Engineering Daily 18 [Home Practice] EV

Trigonometric Ratios + Trigonometric Ratios of Associated Angles + Inverse Trigonometric Functions and Trigonometric Equations

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. If $0 < \theta < \frac{\pi}{4}$ and $\tan\theta + \cot\theta = 4$, what is the value of $\cos 8\theta$?

(A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

2. What is the fundamental period of the function $f(x) = \sin^2 3x + \cos^2 5x$?

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{5}$ (C) $\frac{\pi}{2}$ (D) π

3. What is the value of $\prod_{k=1}^{7} \sin\frac{k\pi}{16}$?

(A) $\frac{1}{128}$ (B) $\frac{\sqrt{2}}{64}$ (C) $\frac{\sqrt{2}}{32}$ (D) $\frac{1}{64}$

4. What is the value of $\cos\left(2\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{2}\right)$?

(A) $\frac{2}{5\sqrt{5}}$ (B) $-\frac{2}{5\sqrt{5}}$ (C) $\frac{1}{5\sqrt{5}}$ (D) $-\frac{4}{5\sqrt{5}}$

5. If $u = \sec\theta + \tan\theta$, which of the following is $\cos 4\theta$?

(A) $\frac{u^8-14u^6+36u^4-14u^2+1}{(u^2+1)^4}$ (B) $\frac{u^4-6u^2+1}{(u^2+1)^2}$

(C) $\frac{u^8+28u^6+70u^4+28u^2+1}{(u^2+1)^4}$ (D) $\frac{u^8-28u^6+70u^4-28u^2+1}{(u^2+1)^4}$

6. What is the maximum value of $\frac{2+3\sin x}{5-4\cos x}$?

(A) $\frac{10+\sqrt{145}}{9}$ (B) $\frac{5+\sqrt{145}}{9}$

(C) $\frac{10+\sqrt{109}}{9}$ (D) $\frac{10+\sqrt{145}}{18}$

7. If $\sin x + \cos x = \frac{1}{3}$, what is the value of $\sin^6 x + \cos^6 x$?

(A) $\frac{19}{27}$ (B) $\frac{13}{27}$ (C) $\frac{11}{27}$ (D) $\frac{17}{27}$

8. What is the value of $\cos\frac{\pi}{31}\cos\frac{2\pi}{31}\cos\frac{4\pi}{31}\cos\frac{8\pi}{31}\cos\frac{16\pi}{31}$?

(A) $-\frac{1}{16}$ (B) $\frac{1}{16}$ (C) $-\frac{1}{32}$ (D) $\frac{1}{32}$

9. What is the value of $\tan 5^\circ \tan 55^\circ \tan 65^\circ$?

(A) $\sqrt{3}-1$ (B) $2+\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $2-\sqrt{3}$

10. What is the value of $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}$?

(A) $\frac{1}{16}$ (B) $\frac{\sqrt{7}}{16}$ (C) $\frac{\sqrt{7}}{8}$ (D) $\frac{1}{8}$

11. In $\triangle ABC$, if $\tan A : \tan B : \tan C = 1 : 2 : 4$, what is the value of $\cos C$?

(A) $\frac{1}{\sqrt{7}}$ (B) $\frac{1}{\sqrt{15}}$ (C) $\frac{1}{\sqrt{14}}$ (D) $\frac{2}{\sqrt{15}}$

12. The sides of a triangle are 13, 14, 15. If the opposite angles are A, B, C respectively, what is the value of $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2}$?

(A) $\frac{71}{42}$ (B) $\frac{37}{21}$ (C) $\frac{73}{42}$ (D) $\frac{41}{24}$

13. In $\triangle ABC$, the semi-perimeter is $s = 24$ and the circumradius is $R = 10$. What is the value of $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$?

(A) $\frac{3}{5}$ (B) $\frac{2}{5}$ (C) $\frac{4}{5}$ (D) $\frac{1}{2}$

14. In $\triangle ABC$, if $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{2}{7}$, which relation among the sides must be true?

(A) $5a + 5c = 9b$ (B) $7a + 7b = 10c$

(C) $9a + 9b = 5c$ (D) $5a + 5b = 9c$

15. If the ratio of the sides of a triangle is $a : b : c = 7 : 8 : 9$, what is the value of $\cos A + \cos B + \cos C$?

(A) $\frac{10}{7}$ (B) $\frac{32}{21}$ (C) $\frac{31}{21}$ (D) $\frac{29}{21}$

16. In $\triangle ABC$, if $\tan\frac{A}{2} : \tan\frac{B}{2} : \tan\frac{C}{2} = 1 : 2 : 3$, what is the value of $\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}$?

(A) $\frac{6}{121}$ (B) $\frac{1}{\sqrt{11}}$

(C) $\frac{3}{11\sqrt{11}}$ (D) $\frac{6}{11\sqrt{11}}$

17. The sides of a triangle are 4, 5, 6. If the opposite angles are A, B, C, what is the value of $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2}$?

(A) $\frac{71\sqrt{7}}{105}$ (B) $\frac{71}{105\sqrt{7}}$

(C) $\frac{73\sqrt{7}}{105}$ (D) $\frac{7\sqrt{7}}{15}$

18. What is the principal value of $\sin^{-1}\left(\sin\frac{17\pi}{11}\right) + \cos^{-1}\left(\cos\frac{19\pi}{7}\right)$?

(A) $\frac{25\pi}{77}$ (B) $\frac{30\pi}{77}$

(C) $\frac{15\pi}{77}$ (D) $\frac{20\pi}{77}$

19. If $x > 0$ and $\tan^{-1}x + \tan^{-1}(2x) = \frac{\pi}{4}$, what is the value of x?

(A) $\frac{\sqrt{5}-1}{2}$ (B) $\frac{\sqrt{17}-3}{4}$

(C) $\frac{\sqrt{17}+3}{4}$ (D) $\frac{\sqrt{13}-3}{2}$

20. If $x > 0$ and $\tan^{-1}x + \tan^{-1}(2x) + \tan^{-1}(5x) = \pi$, what is the value of x?

(A) $\frac{2}{\sqrt{5}}$ (B) $\sqrt{\frac{2}{5}}$ (C) $\frac{1}{\sqrt{5}}$ (D) $\sqrt{\frac{5}{2}}$

21. If $\cos^{-1}x + \cos^{-1}(2x) = \frac{\pi}{2}$, what is the value of x?

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{5}}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{1}{\sqrt{6}}$

22. What is the value of $\tan^{-1}\left(\tan\frac{17\pi}{12}\right) + \sin^{-1}\left(\sin\frac{19\pi}{12}\right) + \cos^{-1}\left(\cos\frac{23\pi}{12}\right)$?

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) 0 (D) $\frac{\pi}{12}$

23. What is the value of $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}$?

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

24. What is the value of $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$?

(A) $\frac{\pi}{3}$ (B) $\tan^{-1}\frac{4}{3}$

(C) $\frac{\pi}{4}$ (D) $\tan^{-1}\frac{3}{4}$

25. In the interval $[0, 2\pi]$, how many real solutions does the equation $\sin x + \sin 2x + \sin 3x = 1$ have?

(A) 4 (B) 2 (C) 1 (D) 3

26. In the interval $[0, 2\pi)$, how many solutions does the equation $2^{\sin x} + 2^{\cos x} = 3$ have?

(A) 4 (B) 3 (C) 2 (D) 1

27. In the interval $[0, 2\pi)$, how many solutions does the equation $\sin x + \cos x = \sin 2x$ have?

(A) 3 (B) 1 (C) 2 (D) 4

28. In the interval $[0, 2\pi)$, how many solutions does the equation $\sin 3x = \cos 5x$ have?

(A) 9 (B) 12 (C) 8 (D) 10

29. In the interval $[0, 2\pi)$, how many solutions does the equation $\cos x + \cos 3x + \cos 5x + \cos 7x = 0$ have?

(A) 14 (B) 10 (C) 12 (D) 16

30. In the interval $[0, 2\pi)$, how many solutions does the equation $|\sin x| = |\cos 3x|$ have?

(A) 16 (B) 10 (C) 12 (D) 8

31. What is the value of $\prod_{k=1}^{2025} \sin \frac{k\pi}{2026}$?

(A) $\frac{2025}{2^{2025}}$ (B) $\frac{2026}{2^{2026}}$ (C) $\frac{2026}{2^{2025}}$ (D) $\frac{2025}{2^{2026}}$

32. If $x = \frac{\pi}{2048}$, what is the value of $\tan x + 2\tan 2x + 4\tan 4x + \cdots + 512\tan 512x$?

(A) $\frac{1}{2}\cot\frac{\pi}{2048}$ (B) $\cot\frac{\pi}{1024}$

(C) $2\cot\frac{\pi}{2048}$ (D) $\cot\frac{\pi}{2048}$

33. In the interval $[0, 2\pi)$, how many solutions does the equation $16\cos^5 x - 20\cos^3 x + 5\cos x = \frac{1}{2}$ have?

(A) 12 (B) 5 (C) 8 (D) 10

34. What is the maximum value of $F(x) = \frac{(\sin x + 2\cos x)^2}{3 + \sin 2x}$?

(A) $\frac{13}{8}$ (B) $\frac{11}{8}$ (C) $\frac{9}{8}$ (D) $\frac{3}{2}$

35. In a clock, when the time is 7:24, what is the smaller angle between the hour hand and the minute hand?

(A) 84° (B) 68° (C) 78° (D) 72°

36. What is the fundamental period of the function $f(x) = 4\sin\left(\frac{5x}{2} - \frac{\pi}{7}\right) - 3$?

(A) $\frac{4\pi}{5}$ (B) $\frac{2\pi}{5}$ (C) $\frac{5\pi}{4}$ (D) $\frac{5\pi}{2}$

37. A circle has radius 18 cm. If an arc subtends an angle of $\frac{5\pi}{12}$ radians at the centre, what is the area of the sector formed by the arc?

(A) $\frac{81\pi}{2}\ \text{cm}^2$ (B) $\frac{135\pi}{2}\ \text{cm}^2$

(C) $45\pi\ \text{cm}^2$ (D) $135\ \text{cm}^2$

38. Which is the range of the function $y = 4 + 7\sin\left(3x - \frac{\pi}{5}\right)$?

(A) $[4, 11]$ (B) $[-3, 11]$

(C) $[-3, 7]$ (D) $[-7, 7]$

39. A wheel rotates 720 times per minute. What is the angular velocity of any point on the wheel?

(A) $12\pi\ \text{rads}^{-1}$ (B) $18\pi\ \text{rads}^{-1}$

(C) $24\pi\ \text{rads}^{-1}$ (D) $36\pi\ \text{rads}^{-1}$

40. If $A + B + C = \pi$, which of the following is the value of $\cos 2A + \cos 2B + \cos 2C$?

(A) $-1 - 4\cos A\cos B\cos C$

(B) $1 - 4\cos A\cos B\cos C$

(C) $-1 + 4\cos A\cos B\cos C$

(D) $1 + 4\cos A\cos B\cos C$

41. In $\triangle ABC$, if $\angle C = 90°$, what is the value of $\sec^2 A + \sec^2 B$?

(A) $\frac{c^2}{a^2b^2}$ (B) $\frac{c^4}{a^2b^2}$

(C) $\frac{a^2b^2}{c^4}$ (D) $\frac{c^2}{ab}$

42. If $\tan\theta = 2 - \sqrt{3}$ and $\tan\phi = 2 + \sqrt{3}$, what is the value of $\theta + \phi$, where $0 < \theta, \phi < \frac{\pi}{2}$?

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) π

43. Which is the solution of the equation $\sqrt{3}\tan 5\theta - \sqrt{3}\tan 3\theta + \tan 5\theta\tan 3\theta + 1 = 0$ in the interval $0° < \theta < 90°$?

(A) 60° (B) 15° (C) 75° (D) 30°

44. How many solutions does the equation $\sec 4\theta - \sec 2\theta = 2$ have in the interval $0° < \theta < 180°$?

(A) 5 (B) 3 (C) 9 (D) 6

45. If $5\tan^{-1} x + 3\cot^{-1} x = \frac{7\pi}{4}$, what is the value of x?

(A) 1 (B) $\sqrt{2} - 1$ (C) $\sqrt{2}$ (D) $\sqrt{2} + 1$

46. For how many integer values of k does the equation $7\cos x + 5\sin x = 2k + 1$ have a solution?

(A) 6 (B) 3 (C) 4 (D) 8

47. If $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$, where k is a natural number, what is the value of $f_4(x) - f_6(x)$?

(A) $\frac{1}{12}$ (B) $\frac{1}{2}$ (C) $\frac{1}{18}$ (D) $\frac{1}{6}$

48. If A, B, C are the three angles of a triangle and $\cot B = \frac{\sin A - \sin C}{\cos C - \cos A}$, what is the value of $\cot B$?

(A) $\frac{2}{\sqrt{3}}$ (B) 1 (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}$

49. If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, what is the value of $\cos\alpha + \cos\beta + \cos\gamma$?

(A) -1 (B) 0 (C) 2 (D) 1

50. What is the general solution of the equation $\sin^3\theta\cos\theta - \cos^3\theta\sin\theta = \frac{1}{4}$, where n is an integer?

(A) $(4n - 1)\frac{\pi}{8}$ (B) $2n\pi + \frac{\pi}{3}$

(C) $(2n - 1)\pi + \frac{\pi}{3}$ (D) $n\pi + \frac{\pi}{8}$