MCQ 50 marks

1. **A.** $\frac{3\sqrt{11}}{10}$

Explanation: Let $t = \tan\alpha$

Then $x = \frac{t}{1+t^2} = \frac{1}{2}\sin 2\alpha \Rightarrow 2x = \sin 2\alpha$

And $y = 5\cos 2\alpha \Rightarrow \frac{y}{5} = \cos 2\alpha$

Squaring and adding, $4x^2 + \frac{y^2}{25} = 1 \Rightarrow \frac{x^2}{\frac{1}{4}} + \frac{y^2}{25} = 1$

Here $a^2 = 25, b^2 = \frac{1}{4}$

So $e = \sqrt{1 - \frac{1/4}{25}} = \sqrt{\frac{99}{100}} = \frac{3\sqrt{11}}{10}$

2. **B.** $5x^2 - 4y^2 = 20$

Explanation: Let the point be $P(x, y)$

$SP = \sqrt{(x-3)^2 + y^2}$, distance from the line $= \frac{|3x-4|}{3}$

Given, $\sqrt{(x-3)^2 + y^2} = \frac{3}{2} \times \frac{|3x-4|}{3} = \frac{|3x-4|}{2}$

Squaring, $4(x-3)^2 + 4y^2 = (3x-4)^2$

$4x^2 - 24x + 36 + 4y^2 = 9x^2 - 24x + 16$

So $5x^2 - 4y^2 = 20$

3. **C.** $\frac{\sqrt{3}}{2}$

Explanation: $x^2 + 4y^2 + 2x - 24y + 33 = 0$

$(x+1)^2 + 4(y-3)^2 = 4 + 1 + 36 - 33 = 4$

$\frac{(x+1)^2}{4} + \frac{(y-3)^2}{1} = 1$

Here $a^2 = 4, b^2 = 1$

$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

4. **D.** -1

Explanation: The chord joining $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is: $2x - (t_1 + t_2)y + 2at_1t_2 = 0$

The focus of $y^2 = 4ax$ is $(a, 0)$

Since the chord passes through the focus, $2a + 2at_1t_2 = 0$

So $t_1t_2 = -1$

5. **A.** $2x^2 - y^2 = 6$

Explanation: For the ellipse, $a^2 = 16, b^2 = 7$;

$e_1 = \sqrt{1 - \frac{7}{16}} = \frac{3}{4}$

Foci $(\pm 4 \times \frac{3}{4}, 0) = (\pm 3, 0)$

For the hyperbola, $ae_2 = 3$, with $e_2 = \sqrt{3}$ gives $a = \sqrt{3}$

$b^2 = a^2(e_2^2 - 1) = 3(3-1) = 6$

Equation: $\frac{x^2}{3} - \frac{y^2}{6} = 1 \Rightarrow 2x^2 - y^2 = 6$

6. **B.** $4x^2 - 9y^2 = 36$

Explanation: Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given, $(2a)^2 + (2b)^2 = 52 \Rightarrow a^2 + b^2 = 13$

$e^2 = 1 + \frac{b^2}{a^2} = \frac{13}{9} \Rightarrow b^2 = \frac{4}{9}a^2$

Solving, $a^2 = 9, b^2 = 4$

Equation: $\frac{x^2}{9} - \frac{y^2}{4} = 1 \Rightarrow 4x^2 - 9y^2 = 36$

7. **C.** $(5, \pm 2\sqrt{15})$

Explanation: $y^2 = 12x \Rightarrow 4a = 12 \Rightarrow a = 3$

Focal distance $= x + a = 8 \Rightarrow x = 5$

$y^2 = 12 \times 5 = 60 \Rightarrow y = \pm 2\sqrt{15}$

The point is $(5, \pm 2\sqrt{15})$

8. **D.** $(3, 0)$

Explanation: Passes through $(3, -6)$: $(-6)^2 = 6p(3) \Rightarrow 36 = 18p \Rightarrow p = 2$

Equation: $y^2 = 12x \Rightarrow 4a = 12 \Rightarrow a = 3$

Focus $(3, 0)$

9. **A.** 0

Explanation: Here $a^2 = 9, b^2 = 16$; since $b^2 > a^2$ the major axis lies along the y-axis, so the foci lie on the y-axis, the same line as the origin

Since the origin and both foci are collinear, the three points are collinear

So the area of the triangle $= 0$

10. **B.** $\frac{x^2}{a^2} + \frac{9y^2}{b^2} = 1$

Explanation: Let $P(a\cos\theta, b\sin\theta)$ and the double ordinate $Q(a\cos\theta, -b\sin\theta)$

The trisection point nearer P is (h, k):

$h = \frac{1\cdot(a\cos\theta) + 2\cdot(a\cos\theta)}{3} = a\cos\theta$

$k = \frac{1\cdot(-b\sin\theta) + 2\cdot(b\sin\theta)}{3} = \frac{b\sin\theta}{3}$

$\cos\theta = \frac{h}{a}, \sin\theta = \frac{3k}{b}$

So the locus is $\frac{x^2}{a^2} + \frac{9y^2}{b^2} = 1$

11. **C.** $41x^2 + 24xy + 34y^2 - 6x + 308y + 449 = 0$

Explanation: Let the point be $P(x, y)$

$\sqrt{x^2 + (y+3)^2} = \frac{1}{\sqrt{2}} \times \frac{|3x-4y+1|}{5}$

Squaring, $50[x^2 + (y+3)^2] = (3x - 4y + 1)^2$

$50x^2 + 50y^2 + 300y + 450 = 9x^2 + 16y^2 + 1 - 24xy + 6x - 8y$
So $41x^2 + 24xy + 34y^2 - 6x + 308y + 449 = 0$

12. **D. Hyperbola**
Explanation: $y = \sqrt{x^2 - 1}$
Squaring, $y^2 = x^2 - 1 \Rightarrow x^2 - y^2 = 1$
$\frac{x^2}{1^2} - \frac{y^2}{1^2} = 1$
This is a hyperbola

13. **A.** $\frac{1}{\sqrt{5}}$
Explanation: Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$)
Given, $2b = 2 \times (2ae) \Rightarrow b = 2ae$
From $b^2 = a^2(1 - e^2)$: $4a^2e^2 = a^2(1 - e^2)$
$5e^2 = 1 \Rightarrow e = \frac{1}{\sqrt{5}}$

14. **B.** $\frac{1}{\sqrt{3}}$
Explanation: $2x^2 + 3y^2 - 4x = 4$
$2(x - 1)^2 + 3y^2 = 6$
$\frac{(x-1)^2}{3} + \frac{y^2}{2} = 1$
$e = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$

15. **C.** $\frac{\sqrt{5}}{3}$
Explanation: Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Through $(2, 2)$: $\frac{4}{a^2} + \frac{4}{b^2} = 1$
Through $(3, \frac{4}{3})$: $\frac{9}{a^2} + \frac{16}{9b^2} = 1$
Solving, $a^2 = 13$, $b^2 = \frac{52}{9}$
$e = \sqrt{1 - \frac{52/9}{13}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

16. **D.** $\left(\frac{4}{3}, 0\right)$
Explanation: $3r^2 \sin^2\theta = 16r \cos\theta$
$3(r \sin\theta)^2 = 16(r \cos\theta) \Rightarrow 3y^2 = 16x$
$y^2 = \frac{16}{3}x \Rightarrow 4a = \frac{16}{3} \Rightarrow a = \frac{4}{3}$
Focus $\left(\frac{4}{3}, 0\right)$

17. **A.** $\lambda > \frac{1}{2}$
Explanation: $y^2 = 6x \Rightarrow 4a = 6 \Rightarrow a = \frac{3}{2}$
From $y = 3x + \lambda$: $m = 3, c = \lambda$
Condition for no contact: $c > \frac{a}{m}$
$\lambda > \frac{3/2}{3} = \frac{1}{2}$

18. **B.** $3x^2 + 4y^2 = 48$
Explanation: $SP = e \cdot PM$
$\sqrt{(x - 2)^2 + y^2} = \frac{1}{2} \times |x - 8|$
Squaring, $4(x^2 - 4x + 4 + y^2) = x^2 - 16x + 64$
So $3x^2 + 4y^2 = 48$

19. **C.** $\frac{4\sqrt{3}}{3}$
Explanation: Through $(-3, 2)$: $\frac{9}{a^2} - \frac{4}{b^2} = 1$
$e^2 = \frac{5}{3} \Rightarrow 1 + \frac{b^2}{a^2} = \frac{5}{3} \Rightarrow b^2 = \frac{2}{3}a^2$
Solving, $a^2 = 3, b^2 = 2$
Latus rectum $= \frac{2b^2}{a} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

20. **D.** $\frac{\sqrt{7}}{2}$
Explanation: Through $(8, -6)$: $3(64) - m(36) = 48 \Rightarrow 192 - 36m = 48 \Rightarrow m = 4$
Equation: $3x^2 - 4y^2 = 48 \Rightarrow \frac{x^2}{16} - \frac{y^2}{12} = 1$
$e = \sqrt{1 + \frac{12}{16}} = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

21. **A.** $2\sqrt{10}$
Explanation: $x^2 - 9y^2 - 4x = 5$
$(x - 2)^2 - 9y^2 = 9$
$\frac{(x-2)^2}{9} - \frac{y^2}{1} = 1 \Rightarrow a = 3, b = 1$
$e = \sqrt{1 + \frac{1}{9}} = \frac{\sqrt{10}}{3}$
Distance between foci $= 2ae = 2 \times 3 \times \frac{\sqrt{10}}{3} = 2\sqrt{10}$

22. **B.** $2\sqrt{3}$
Explanation: Here $a^2 = 3\sqrt{3}$, $b = 3$ (major axis)
Latus rectum $= \frac{2a^2}{b} = \frac{2 \times 3\sqrt{3}}{3} = 2\sqrt{3}$

23. **B.** $\left(\frac{3}{m^2}, \frac{6}{m}\right)$
Explanation: $y^2 = 12x \Rightarrow 4a = 12 \Rightarrow a = 3$
For $y = mx + c$ touching $y^2 = 4ax$, point of contact $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
With $a = 3$, point of contact $\left(\frac{3}{m^2}, \frac{6}{m}\right)$

24. **C. Hyperbola**
Explanation: $\operatorname{cosec} t + \sin t = \frac{x}{6}$ and $\operatorname{cosec} t - \sin t = \frac{y}{15}$
Subtracting the square of the second from the square of the first, $4 = \frac{x^2}{36} - \frac{y^2}{225}$
$\frac{x^2}{36} - \frac{y^2}{225} = 4$, which is a hyperbola

25. **D.** $\left(\frac{1}{2}, 2\right)$
Explanation: $(x - 1)^3 + y^2 - 4y + 2 = x^3 - 1$
$x^3 - 3x^2 + 3x - 1 + y^2 - 4y + 2 = x^3 - 1$
$-3x^2 + 3x + y^2 - 4y + 2 = 0 \Rightarrow 3x^2 - 3x - y^2 + 4y - 2 = 0$
$3\left(x - \frac{1}{2}\right)^2 - (y - 2)^2 = -\frac{21}{4}$
Centre $\left(\frac{1}{2}, 2\right)$

26. **A.** 4π

Explanation: $\frac{x-2}{4} = \cos\theta$, $y - 1 = \sin\theta$

Squaring and adding, $\frac{(x-2)^2}{16} + (y-1)^2 = 1$

Here $a = 4, b = 1$

Area $= \pi ab = 4\pi$

27. **B.** $4\sqrt{5}\pi$

Explanation: $5(x-1)^2 + (y+2)^2 = 20$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{20} = 1$

Semi-axes are $\sqrt{4} = 2$ and $\sqrt{20} = 2\sqrt{5}$

Area $= \pi \times 2 \times 2\sqrt{5} = 4\sqrt{5}\pi$

28. **C.** ± 5

Explanation: $\frac{x^2}{4} + \frac{y^2}{9} = 1 \Rightarrow a^2 = 4, b^2 = 9$

Tangency condition: $c^2 = a^2m^2 + b^2$

With $m = 2$, $c^2 = 4 \times 4 + 9 = 25$

$c = \pm 5$

29. **D.** $9x^2 + y^2 = 36$

Explanation: $y^2 = 8x \Rightarrow a = 2$, focus $(2, 0)$

$x^2 = 24y \Rightarrow a = 6$, focus $(0, 6)$

Let the ellipse be $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

From $(2, 0)$: $A^2 = 4$; from $(0, 6)$: $B^2 = 36$

Equation: $\frac{x^2}{4} + \frac{y^2}{36} = 1 \Rightarrow 9x^2 + y^2 = 36$

30. **A.** $x^2 = 2ay$

Explanation: Vertex $(0, 0)$; a point on the parabola $P(2at, at^2)$

Midpoint $(x, y) = \left(at, \frac{at^2}{2}\right)$

$x = at \Rightarrow t = \frac{x}{a}$

$y = \frac{a}{2}\left(\frac{x}{a}\right)^2 \Rightarrow 2y = \frac{x^2}{a}$

So $x^2 = 2ay$

31. **B.** $4a\cos\theta\,\text{cosec}^2\theta$

Explanation: Equation of the chord: $y = x\tan\theta$

$(x\tan\theta)^2 = 4ax \Rightarrow x = 4a\cot^2\theta$

$y = x\tan\theta = 4a\cot\theta$

Length $OP = \sqrt{x^2 + y^2} = 4a\sqrt{\cot^4\theta + \cot^2\theta} = 4a\cot\theta\,\text{cosec}\,\theta$

Simplifying, $OP = 4a\cos\theta\,\text{cosec}^2\theta$

32. **D.** $(0, 9)$

Explanation: Let the parabola be $y = ax^2 + bx + c$

$(-3, 0)$: $0 = 9a - 3b + c$

$(3, 0)$: $0 = 9a + 3b + c$

$(2, 5)$: $5 = 4a + 2b + c$

Solving, $a = -1, b = 0, c = 9$

$y = -x^2 + 9 \Rightarrow x^2 = -(y - 9)$

Vertex $(0, 9)$

33. **A.** $3x + 4y + 25 = 0$

Explanation: Slope of axis $= \frac{-4-4}{-3-3} = \frac{4}{3}$

Slope of directrix, perpendicular to axis, $= -\frac{3}{4}$

Directrix through $(-3, -4)$: $(y + 4) = -\frac{3}{4}(x + 3)$

Simplifying, $3x + 4y + 25 = 0$

34. **B.** $2x + 2y - a = 0$

Explanation: $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Squaring, $x + y + 2\sqrt{xy} = a \Rightarrow (x + y - a)^2 = 4xy$

Expanding, $(x - y)^2 = a(2x + 2y - a)$

So the tangent: $2x + 2y - a = 0$

35. **C.** $x + 2y = 0$

Explanation: $\frac{x^2}{3} - \frac{y^2}{1} = 1 \Rightarrow a = \sqrt{3}, b = 1$

Asymptotes: $\frac{x}{a} \pm \frac{y}{b} = 0 \Rightarrow \frac{x}{\sqrt{3}} \pm y = 0 \Rightarrow x \pm \sqrt{3}y = 0$

So both $x - \sqrt{3}y = 0$ and $x + \sqrt{3}y = 0$ are asymptotes, hence $x + 2y = 0$ is NOT an asymptote

36. **C.** 2

Explanation: Let $y^2 = 4ax$; since radius $= 4$ and depth $= 2$, the curve passes through $(2, 4)$

$4^2 = 4a(2) \Rightarrow 16 = 8a \Rightarrow a = 2$

Distance of the receiver (focus) from the vertex $= a = 2$ ft

37. **D.** 16

Explanation: $\left|\frac{a}{e} - ae\right| = 9$

$a\left(\frac{5}{4} - \frac{4}{5}\right) = 9 \Rightarrow a \times \frac{9}{20} = 9 \Rightarrow a = 20$

Major axis $= 2a = 40$

$b^2 = a^2(1 - e^2) = 400 \times \frac{9}{25} = 144 \Rightarrow b = 12$, minor axis $= 24$

Difference $= 40 - 24 = 16$ cm

38. **A.** $\frac{\sqrt{5}}{6}$

Explanation: $9x^2 + 5y^2 = 1 \Rightarrow \frac{x^2}{1/9} + \frac{y^2}{1/5} = 1$

Since $\frac{1}{5} > \frac{1}{9}$, the major axis is along y; $a^2 = \frac{1}{5}$ (major), $b^2 = \frac{1}{9}$

$e = \sqrt{1 - \frac{1/9}{1/5}} = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$

Distance $= a\left(\frac{1}{e} - e\right) = \frac{1}{\sqrt{5}}\left(\frac{3}{2} - \frac{2}{3}\right) = \frac{\sqrt{5}}{6}$

39. **B. Ellipse**

Explanation: When $|r| < 1$: $1 + r > 0$ and $r - 1 < 0$

$\frac{x^2}{1+r} - \frac{y^2}{r-1} = \frac{x^2}{1+r} + \frac{y^2}{1-r} = 1$

Since both denominators are positive, this is an ellipse

40. **C.** 1 m

Explanation: $a^2 = 0.0625, b^2 = 0.1875$

Latus rectum $= \frac{2b^2}{a} = \frac{2\times0.1875}{0.25} = 1.5$ m

At this height, setting $y = \pm0.75$: $\frac{x^2}{0.0625} - \frac{(0.75)^2}{0.1875} = 1$

$x = \sqrt{0.25} = 0.5$ m

Diameter $= 2 \times 0.5 = 1$ m

41. **D.** 1.5π

Explanation: $\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$

Area of the full ellipse $= \pi \times 2 \times 3 = 6\pi$

The region bounded by the positive axes (one quarter) $= \frac{6\pi}{4} = 1.5\pi$

42. **A.** $(3, \pm2\sqrt{6})$

Explanation: $(y-3)^2 + 6y - 9 = 8x \Rightarrow y^2 - 6y + 9 + 6y - 9 = 8x \Rightarrow y^2 = 8x$

$4a = 8 \Rightarrow a = 2$

Focal distance $= x + a = 5 \Rightarrow x = 3$

$y^2 = 8 \times 3 = 24 \Rightarrow y = \pm2\sqrt{6}$

The point is $(3, \pm2\sqrt{6})$

43. **B.** $\frac{\sqrt{3}}{2}$

Explanation: $x^2 + 4y^2 - 2x = 15$

$(x-1)^2 + 4y^2 = 16$

$\frac{(x-1)^2}{16} + \frac{y^2}{4} = 1$

$e = \sqrt{1 - \frac{4}{16}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

44. **C.** $(y-2)^2 = 8(x-3)$

Explanation: Let the equation be $(y-2)^2 = 4a(x-3)$

Perpendicular distance from vertex to directrix $= a$

$|a| = |3 - 1| = 2$

Equation: $(y-2)^2 = 8(x-3)$

45. **A.** 3

Explanation: Given, $\frac{2b^2}{a} = b \Rightarrow \frac{b}{a} = \frac{1}{2}$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

$e^2 = \frac{3}{4} \Rightarrow 4e^2 = 3$

46. **D.** $\left(\frac{1}{3}, 2\right)$

Explanation: $y^2 = 12x \Rightarrow 4a = 12 \Rightarrow a = 3$

From $y = mx + c$: $m = 3, c = 1$; tangency condition $c = \frac{a}{m}$: $1 = \frac{3}{3} = 1$ (holds)

Point of contact $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{3}{9}, \frac{6}{3}\right) = \left(\frac{1}{3}, 2\right)$

47. **B.** 6.25

Explanation: Through $(2, 1)$: $(1+4)^2 = 4k(2-1)$

$25 = 4k \Rightarrow k = 6.25$

48. **C.** $a\left(t + \frac{1}{t}\right)^2$

Explanation: $P(at^2, 2at)$; for the other end Q of the focal chord, $t_1 t = -1 \Rightarrow t_1 = -\frac{1}{t}$, so $Q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

$PQ^2 = a^2\left(t^2 - \frac{1}{t^2}\right)^2 + 4a^2\left(t + \frac{1}{t}\right)^2$

$= a^2\left(t + \frac{1}{t}\right)^2\left[\left(t - \frac{1}{t}\right)^2 + 4\right] = a^2\left(t + \frac{1}{t}\right)^4$

So $PQ = a\left(t + \frac{1}{t}\right)^2$

49. **D.** $2\sqrt{14}$

Explanation: Intersection of $x + y = 3$ and $2x - 3y = 1$ is $(2, 1)$

$e^2 = 3 \Rightarrow \frac{b^2}{a^2} = 2 \Rightarrow b^2 = 2a^2$

Through $(2, 1)$: $\frac{4}{a^2} - \frac{1}{2a^2} = 1 \Rightarrow \frac{7}{2a^2} = 1 \Rightarrow a^2 = \frac{7}{2}, b^2 = 7$

Latus rectum $= \frac{2b^2}{a} = \frac{14}{\sqrt{7/2}} = 2\sqrt{14}$

50. **A.** $5y^2 = 18ax$

Explanation: Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$

Given, $2at_1 = 2 \times 2at_2 \Rightarrow t_1 = 2t_2$

Midpoint (h, k): $k = at_1 + at_2 = 3at_2 \Rightarrow t_2 = \frac{k}{3a}$

$h = \frac{at_1^2 + at_2^2}{2} = \frac{5at_2^2}{2}$

$2h = 5a \cdot \frac{k^2}{9a^2} \Rightarrow 5k^2 = 18ah \Rightarrow 5y^2 = 18ax$