐ $\frac{n\pi}{12}$ Ⓑ $\frac{n\pi}{3}$ Ⓒ $\frac{n\pi}{4}$ Ⓓ $\frac{n\pi}{5}$

49. If $\theta = \frac{\pi}{36}$, what is the value of $\sin^2 3\theta + \sin^2 4\theta + \sin^2 5\theta + \cdots + \sin^2 15\theta$?

Ⓐ $\frac{5}{2}$ Ⓑ $\frac{19}{2}$ Ⓒ $\frac{7}{2}$ Ⓓ $\frac{13}{2}$

50. If $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$, what is the value of x?

Ⓐ $\frac{\sqrt{7}}{14}$ Ⓑ $\frac{1}{2}\sqrt{\frac{7}{3}}$

Ⓒ $\frac{\sqrt{3}}{14}$ Ⓓ $\frac{1}{2}\sqrt{\frac{3}{7}}$