Engineering Daily 15 [Set 04] EV

Conics

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. For all values of t, the point $x = \frac{t}{1+t^2}$, $y = \frac{5(1-t^2)}{1+t^2}$ lies on an ellipse. What is the eccentricity of the ellipse?

(A) $\frac{3\sqrt{11}}{10}$ (B) $\frac{13}{5}$ (C) $\frac{3\sqrt{11}}{13}$ (D) $\frac{3\sqrt{13}}{11}$

2. A point moves so that its distance from the point $(3, 0)$ is always $\frac{3}{2}$ times its distance from the line $3x - 4 = 0$; the locus of the point is -

(A) $5x^2 - y^2 = 10$ (B) $5x^2 - 4y^2 = 20$

(C) $5x^2 - 4y^2 = 10$ (D) $5x^2 - y^2 = 20$

3. The eccentricity of the conic represented by the equation $x^2 + 4y^2 + 2x - 24y + 33 = 0$ is -

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{3}{4}$ (C) $\frac{\sqrt{3}}{2}$ (D) 2

4. For the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$, if the chord joining them passes through the focus, what is t_1t_2?

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) -1

5. A hyperbola has the same foci as the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$. If the eccentricity of the hyperbola is $\sqrt{3}$, its equation is -

(A) $2x^2 - y^2 = 6$ (B) $x^2 - 2y^2 = -6$

(C) $2x^2 - y^2 = -6$ (D) $x^2 - 2y^2 = 6$

6. Find the equation of the hyperbola whose transverse and conjugate axis lengths have squares summing to 52, and whose eccentricity is $\frac{\sqrt{13}}{3}$.

(A) $x^2 - 9y^2 = 9$ (B) $4x^2 - 9y^2 = 36$

(C) $16x^2 - 4y^2 = 64$ (D) $2x^2 - 9y^2 = 18$

7. If the focal distance of a point on the parabola $y^2 = 12x$ is 8, what are the coordinates of the point?

(A) $(5, \pm 2\sqrt{13})$ (B) $(\pm 2\sqrt{15}, 5)$

(C) $(5, \pm 2\sqrt{15})$ (D) $(\pm 13, 5)$

8. If the parabola $y^2 = 6px$ passes through the point $(3, -6)$, what are the coordinates of its focus?

(A) $(4, 0)$ (B) $(2, 0)$ (C) $(1, 0)$ (D) $(3, 0)$

9. The vertices of a triangle are the origin and the two foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$. What is the area of the triangle in square units?

(A) 0 (B) 12 (C) 6π (D) 6

10. PQ is any double ordinate of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The locus of the point trisecting PQ nearer to P is -

(A) $\frac{x^2}{a^2} + \frac{2y^2}{b^2} = 1$ (B) $\frac{x^2}{a^2} + \frac{9y^2}{b^2} = 1$

(C) $\frac{x^2}{a^2} + \frac{4y^2}{b^2} = 1$ (D) $\frac{x^2}{a^2} + \frac{16y^2}{b^2} = 1$

11. A point moves in the xy-plane such that the ratio of its distance from the point $(0, -3)$ to its distance from the line $3x - 4y + 1 = 0$ is always $\frac{1}{\sqrt{2}}$. Find the equation of the locus of the point.

(A) $41x^2 + 24xy + 34y^2 - 6x + 308y + 444 = 0$

(B) $41x^2 + 24xy + 34y^2 - 6x + 309y + 449 = 0$

(C) $41x^2 + 24xy + 34y^2 - 6x + 308y + 449 = 0$

(D) $41x^2 + 24xy + 34y^2 - x + 308y + 449 = 0$

12. What kind of curve is $y = \sqrt{x^2 - 1}$?

(A) Parabola (B) Ellipse (C) Circle (D) Hyperbola

13. If the minor axis of an ellipse is twice the distance between its two foci, what is its eccentricity?

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{\sqrt{5}}$

14. The eccentricity of the ellipse $2x^2 + 3y^2 - 4x = 4$ is -

(A) $\frac{\sqrt{3}}{5}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{2}{3}$

15. $\left(3, \frac{4}{3}\right)$ and $(2, 2)$ are two points on an ellipse. If the axes of the ellipse lie along the coordinate axes, what is its eccentricity?

(A) $\frac{\sqrt{8}}{5}$ (B) $\frac{\sqrt{5}}{6}$ (C) $\frac{\sqrt{5}}{3}$ (D) $\frac{\sqrt{5}}{4}$

16. If $3r^2 \sin^2\theta = 16r\cos\theta$, what are the coordinates of the focus?

(A) $(4, 0)$ (B) $\left(\frac{3}{4}, 0\right)$

(C) $\left(\frac{2}{3}, 0\right)$ (D) $\left(\frac{4}{3}, 0\right)$

17. The line $y = 3x + \lambda$ will neither touch nor intersect the parabola $y^2 = 6x$ if λ satisfies -

(A) $\lambda > \frac{1}{2}$ (B) $\lambda > \frac{1}{4}$ (C) $\lambda > \frac{1}{3}$ (D) $\lambda > 2$

18. Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, with focus $(2, 0)$ and directrix $x - 8 = 0$.

(A) $3x^2 + 10y^2 = 48$ (B) $3x^2 + 4y^2 = 48$

(C) $3x^2 + 2y^2 = 48$ (D) $4x^2 + 3y^2 = 48$

19. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through $(-3, 2)$ and has eccentricity $\sqrt{\frac{5}{3}}$. Find the length of its latus rectum.

(A) $\frac{7}{3}$ (B) $\frac{5}{3}$ (C) $\frac{4\sqrt{3}}{3}$ (D) $\frac{3}{2}$

20. If the hyperbola $3x^2 - my^2 = 48$ passes through $(8, -6)$, what is its eccentricity?

(A) $\frac{5}{2}$ (B) $\frac{5}{3}$ (C) $\frac{7}{2}$ (D) $\frac{\sqrt{7}}{2}$

21. What is the distance between the two foci of the hyperbola $x^2 - 9y^2 - 4x = 5$?

(A) $2\sqrt{10}$ (B) $2\sqrt{5}$ (C) 10 (D) $\frac{10}{3}$

22. What is the latus rectum of the ellipse $\frac{x^2}{3\sqrt{3}} + \frac{y^2}{9} = 1$?

Ⓐ $2\sqrt{5}$ Ⓑ $2\sqrt{3}$ Ⓒ 3 Ⓓ $3\sqrt{2}$

23. If the line $y = mx + c$ touches the parabola $y^2 = 12x$, what are the coordinates of the point of contact?

Ⓐ $\left(\frac{3}{m^2}, \frac{4}{m}\right)$ Ⓑ $\left(\frac{3}{m^2}, \frac{6}{m}\right)$ Ⓒ $\left(\frac{2}{m^2}, \frac{4}{m}\right)$ Ⓓ $\left(\frac{3}{m^2}, \frac{2}{m}\right)$

24. What curve is represented by $x = 6(\operatorname{cosec} t + \sin t)$ and $y = 15(\operatorname{cosec} t - \sin t)$, where t is a parameter?

Ⓐ Parabola Ⓑ Ellipse Ⓒ Hyperbola Ⓓ Circle

25. If $(x-1)^3 + (y^2 - 4y + 2) = (x^3 - 1)$, what are the coordinates of the centre of the conic?

Ⓐ $(0, 12)$ Ⓑ It is not a conic

Ⓒ $(2, 12)$ Ⓓ $\left(\frac{1}{2}, 2\right)$

26. What is the area, in square units, of the ellipse $x = 4\cos\theta + 2, y = \sin\theta + 1$?

Ⓐ 4π Ⓑ 4 Ⓒ 3π Ⓓ 8π

27. What is the area, in square units, enclosed by the curve $5(x-1)^2 + (y+2)^2 = 20$?

Ⓐ 4π Ⓑ $4\sqrt{5}\pi$ Ⓒ 25π Ⓓ 5π

28. If $y = 2x + c$ is tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, what is the value of c?

Ⓐ ± 4 Ⓑ ± 2 Ⓒ ± 5 Ⓓ ± 3

29. Find the equation of the ellipse that passes through the foci of the parabolas $y^2 = 8x$ and $x^2 = 24y$.

Ⓐ $x^2 + 3y^2 = 36$ Ⓑ $16x^2 + 9y^2 = 144$

Ⓒ $9x^2 + 16y^2 = 144$ Ⓓ $9x^2 + y^2 = 36$

30. Find the locus of the midpoint of a chord through the vertex of the parabola $x^2 = 4ay$.

Ⓐ $x^2 = 2ay$ Ⓑ $x^2 = 4ay$

Ⓒ $y^2 = 2ax$ Ⓓ $y^2 = 4ax$

31. A chord of the parabola $y^2 = 4ax$ through the vertex makes an angle θ with its axis. The length of the chord is -

Ⓐ $4a\cos\theta\tan^2\theta$ Ⓑ $4a\cos\theta\operatorname{cosec}^2\theta$

Ⓒ $4a\sin\theta\operatorname{cosec}^2\theta$ Ⓓ $4a\cos\theta\sec^2\theta$

32. A parabola with axis parallel to the y-axis passes through $(-3, 0)$, $(3, 0)$ and $(2, 5)$. Its vertex is -

Ⓐ $(-1, 0)$ Ⓑ $(9, 0)$ Ⓒ $(8, 0)$ Ⓓ $(0, 9)$

33. A parabola has focus $(3, 4)$ and the foot of the directrix is $(-3, -4)$. The equation of the directrix is -

Ⓐ $3x + 4y + 25 = 0$ Ⓑ $3x + 4y + 20 = 0$

Ⓒ $4x + 3y + 25 = 0$ Ⓓ $3x + 4y - 20 = 0$

34. What is the equation of the tangent at the vertex of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$?

Ⓐ $x - y = 0$ Ⓑ $2x + 2y - a = 0$

Ⓒ $x - y - a = 0$ Ⓓ $x + y + a = 0$

35. Which of the following is NOT an asymptote of the hyperbola $\frac{x^2}{3} - \frac{y^2}{1} = 1$?

Ⓐ $x - \sqrt{3}y = 0$

Ⓑ $x + \sqrt{3}y = 0$

Ⓒ $x + 2y = 0$

Ⓓ Both A and B are asymptotes

36. A parabolic dish has a diameter of 8 ft and a depth of 2 ft. If the receiver is placed at the focus of the parabola, what is the distance of the receiver from the vertex?

Ⓐ 3 Ⓑ 4 Ⓒ 2 Ⓓ 0.5

37. For an ellipse, the distance between a focus and its nearest directrix is 9 cm and the eccentricity is $\frac{4}{5}$. What is the difference between its major and minor axis?

Ⓐ 12 Ⓑ 10 Ⓒ 44 Ⓓ 16

38. Find the distance between a focus and its corresponding directrix of the ellipse $9x^2 + 5y^2 = 1$.

Ⓐ $\frac{\sqrt{5}}{6}$ Ⓑ $\frac{\sqrt{5}}{3}$ Ⓒ $\frac{\sqrt{6}}{5}$ Ⓓ $\frac{3}{2}$

39. If $\frac{x^2}{1+r} - \frac{y^2}{r-1} = 1$ with $|r| < 1$, what does this equation represent?

Ⓐ Circle Ⓑ Ellipse

Ⓒ Hyperbola Ⓓ Parabola

40. An architect's building design includes pillars shaped like a hyperbola, modeled by $\frac{x^2}{0.0625} - \frac{y^2}{0.1875} = 1$, with units in meters. If the height of the pillars equals the length of the hyperbola's latus rectum, what is the diameter at the top of the pillars?

Ⓐ 0.5 m Ⓑ 2 m Ⓒ 1 m Ⓓ 1.5 m

41. For the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, what is the area, in square units, of the region bounded by the positive major and minor axes?

Ⓐ π Ⓑ 0.5π Ⓒ 2π Ⓓ 1.5π

42. If the focal distance of a point on the parabola $(y-3)^2 + 6y - 9 = 8x$ is 5, what are the coordinates of that point?

Ⓐ $(3, \pm 2\sqrt{6})$ Ⓑ $(3, \pm 2\sqrt{5})$

Ⓒ $(3, \pm 6)$ Ⓓ $(3, \pm 5)$

43. The eccentricity of the ellipse $x^2 + 4y^2 - 2x = 15$ is -

Ⓐ $\frac{\sqrt{3}}{4}$ Ⓑ $\frac{\sqrt{3}}{2}$ Ⓒ $\frac{\sqrt{5}}{4}$ Ⓓ $\frac{3}{5}$

44. A parabola has vertex $(3, 2)$ and directrix $x = 1$. Which of the following is its equation?

Ⓐ $(y-2)^2 = 4(x-3)$ Ⓑ $(y-2)^2 = -8(x-3)$

Ⓒ $(y-2)^2 = 8(x-3)$ Ⓓ $(y-2)^2 = -4(x-3)$

45. If the latus rectum of an ellipse equals half the minor axis, what is the value of $4e^2$?

Ⓐ 3 Ⓑ $\frac{3}{4}$ Ⓒ 4 Ⓓ $\frac{1}{4}$

46. If the line $y = 3x + 1$ touches the parabola $y^2 = 12x$, what is the point of contact?

Ⓐ $\left(\frac{1}{2}, 3\right)$ Ⓑ $\left(\frac{1}{3}, 3\right)$ Ⓒ $\left(\frac{1}{3}, 1\right)$ Ⓓ $\left(\frac{1}{3}, 2\right)$

47. If the curve $(y + 4)^2 = 4k(x - 1)$ passes through the point $(2, 1)$, what is k?

Ⓐ 0.5 Ⓑ 6.25 Ⓒ 1.5 Ⓓ 2.5

48. For the parabola $y^2 = 4ax$, if one end of a focal chord is $(at^2, 2at)$, what is the length of the chord?

Ⓐ $2a\left(t + \frac{1}{t}\right)^2$ Ⓑ $a\left(t + \frac{1}{t}\right)^3$

Ⓒ $a\left(t + \frac{1}{t}\right)^2$ Ⓓ $4a\left(t + \frac{1}{t}\right)^2$

49. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the intersection of $x + y = 3$ and $2x - 3y = 1$, and has eccentricity $\sqrt{3}$. What is the length of its latus rectum?

Ⓐ 13 Ⓑ 14 Ⓒ $2\sqrt{13}$ Ⓓ $2\sqrt{14}$

50. PQ is a chord of the parabola $y^2 = 4ax$. If the ordinate of P is twice that of Q, what is the equation of the locus of the midpoint of PQ?

Ⓐ $5y^2 = 18ax$ Ⓑ $5y^2 = 16ax$

Ⓒ $y^2 = 16ax$ Ⓓ $5y^2 = 12ax$