# Engineering Daily 18 [Home Practice] EV - Answer Sheet

Trigonometric Ratios + Trigonometric Ratios of Associated Angles + Inverse Trigonometric Functions and Trigonometric Equations

**Total Marks:** 50 **Negative Marks:** 0.25 **Time:** 1 Hour

## MCQ 50 marks

**1.** **A.** $-\frac{1}{2}$

**Explanation:** $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}$

Given, $\frac{2}{\sin 2\theta} = 4$

$\sin 2\theta = \frac{1}{2}$

Since $0 < 2\theta < \frac{\pi}{2}$, $2\theta = \frac{\pi}{6}$, i.e. $\theta = \frac{\pi}{12}$

$\cos 8\theta = \cos\frac{2\pi}{3}$

Therefore, the value is $-\frac{1}{2}$

**2.** **D.** $\pi$

**Explanation:** $f(x) = \frac{1-\cos 6x}{2} + \frac{1+\cos 10x}{2} = 1 - \frac{1}{2}\cos 6x + \frac{1}{2}\cos 10x$

The period of $\cos 6x$ is $\frac{2\pi}{6} = \frac{\pi}{3}$ and the period of $\cos 10x$ is $\frac{2\pi}{10} = \frac{\pi}{5}$

For the common period $T = m\frac{\pi}{3} = n\frac{\pi}{5}$, the smallest positive integers are $m = 3$, $n = 5$

Therefore, the fundamental period is $\pi$

**3.** **B.** $\frac{\sqrt{2}}{64}$

**Explanation:** Let $P = \prod_{k=1}^{7} \sin\frac{k\pi}{16}$

Formula: $\prod_{k=1}^{m-1} \sin\frac{k\pi}{m} = \frac{m}{2^{m-1}}$; for $m = 16$ the product is $\frac{16}{2^{15}}$

Here, $\sin\frac{(16-k)\pi}{16} = \sin\frac{k\pi}{16}$ and $\sin\frac{8\pi}{16} = 1$

So, $P^2 = \frac{16}{2^{15}}$

$P = \frac{4}{2^7\sqrt{2}} = \frac{\sqrt{2}}{64}$

Therefore, the value is $\frac{\sqrt{2}}{64}$

**4.** **B.** $-\frac{2}{5\sqrt{5}}$

**Explanation:** Let $\alpha = \sin^{-1}\frac{3}{5}$, so $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

$\cos 2\alpha = \frac{7}{25}$, $\sin 2\alpha = \frac{24}{25}$

Again, for $\beta = \tan^{-1}\frac{1}{2}$, $\cos\beta = \frac{2}{\sqrt{5}}$, $\sin\beta = \frac{1}{\sqrt{5}}$

$\cos(2\alpha + \beta) = \cos 2\alpha\cos\beta - \sin 2\alpha\sin\beta$

$= \frac{7}{25}\times\frac{2}{\sqrt{5}} - \frac{24}{25}\times\frac{1}{\sqrt{5}}$

Therefore, the value is $-\frac{2}{5\sqrt{5}}$

**5.** **D.** $\frac{u^8-28u^6+70u^4-28u^2+1}{(u^2+1)^4}$

**Explanation:** If $\sec\theta + \tan\theta = u$ then $\sec\theta - \tan\theta = \frac{1}{u}$

So, $\cos\theta = \frac{2u}{u^2+1}$ and $\sin\theta = \frac{u^2-1}{u^2+1}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{-u^4+6u^2-1}{(u^2+1)^2}$

Again, $\cos 4\theta = 2\cos^2 2\theta - 1$

$= \frac{2(u^4-6u^2+1)^2-(u^2+1)^4}{(u^2+1)^4}$

Therefore, $\cos 4\theta = \frac{u^8-28u^6+70u^4-28u^2+1}{(u^2+1)^4}$

**6.** **A.** $\frac{10+\sqrt{145}}{9}$

**Explanation:** Let $y = \frac{2+3\sin x}{5-4\cos x}$; the denominator $5 - 4\cos x > 0$

$3\sin x + 4y\cos x = 5y - 2$

For real $x$, $(5y-2)^2 \le 3^2 + (4y)^2$

$25y^2 - 20y + 4 \le 9 + 16y^2$

$9y^2 - 20y - 5 \le 0$

$y = \frac{20\pm\sqrt{400+180}}{18} = \frac{10\pm\sqrt{145}}{9}$

So, $\frac{10-\sqrt{145}}{9} \le y \le \frac{10+\sqrt{145}}{9}$

Therefore, the maximum value is $\frac{10+\sqrt{145}}{9}$

**7.** **C.** $\frac{11}{27}$

**Explanation:** Given, $(\sin x + \cos x)^2 = 1 + 2\sin x\cos x = \frac{1}{9}$

So, $\sin x\cos x = -\frac{4}{9}$

$\sin^6 x + \cos^6 x = 1 - 3\sin^2 x\cos^2 x$

$= 1 - 3\times\frac{16}{81} = \frac{33}{81}$

Therefore, the value is $\frac{11}{27}$

**8.** **C.** $-\frac{1}{32}$

**Explanation:** Let $x = \frac{\pi}{31}$ and $P = \cos x\cos 2x\cos 4x\cos 8x\cos 16x$

$\sin x\cdot P = \frac{1}{32}\sin 32x$

$\sin 32x = \sin\frac{32\pi}{31} = \sin\left(\pi + \frac{\pi}{31}\right) = -\sin x$

$P = -\frac{1}{32}$

Therefore, the value is $-\frac{1}{32}$

**9.** **D.** $2 - \sqrt{3}$

**Explanation:** Formula: $\tan\theta\tan(60° - \theta)\tan(60° + \theta) = \tan 3\theta$

Here, for $\theta = 5°$, $60° - \theta = 55°$ and $60° + \theta = 65°$

$\tan 5°\tan 55°\tan 65° = \tan 15°$

Therefore, the value is $2 - \sqrt{3}$

**10.** **D.** $\frac{1}{8}$

**Explanation:** Let $P = \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$
$\sin\frac{\pi}{7}\cdot P = \frac{1}{8}\sin\frac{8\pi}{7} = -\frac{1}{8}\sin\frac{\pi}{7}$
So, $P = -\frac{1}{8}$
Again, $\cos\frac{4\pi}{7} = \cos\left(\pi - \frac{3\pi}{7}\right) = -\cos\frac{3\pi}{7}$
$\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7} = -P$
Therefore, the value is $\frac{1}{8}$

**11.** **B.** $\frac{1}{\sqrt{15}}$
**Explanation:** Let $\tan A = k$, $\tan B = 2k$, $\tan C = 4k$
The three tangents have the same sign and a triangle has at most one obtuse angle, so $k > 0$
As $A + B + C = \pi$, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
$7k = 8k^3$
So, $k^2 = \frac{7}{8}$
$\tan^2 C = 16k^2 = 14$
Since C is acute, $\cos C = \frac{1}{\sqrt{1+\tan^2 C}} = \frac{1}{\sqrt{15}}$
Therefore, $\cos C = \frac{1}{\sqrt{15}}$

**12.** **C.** $\frac{73}{42}$
**Explanation:** Here, $s = \frac{13+14+15}{2} = 21$
$\Delta = \sqrt{21\times 8\times 7\times 6} = 84$, so $r = \frac{\Delta}{s} = 4$
$\tan\frac{A}{2} = \frac{r}{s-a} = \frac{4}{8} = \frac{1}{2}$, $\tan\frac{B}{2} = \frac{4}{7}$, $\tan\frac{C}{2} = \frac{4}{6} = \frac{2}{3}$
$\frac{1}{2} + \frac{4}{7} + \frac{2}{3} = \frac{21+24+28}{42}$
Therefore, the value is $\frac{73}{42}$

**13.** **A.** $\frac{3}{5}$
**Explanation:** Formula: $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = \frac{s}{4R}$
Here, $s = 24$ and $R = 10$
$\frac{s}{4R} = \frac{24}{4\times 10} = \frac{3}{5}$
Therefore, the value is $\frac{3}{5}$

**14.** **D.** $5a + 5b = 9c$
**Explanation:** $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{s-c}{s}$
Given, $\frac{s-c}{s} = \frac{2}{7}$
$7s - 7c = 2s$
$5s = 7c$
Putting $s = \frac{a+b+c}{2}$, $5(a + b + c) = 14c$
Therefore, $5a + 5b = 9c$

**15.** **C.** $\frac{31}{21}$
**Explanation:** By the cosine rule, $\cos A = \frac{8^2+9^2-7^2}{2\times 8\times 9} = \frac{2}{3}$
$\cos B = \frac{7^2+9^2-8^2}{2\times 7\times 9} = \frac{11}{21}$
$\cos C = \frac{7^2+8^2-9^2}{2\times 7\times 8} = \frac{2}{7}$
So, the sum $= \frac{14}{21} + \frac{11}{21} + \frac{6}{21}$
Therefore, the value is $\frac{31}{21}$

**16.** **D.** $\frac{6}{11\sqrt{11}}$
**Explanation:** Let $\tan\frac{A}{2} = k$, $\tan\frac{B}{2} = 2k$, $\tan\frac{C}{2} = 3k$ $(k > 0)$
Since $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$, the sum of pairwise products is 1
$2k^2 + 6k^2 + 3k^2 = 1$
So, $k^2 = \frac{1}{11}$, i.e. $k = \frac{1}{\sqrt{11}}$
$\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2} = 6k^3$
Therefore, the value is $\frac{6}{11\sqrt{11}}$

**17.** **A.** $\frac{71\sqrt{7}}{105}$
**Explanation:** Here, $s = \frac{15}{2}$, $s - a = \frac{7}{2}$, $s - b = \frac{5}{2}$, $s - c = \frac{3}{2}$
$\Delta = \sqrt{\frac{15}{2}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}} = \frac{15\sqrt{7}}{4}$
So, $r = \frac{\Delta}{s} = \frac{\sqrt{7}}{2}$
$\tan\frac{A}{2} = \frac{r}{s-a} = \frac{1}{\sqrt{7}}$, $\tan\frac{B}{2} = \frac{\sqrt{7}}{5}$, $\tan\frac{C}{2} = \frac{\sqrt{7}}{3}$
$\sqrt{7}\left(\frac{1}{7} + \frac{1}{5} + \frac{1}{3}\right) = \sqrt{7}\times\frac{71}{105}$
Therefore, the value is $\frac{71\sqrt{7}}{105}$

**18.** **D.** $\frac{20\pi}{77}$
**Explanation:** $\frac{17\pi}{11} = 2\pi - \frac{5\pi}{11}$, so $\sin\frac{17\pi}{11} = \sin\left(-\frac{5\pi}{11}\right)$ and the first term is $-\frac{5\pi}{11}$
$\frac{19\pi}{7} = 2\pi + \frac{5\pi}{7}$ and $\frac{5\pi}{7} \in [0, \pi]$, so the second term is $\frac{5\pi}{7}$
$-\frac{5\pi}{11} + \frac{5\pi}{7} = \frac{5\pi(11-7)}{77}$
Therefore, the value is $\frac{20\pi}{77}$

**19.** **B.** $\frac{\sqrt{17}-3}{4}$
**Explanation:** $\tan\left(\tan^{-1}x + \tan^{-1}2x\right) = \frac{x+2x}{1-2x^2} = \tan\frac{\pi}{4} = 1$
$3x = 1 - 2x^2$
$2x^2 + 3x - 1 = 0$
$x = \frac{-3\pm\sqrt{17}}{4}$
Since $x > 0$, $x = \frac{\sqrt{17}-3}{4}$
Therefore, $x = \frac{\sqrt{17}-3}{4}$

**20.** **A.** $\frac{2}{\sqrt{5}}$
**Explanation:** If three angles sum to $\pi$, then $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
$x + 2x + 5x = x\times 2x\times 5x$
$8x = 10x^3$
$x^2 = \frac{4}{5}$

Since $x > 0$, $x = \frac{2}{\sqrt{5}}$

Therefore, $x = \frac{2}{\sqrt{5}}$

**21.** **B.** $\frac{1}{\sqrt{5}}$

**Explanation:** $\cos^{-1}(2x) = \frac{\pi}{2} - \cos^{-1} x$

Taking cos on both sides, $2x = \sin\left(\cos^{-1} x\right) = \sqrt{1 - x^2}$

$4x^2 = 1 - x^2$

$5x^2 = 1$

The right side is non-negative, so $x > 0$

Therefore, $x = \frac{1}{\sqrt{5}}$

**22.** **D.** $\frac{\pi}{12}$

**Explanation:** $\tan \frac{17\pi}{12} = \tan\left(\pi + \frac{5\pi}{12}\right) = \tan \frac{5\pi}{12}$, so the first term is $\frac{5\pi}{12}$

$\sin \frac{19\pi}{12} = \sin\left(2\pi - \frac{5\pi}{12}\right) = \sin\left(-\frac{5\pi}{12}\right)$, so the second term is $-\frac{5\pi}{12}$

$\cos \frac{23\pi}{12} = \cos\left(2\pi - \frac{\pi}{12}\right) = \cos \frac{\pi}{12}$, so the third term is $\frac{\pi}{12}$

$\frac{5\pi}{12} - \frac{5\pi}{12} + \frac{\pi}{12}$

Therefore, the value is $\frac{\pi}{12}$

**23.** **A.** $\frac{\pi}{2}$

**Explanation:** Let $\alpha = \sin^{-1} \frac{3}{5}$ and $\beta = \sin^{-1} \frac{5}{13}$, so $\cos \alpha = \frac{4}{5}$ and $\cos \beta = \frac{12}{13}$

$\cos(\alpha + \beta) = \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} = \frac{33}{65}$

Since $\cos(\alpha + \beta) > 0$ and $0 < \alpha + \beta < \pi$, $\alpha + \beta$ is acute

So, $\sin^{-1} \frac{33}{65} = \frac{\pi}{2} - (\alpha + \beta)$

$\alpha + \beta + \frac{\pi}{2} - (\alpha + \beta) = \frac{\pi}{2}$

Therefore, the value is $\frac{\pi}{2}$

**24.** **B.** $\tan^{-1} \frac{4}{3}$

**Explanation:** $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1} \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \tan^{-1} 1 = \frac{\pi}{4}$

So, the total $= \frac{\pi}{4} + \tan^{-1} \frac{1}{7}$, which lies in $\left(0, \frac{\pi}{2}\right)$

$\tan\left(\frac{\pi}{4} + \tan^{-1} \frac{1}{7}\right) = \frac{1+\frac{1}{7}}{1-\frac{1}{7}} = \frac{4}{3}$

Therefore, the value is $\tan^{-1} \frac{4}{3}$

**25.** **B.** 2

**Explanation:** We know, $\sin x + \sin 3x = 2 \sin 2x \cos x$, so the equation becomes $\sin 2x(2 \cos x + 1) = 1$

Let $P(x) = \sin 2x(2 \cos x + 1)$

For $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, $\cos x \le 0$, so $|2 \cos x + 1| \le 1$ and $|P(x)| \le 1$

Equality would need $|\sin 2x| = 1$ and $|2 \cos x + 1| = 1$ together, which is impossible; so $P(x) < 1$

For $\frac{3\pi}{2} \le x \le 2\pi$, $\sin 2x \le 0$ and $2 \cos x + 1 > 0$, so $P(x) \le 0$

So, all roots lie in $\left(0, \frac{\pi}{2}\right)$

Here, $P(0) = P\left(\frac{\pi}{2}\right) = 0$ and $P\left(\frac{\pi}{4}\right) = \sqrt{2} + 1 > 1$

With $c = \cos x$, $P'(x) = 2\left(6c^3 + 2c^2 - 4c - 1\right)$; this cubic changes sign exactly once on $[0, 1]$

Therefore, $P$ first increases then decreases with a maximum greater than 1; so $P(x) = 1$ has exactly 2 roots

**26.** **C.** 2

**Explanation:** At $x = 0$, $2^0 + 2^1 = 3$ and at $x = \frac{\pi}{2}$, $2^1 + 2^0 = 3$; both are solutions

On $\left(\frac{\pi}{2}, \pi\right)$, $0 < \sin x < 1$ and $\cos x < 0$, so the left side is $< 2 + 1 = 3$

On $\left[\pi, \frac{3\pi}{2}\right]$, $\sin x \le 0$ and $\cos x \le 0$, so the left side is $\le 2 < 3$

On $\left(\frac{3\pi}{2}, 2\pi\right)$, $\sin x < 0$ and $\cos x < 1$, so the left side is $< 1 + 2 = 3$

On $\left(0, \frac{\pi}{2}\right)$ let $G(x) = 2^{\sin x} + 2^{\cos x} - 3$; $G(0) = G\left(\frac{\pi}{2}\right) = 0$

$G'(x) = \ln 2\left(\cos x \cdot 2^{\sin x} - \sin x \cdot 2^{\cos x}\right)$

If $G'(x) = 0$ then $\frac{\cos x}{2^{\cos x}} = \frac{\sin x}{2^{\sin x}}$; $\frac{t}{2^t}$ is increasing on $(0, 1)$, so $\sin x = \cos x$, i.e. the only critical point is $x = \frac{\pi}{4}$

Since $G'(0) > 0$, $G$ increases first and decreases after $\frac{\pi}{4}$; so $G > 0$ on $\left(0, \frac{\pi}{2}\right)$

Therefore, the solutions are $x = 0$ and $x = \frac{\pi}{2}$, in total 2

**27.** **C.** 2

**Explanation:** Let $u = \sin x + \cos x$, so $u^2 = 1 + \sin 2x$, i.e. $\sin 2x = u^2 - 1$

Given, $u = u^2 - 1$ or $u^2 - u - 1 = 0$

$u = \frac{1 \pm \sqrt{5}}{2}$

But $|u| \le \sqrt{2}$ and $\frac{1+\sqrt{5}}{2} > \sqrt{2}$, so $u = \frac{1-\sqrt{5}}{2}$

$\sin\left(x + \frac{\pi}{4}\right) = \frac{1-\sqrt{5}}{2\sqrt{2}}$

The right side is negative and greater than $-1$, so on $x + \frac{\pi}{4} \in \left[\frac{\pi}{4}, \frac{9\pi}{4}\right)$ this value occurs exactly 2 times

Therefore, there are 2 solutions

**28.** **D.** 10

**Explanation:** $\sin 3x = \cos 5x = \sin\left(\frac{\pi}{2} - 5x\right)$
Case 1: $3x = \frac{\pi}{2} - 5x + 2n\pi$, i.e. $x = \frac{\pi}{16} + \frac{n\pi}{4}$; taking $n = 0, 1, \ldots, 7$ gives 8 roots in $[0, 2\pi)$
Case 2: $3x = \pi - \left(\frac{\pi}{2} - 5x\right) + 2n\pi$, i.e. $x = -\frac{\pi}{4} - n\pi$; in $[0, 2\pi)$ these are $x = \frac{3\pi}{4}, \frac{7\pi}{4}$, i.e. 2 roots
The roots of case 1 are odd multiples of $\frac{\pi}{16}$, while $\frac{3\pi}{4} = \frac{12\pi}{16}$ and $\frac{7\pi}{4} = \frac{28\pi}{16}$ are even multiples; so there is no overlap
Therefore, the total is $8 + 2 = 10$ solutions

**29.** **A.** 14
**Explanation:** $\cos x + \cos 7x = 2\cos 4x \cos 3x$
$\cos 3x + \cos 5x = 2\cos 4x \cos x$
$2\cos 4x(\cos 3x + \cos x) =$
$4\cos 4x \cos 2x \cos x = 0$
$\cos 4x = 0$ gives odd multiples of $\frac{\pi}{8}$, i.e. 8 roots
$\cos 2x = 0$ gives $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, i.e. 4 roots
$\cos x = 0$ gives $x = \frac{\pi}{2}, \frac{3\pi}{2}$, i.e. 2 roots
These three sets have no common elements
Therefore, the total is $8 + 4 + 2 = 14$ solutions

**30.** **C.** 12
**Explanation:** Squaring both sides, $\sin^2 x = \cos^2 3x$
$1 - \cos 2x = 1 + \cos 6x$
$\cos 6x + \cos 2x = 0$
$2\cos 4x \cos 2x = 0$
$\cos 4x = 0$ gives 8 roots and $\cos 2x = 0$ gives 4 roots; the two sets do not overlap
Therefore, there are 12 solutions

**31.** **C.** $\frac{2026}{2^{2025}}$
**Explanation:** Formula: $\prod_{k=1}^{n-1} \sin\frac{k\pi}{n} = \frac{n}{2^{n-1}}$
Here, $n = 2026$
Therefore, the product is $\frac{2026}{2^{2025}}$

**32.** **D.** $\cot\frac{\pi}{2048}$
**Explanation:** $\tan t = \cot t - 2\cot 2t$
So, $2^k \tan\left(2^k x\right) = 2^k \cot\left(2^k x\right) - 2^{k+1}\cot\left(2^{k+1}x\right)$
Summing $k = 0$ to 9 (telescoping), the sum $= \cot x - 2^{10}\cot\left(2^{10}x\right)$
Here, $2^{10}x = \frac{1024\pi}{2048} = \frac{\pi}{2}$ and $\cot\frac{\pi}{2} = 0$
Therefore, the value is $\cot\frac{\pi}{2048}$

**33.** **D.** 10
**Explanation:** $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$
So, the equation becomes $\cos 5x = \frac{1}{2}$
Here, $5x \in [0, 10\pi)$, which covers 5 full periods of $2\pi$
In each $2\pi$ there are 2 values with $\cos 5x = \frac{1}{2}$
Therefore, there are $5 \times 2 = 10$ solutions

**34.** **B.** $\frac{11}{8}$
**Explanation:** Let $F(x) = \lambda$, so $(\sin x + 2\cos x)^2 = \lambda(3 + 2\sin x \cos x)$
$\sin^2 x + 4\cos^2 x + 4\sin x \cos x = 3\lambda\left(\sin^2 x + \cos^2 x\right) + 2\lambda \sin x \cos x$
Dividing by $\cos^2 x$ and putting $t = \tan x$, $(1 - 3\lambda)t^2 + (4 - 2\lambda)t + (4 - 3\lambda) = 0$
As $t$ is real, the discriminant $D \geq 0$
$(4 - 2\lambda)^2 - 4(1 - 3\lambda)(4 - 3\lambda) \geq 0$
$16 - 16\lambda + 4\lambda^2 - 16 + 60\lambda - 36\lambda^2 \geq 0$
$4\lambda(11 - 8\lambda) \geq 0$
So, $0 \leq \lambda \leq \frac{11}{8}$
Therefore, the maximum value is $\frac{11}{8}$

**35.** **C.** 78°
**Explanation:** Formula for the angle between clock hands: $\theta = \left|30H - \frac{11M}{2}\right|$
Here, $H = 7$ and $M = 24$
$\theta = \left|30 \times 7 - \frac{11 \times 24}{2}\right| = |210 - 132|$
Therefore, the smaller angle is 78°

**36.** **A.** $\frac{4\pi}{5}$
**Explanation:** The period of $f(x) = A\sin(bx + c) + d$ is $T = \frac{2\pi}{|b|}$
Here, $b = \frac{5}{2}$
$T = \frac{2\pi}{\frac{5}{2}} = \frac{4\pi}{5}$
Therefore, the fundamental period is $\frac{4\pi}{5}$

**37.** **B.** $\frac{135\pi}{2}$ cm$^2$
**Explanation:** Area of a sector $A = \frac{1}{2}r^2\theta$
Here, $r = 18$ cm and $\theta = \frac{5\pi}{12}$
$A = \frac{1}{2} \times 18^2 \times \frac{5\pi}{12} = \frac{1}{2} \times 324 \times \frac{5\pi}{12}$
Therefore, the area is $\frac{135\pi}{2}$ cm$^2$

**38.** **B.** $[-3, 11]$
**Explanation:** We know, $-1 \leq \sin\left(3x - \frac{\pi}{5}\right) \leq 1$
$-7 \leq 7\sin\left(3x - \frac{\pi}{5}\right) \leq 7$
$-3 \leq y \leq 11$
Therefore, the range is $[-3, 11]$

**39.** **C.** $24\pi$ rads$^{-1}$

**Explanation:** One rotation covers an angle of $2\pi$ radians
Here, in 60 seconds the wheel rotates 720 times
$\omega = \frac{2\pi \times 720}{60} = 24\pi \text{ rads}^{-1}$
Therefore, the angular velocity is $24\pi \text{ rads}^{-1}$

**40.** **A.** $-1 - 4\cos A\cos B\cos C$
**Explanation:** If $A + B + C = \pi$, then $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$
We know, $\cos 2A + \cos 2B + \cos 2C = 2\left(\cos^2 A + \cos^2 B + \cos^2 C\right) - 3$
$= 2(1 - 2\cos A\cos B\cos C) - 3$
Therefore, the value is $-1 - 4\cos A\cos B\cos C$

**41.** **B.** $\frac{c^4}{a^2b^2}$
**Explanation:** In the right-angled triangle, $\cos A = \frac{b}{c}$ and $\cos B = \frac{a}{c}$
$\sec^2 A + \sec^2 B = \frac{c^2}{b^2} + \frac{c^2}{a^2} = \frac{c^2(a^2+b^2)}{a^2b^2}$
Since $a^2 + b^2 = c^2$
Therefore, the value is $\frac{c^4}{a^2b^2}$

**42.** **A.** $\frac{\pi}{2}$
**Explanation:** $\tan\theta\tan\phi = (2 - \sqrt{3})(2 + \sqrt{3}) = 4 - 3 = 1$
So, the denominator of $\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$ is 0, i.e. $\tan(\theta + \phi)$ is undefined
Since $0 < \theta + \phi < \pi$, $\theta + \phi = \frac{\pi}{2}$
Therefore, the value is $\frac{\pi}{2}$

**43.** **C.** $75°$
**Explanation:** Given, $\sqrt{3}\tan 5\theta - \sqrt{3}\tan 3\theta + \tan 5\theta\tan 3\theta + 1 = 0$
or, $\sqrt{3}(\tan 5\theta - \tan 3\theta) = -(1 + \tan 3\theta\tan 5\theta)$
or, $\frac{\tan 5\theta - \tan 3\theta}{1 + \tan 3\theta\tan 5\theta} = -\frac{1}{\sqrt{3}}$
or, $\tan 2\theta = \tan 150°$
or, $2\theta = n \cdot 180° + 150°$
So, $\theta = n \cdot 90° + 75°$
Therefore, in $0° < \theta < 90°$, $\theta = 75°$

**44.** **A.** 5
**Explanation:** Here, $\frac{1}{\cos 4\theta} - \frac{1}{\cos 2\theta} = 2$
or, $\cos 2\theta - \cos 4\theta = 2\cos 4\theta\cos 2\theta = \cos 6\theta + \cos 2\theta$
or, $\cos 6\theta + \cos 4\theta = 0$
or, $2\cos 5\theta\cos\theta = 0$
So, $\cos 5\theta = 0$ or $\cos\theta = 0$
If $\cos 5\theta = 0$, then $\theta = (2n + 1) \cdot 18°$, i.e. $\theta = 18°, 54°, 90°, 126°, 162°$
If $\cos\theta = 0$, then $\theta = 90°$, which is already included
Therefore, the number of solutions is 5

**45.** **B.** $\sqrt{2} - 1$
**Explanation:** We know, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
So, $2\tan^{-1} x + 3\left(\tan^{-1} x + \cot^{-1} x\right) = \frac{7\pi}{4}$
or, $2\tan^{-1} x + \frac{3\pi}{2} = \frac{7\pi}{4}$
or, $2\tan^{-1} x = \frac{\pi}{4}$
or, $\tan^{-1} x = \frac{\pi}{8}$
Therefore, $x = \tan\frac{\pi}{8} = \sqrt{2} - 1$

**46.** **D.** 8
**Explanation:** We know, $-\sqrt{7^2 + 5^2} \le 7\cos x + 5\sin x \le \sqrt{7^2 + 5^2}$
So, for a solution to exist, $-\sqrt{74} \le 2k + 1 \le \sqrt{74}$
or, $-4.8 \le k \le 3.8$
Here, k is an integer, so $k = -4, -3, -2, -1, 0, 1, 2, 3$
Therefore, there are 8 values of k

**47.** **A.** $\frac{1}{12}$
**Explanation:** Here, $f_4(x) = \frac{1}{4}\left\{\left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x\cos^2 x\right\} = \frac{1}{4} - \frac{1}{8}\sin^2 2x$
And, $f_6(x) = \frac{1}{6}\left\{\left(\sin^2 x + \cos^2 x\right)^3 - 3\sin^2 x\cos^2 x\left(\sin^2 x + \cos^2 x\right)\right\} = \frac{1}{6} - \frac{1}{8}\sin^2 2x$
Therefore, $f_4(x) - f_6(x) = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

**48.** **C.** $\frac{1}{\sqrt{3}}$
**Explanation:** Here, $\frac{\sin A - \sin C}{\cos C - \cos A} = \frac{2\cos\frac{A+C}{2}\sin\frac{A-C}{2}}{2\sin\frac{A+C}{2}\sin\frac{A-C}{2}} = \cot\frac{A+C}{2}$
So, $\cot B = \cot\frac{A+C}{2}$
or, $A + C = 2B$
Again, $A + B + C = 180°$
or, $3B = 180°$
or, $B = 60°$
Therefore, $\cot B = \cot 60° = \frac{1}{\sqrt{3}}$

**49.** **B.** 0

**Explanation:** Given, $2\cos(\beta - \gamma) + 2\cos(\gamma - \alpha) + 2\cos(\alpha - \beta) + 3 = 0$
or, $\sum(2\cos\beta\cos\gamma + 2\sin\beta\sin\gamma) + \sum\left(\sin^2\alpha + \cos^2\alpha\right) = 0$
or, $(\sin\alpha + \sin\beta + \sin\gamma)^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$
So, when the sum of two squares is zero, each is zero
Therefore, $\cos\alpha + \cos\beta + \cos\gamma = 0$

**50.** **A.** $(4n - 1)\frac{\pi}{8}$

**Explanation:** Here, $\sin\theta\cos\theta(\sin^2\theta - \cos^2\theta) = \frac{1}{4}$
or, $-\sin\theta\cos\theta\cos 2\theta = \frac{1}{4}$
or, $4\sin\theta\cos\theta\cos 2\theta = -1$
or, $2\sin 2\theta\cos 2\theta = -1$
or, $\sin 4\theta = -1$
So, $4\theta = (4n - 1)\frac{\pi}{2}$
Therefore, $\theta = (4n - 1)\frac{\pi}{8}$