Engineering Daily 15 [Set 03] EV - Answer Sheet

Conics

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. **B.** -2

Explanation: $x_1^2 = ay_1$, $x_2^2 = ay_2 \Rightarrow x_1 + x_2 = 2a$ (using the line's slope)
$(x_1 - x_2)^2(1 + 4) = 40 \Rightarrow (x_1 - x_2)^2 = 8$
From $x^2 - 2ax - a = 0$: $(x_1 - x_2)^2 = 4a^2 + 4a = 8$
$a^2 + a - 2 = 0 \Rightarrow (a - 1)(a + 2) = 0$
Matching the given options, $a = -2$

2. **D.** $e > \frac{2}{\sqrt{3}}$

Explanation: Let P $(a \sec \phi, b \tan \phi)$.
Since $\angle POQ = 60°$, $\angle PON = 30°$.
$\tan 30° = \frac{b \sin \phi}{a} = \frac{1}{\sqrt{3}}$
$\sin^2 \phi = \frac{a^2}{3b^2}$
Since $\sin \phi < 1$, $\frac{b^2}{a^2} > \frac{1}{3}$
$e^2 = 1 + \frac{b^2}{a^2} > \frac{4}{3}$
Therefore $e > \frac{2}{\sqrt{3}}$.

3. **D.** $4ly = 4ln - (m^2 + 1)$

Explanation: $x^2 + \frac{m}{l}x + \frac{n}{l} = \frac{y}{l}$
$\left(x + \frac{m}{2l}\right)^2 = \frac{1}{l}\left(y - \frac{4ln - m^2}{4l}\right)$
Vertex: $\left(-\frac{m}{2l}, \frac{4ln - m^2}{4l}\right)$, $4a = \frac{1}{l} \Rightarrow a = \frac{1}{4l}$
Directrix: $y = \frac{4ln - m^2}{4l} - \frac{1}{4l}$
$\therefore 4ly = 4ln - (m^2 + 1)$

4. **A.** $\frac{\sqrt{13}}{3}$

Explanation: Let $t = \tan \theta$.
$x = 6 \sec 2\theta$, $y = 4 \tan 2\theta$
$\frac{x}{6} = \sec 2\theta$, $\frac{y}{4} = \tan 2\theta$
$\frac{x^2}{6^2} - \frac{y^2}{4^2} = 1$, so $a = 6, b = 4$
$e = \sqrt{1 + \frac{16}{36}} = \frac{\sqrt{13}}{3}$.

5. **B.** $x + y + 3 = 0$

Explanation: $y^2 = 12x = 4 \cdot 3 \cdot x \Rightarrow$ focus $(3, 0)$
Negative end of latus rectum: $(3, -6)$
Point where directrix meets axis: $(-3, 0)$
Line equation: $y + 6 = \frac{-6}{6}(x - 3) \Rightarrow x + y + 3 = 0$

6. **C.** $16\sqrt{2}$

Explanation: $x^2 = 8y = 4 \cdot 2 \cdot y$, $a = 2$
Focal distance $= y + a = 18 \Rightarrow y = 16$
$x^2 = 8 \times 16 \Rightarrow x = \pm 8\sqrt{2}$
Distance between the two points $= 2 \times 8\sqrt{2} = 16\sqrt{2}$

7. **A.** $\frac{5}{\sqrt{2}}$

Explanation: The intersection point of the lines is $(5, 4)$
$\frac{25}{a^2} - \frac{16}{b^2} = 1$ and $\frac{2b^2}{a} = \frac{32}{25}$
Substituting $b^2 = \frac{16}{25}a$ and solving gives $a = \frac{5}{\sqrt{2}}$.

8. **D.** $3x^2 + 3y^2 - 2xy - 8 = 0$

Explanation: Using $SP = e \cdot PM$ and substituting $(1, 1)$ gives $e = \frac{1}{\sqrt{2}}$.
$(x - 1)^2 + (y + 1)^2 = \frac{1}{4}(x - y - 4)^2$
Simplifying: $3x^2 + 3y^2 - 2xy - 8 = 0$

9. **B.** $111.8°$

Explanation: $y = 3 \sec \theta$, $x = 2 \tan \theta \Rightarrow \tan \theta = -\frac{5}{2}$ (second quadrant, since $x < 0, y > 0$)
$\theta = 180° - \tan^{-1}\left(\frac{5}{2}\right) = 180° - 68.2° = 111.8°$

10. **B.** $2x^2 + 2y^2 - 2xy - 3 = 0$

Explanation: $S(-1, -1)$, $S'(1, 1)$, $2a = 2\sqrt{3}$
From $SP + S'P = 2a$, squaring and simplifying:
$2x^2 + 2y^2 - 2xy - 3 = 0$

11. **A.** 12

Explanation: $\frac{(x-2)^2}{5^2} + \frac{(y-3)^2}{4^2} = 1$, $e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$
Foci: $x = 2 \pm 5 \times \frac{3}{5} = 5, -1$; $y = 3$
Vertices: $(5, 3), (-1, 3), (0, 0)$
Area = 12 square units

12. **D.** $\pm\sqrt{3}$

Explanation: $\frac{x^2}{9} + \frac{y^2}{4} = 1$
With tangent slope m': $(4 - mm')^2 = 9m'^2 + 4$
$(m^2 - 9)m'^2 - 8mm' + 12 = 0$
Perpendicularity gives $m_1'm_2' = -1 = \frac{12}{m^2 - 9}$
$9 - m^2 = 12 \Rightarrow m = \pm\sqrt{3}$

13. **B.** -6

Explanation: Substituting $(0, 2)$ gives $c = 2$
$y' = 10x + 3$, at $x = 0$: $y' = 3$
Tangent: $y - 2 = 3(x - 0) \Rightarrow -6x + 2y - 4 = 0$
Parallel to $ax + 2y + 1 = 0$: $\frac{a}{-6} = \frac{2}{2} \Rightarrow a = -6$

14. **C.** $\frac{9x^2}{80} - \frac{y^2}{5} = 1$

Explanation: Asymptotes give $(4y - 3x)(4y + 3x) + K = 0$
Substituting $(4, 2)$: $(8 - 12)(8 + 12) + K = 0 \Rightarrow K = 80$
$\frac{9x^2}{80} - \frac{y^2}{5} = 1$

15. **B.** $(y^2 - 12x)(x + 3)^2 = \tan^2 \alpha$

Explanation: $a = 3$; tangent: $y = mx + \frac{3}{m} \Rightarrow m^2x - my + 3 = 0$
$m_1 + m_2 = \frac{y}{x}$, $m_1 m_2 = \frac{3}{x}$
$\tan \alpha = \frac{m_1 - m_2}{1 + m_1 m_2}$
$\tan^2 \alpha = \frac{y^2 - 12x}{(x+3)^2}$

16. **C.** $\frac{32}{3\sqrt{3}}\pi$

Explanation: $ae = \frac{19}{3} - 5 = \frac{4}{3} \Rightarrow a = \frac{8}{3}$
$b^2 = a^2(1 - e^2) \Rightarrow b = \frac{4}{\sqrt{3}}$
Area $= \pi ab = \frac{8}{3} \times \frac{4}{\sqrt{3}}\pi = \frac{32}{3\sqrt{3}}\pi$

17. **D.** 1

Explanation: $y^2 = 12z = 4 \cdot 3 \cdot z \Rightarrow$ focus $(0, 0, 3)$
$x^2 = 16z = 4 \cdot 4 \cdot z \Rightarrow$ focus $(0, 0, 4)$
Distance $= \sqrt{(0-0)^2 + (0-0)^2 + (4-3)^2} = 1$

18. **D.** $\pm\frac{3\sqrt{6}}{2}$

Explanation: $\frac{x}{2} + \frac{\sqrt{3}y}{2} = p \Rightarrow y = -\frac{1}{\sqrt{3}}x + \frac{2p}{\sqrt{3}}$;
$m = -\frac{1}{\sqrt{3}}$, $c = \frac{2p}{\sqrt{3}}$
Tangency condition: $c^2 = a^2m^2 - b^2$
$\frac{4p^2}{3} = 81 \times \frac{1}{3} - 9 = 18 \Rightarrow p^2 = \frac{27}{2}$
$p = \pm\sqrt{\frac{27}{2}} = \pm\frac{3\sqrt{6}}{2}$

19. **A.** $-\frac{4}{9}$

Explanation: $4(x - 1)^2 - 9(y - 1)^2 = 36 \Rightarrow \frac{(x-1)^2}{9} - \frac{(y-1)^2}{4} = 1$
Asymptotes: $\frac{x-1}{3} = \pm\frac{y-1}{2}$, slopes $\pm\frac{2}{3}$
Product $= -\frac{4}{9}$

20. **B.** $\frac{5x^2}{36} - \frac{y^2}{4} = 1$

Explanation: $(6, 4)$: $\frac{36}{a^2} - \frac{16}{b^2} = 1$
$(-3, 1)$: $\frac{9}{a^2} - \frac{1}{b^2} = 1$
Solving: $b^2 = 4$, $a^2 = \frac{36}{5}$
$\therefore \frac{5x^2}{36} - \frac{y^2}{4} = 1$

21. **D.** $4a \csc \theta \cot \theta$

Explanation: Chord: $y = x \tan \theta \Rightarrow x = y \cot \theta$
$y^2 = 4a \cot \theta \cdot y \Rightarrow y = 0$ or $4a \cot \theta$
$P(0, 0)$, $Q(4a \cot^2 \theta, 4a \cot \theta)$
$PQ = \sqrt{16a^2 \cot^4 \theta + 16a^2 \cot^2 \theta} = 4a \cot \theta \csc \theta$

22. **A.** $3x^2 - y^2 = 12$

Explanation: For the ellipse $e_1 = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$, foci $(\pm 4, 0)$
For the hyperbola $ae_2 = 4 \Rightarrow a \times 2 = 4 \Rightarrow a = 2$
$b^2 = a^2(e^2 - 1) = 4(4 - 1) = 12$
$\frac{x^2}{4} - \frac{y^2}{12} = 1 \Rightarrow 3x^2 - y^2 = 12$

23. **D.** $2\sqrt{14}$

Explanation: Intersection point $(2, 1)$
$\frac{4}{a^2} - \frac{1}{b^2} = 1$
$e = \sqrt{3} = \sqrt{1 + \frac{b^2}{a^2}} \Rightarrow b^2 = 2a^2$
$\frac{4}{a^2} - \frac{1}{2a^2} = 1 \Rightarrow a^2 = \frac{7}{2}$, $b^2 = 7$
Latus rectum $= \frac{2b^2}{a} = \frac{14}{\sqrt{7/2}} = 2\sqrt{14}$

24. **D.** $(3, 0)$

Explanation: $t = \frac{y}{6} \Rightarrow x = 3\left(\frac{y}{6}\right)^2 \Rightarrow y^2 = 12x$
$4a = 12 \Rightarrow a = 3$
Focus $(3, 0)$

25. **B.** 1

Explanation: Substituting $x = \frac{y^2}{12}$ and solving gives $y = -6, 18$
$y = -6 \Rightarrow x = 3 \Rightarrow A(3, -6)$
$y = 18 \Rightarrow x = 27 \Rightarrow B(27, 18)$
Slope $= \frac{18-(-6)}{27-3} = \frac{24}{24} = 1$

26. **A.** $x = -4$

Explanation: $x^2 + 8x - \frac{4}{3}y + \frac{52}{3} = 0 \Rightarrow (x + 4)^2 = \frac{4}{3}(y - 1)$
Axis: $x + 4 = 0 \Rightarrow x = -4$

27. **B.** 10

Explanation: $a = 2$; $8^2 = 8x \Rightarrow x = 8$, $P(8, 8)$
Focal distance $= x + a = 8 + 2 = 10$

28. **C.** $4x^2 - 4y - 3 = 0$
Explanation: Let P (α, β) on the parabola, $\alpha^2 = 4\beta$
Q(h, k) divides in ratio 1 : 3: $h = \frac{\alpha}{4} \Rightarrow \alpha = 4h$; $k = \frac{\beta - 3}{4} \Rightarrow \beta = 4k + 3$
$(4h)^2 = 4(4k + 3) \Rightarrow 16h^2 = 16k + 12$
$\therefore 4x^2 - 4y - 3 = 0$

29. **C.** 16
Explanation: $(x - 2)^2 = 16(y + 1)$
$4a = 16 \Rightarrow$ latus rectum $= 16$

30. **B.** $\frac{(y-4)^2}{2^2} - \frac{(x-3)^2}{2.5^2} = 1$
Explanation: Vertices $(3, 6)$ and $(3, 2)$, centre $(3, 4)$, $b = 2$ (distance from centre to vertex)
$e = \frac{\sqrt{41}}{4} \Rightarrow 1 + \frac{a^2}{b^2} = \frac{41}{16} \Rightarrow a^2 = \frac{25}{4} \Rightarrow a = 2.5$
$\frac{(y-4)^2}{2^2} - \frac{(x-3)^2}{2.5^2} = 1$

31. **C.** $\pm 60°$
Explanation: P $(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$
$\sqrt{3} = \sqrt{6\cos^2\theta + 2\sin^2\theta} \Rightarrow 3 = 4\cos^2\theta + 2$
$\cos\theta = \pm\frac{1}{2} \Rightarrow \theta = \pm 60°$

32. **C.** $(6 - \sqrt{3}, -\sqrt{3})$
Explanation: $4a = 24 \Rightarrow a = 6$
$x + \sqrt{3} = 6 \Rightarrow x = 6 - \sqrt{3}$
$y + \sqrt{3} = 0 \Rightarrow y = -\sqrt{3}$
Focus $(6 - \sqrt{3}, -\sqrt{3})$

33. **D.** 9.6
Explanation: $\frac{x^2}{10^2} + \frac{y^2}{12^2} = 1$ $(a = 10, b = 12)$
Substituting $x = 6$: $\frac{36}{100} + \frac{h^2}{144} = 1$
$h^2 = 144 \times 0.64 = 92.16 \Rightarrow h = 9.6$ metres

34. **A.** 12
Explanation: Substituting $(2, 4)$: $\frac{4}{P} + \frac{16}{2P} = 1 \Rightarrow \frac{4}{P} + \frac{8}{P} = 1 \Rightarrow \frac{12}{P} = 1$
$\therefore P = 12$

35. **A.** $x = 1, y = 2$
Explanation: $9(x - 1)^2 + 4(y - 2)^2 = 36$
$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$
Centre $(1, 2)$; it is symmetric about $x = 1$ and $y = 2$

36. **D.** $\frac{3}{2}$
Explanation: $\frac{x^2}{1/3} + \frac{y^2}{1/4} = 1 \Rightarrow a = \frac{1}{\sqrt{3}}, b = \frac{1}{2}$
$e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$
Distance $= a\left(\frac{1}{e} - e\right) = \frac{1}{\sqrt{3}}\left(2 - \frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

37. **C.** $2\sqrt{13}$
Explanation: $\frac{y^2}{4} - \frac{x^2}{9} = 1 \Rightarrow a = 2, b = 3$
$e = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$
Distance $= 2ae = 2 \times 2 \times \frac{\sqrt{13}}{2} = 2\sqrt{13}$

38. **C.** 4
Explanation: $4a = 4 \Rightarrow a = 1$; latus rectum length $= 4$
Base of the triangle $= 4$ and height $= 2a = 2$
Area $= \frac{1}{2} \times 4 \times 4 = 8$

39. **D.** $2x - 3y = 0$
Explanation: $\frac{y^2}{4} - \frac{x^2}{9} = 0 \Rightarrow y = \pm\frac{2}{3}x$
i.e. $2x - 3y = 0$ and $2x + 3y = 0$

40. **C.** $(0.5, 4)$
Explanation: $4a = 32 \Rightarrow a = 8$; $m = 4, c = 2$
Point of tangency $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{8}{16}, \frac{16}{4}\right) = (0.5, 4)$

41. **B.** 16 : 9
Explanation: For $y^2 = 8x$, latus rectum $L_1 = 8$
$\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow a = 4, b = 3$, $L_2 = \frac{2b^2}{a} = \frac{18}{4} = \frac{9}{2}$
$\frac{L_1}{L_2} = \frac{8}{9/2} = \frac{16}{9}$

42. **C.** $25x^2 + 16y^2 = 400$
Explanation: $b = 5$ (major axis along y-axis)
Directrix: $y = \pm\frac{b}{e} = \pm\frac{25}{3} \Rightarrow e = \frac{3}{5}$
$a^2 = b^2(1 - e^2) = 25\left(1 - \frac{9}{25}\right) = 16$
$\frac{x^2}{16} + \frac{y^2}{25} = 1 \Rightarrow 25x^2 + 16y^2 = 400$

43. **A.** $4a^2 + 9b^2 - 12a = 0$
Explanation: Vertex A$(3, 0)$, other end of chord (α, β): $\frac{\alpha^2}{9} + \frac{\beta^2}{4} = 1$
Midpoint $(a, b) \Rightarrow \alpha = 2a - 3, \beta = 2b$
$\frac{(2a-3)^2}{9} + b^2 = 1 \Rightarrow 4a^2 + 9b^2 - 12a = 0$

44. **C.** সরলরেখা যুগল
Explanation: $A = 1, B = 4, H = -2, G = \frac{3}{2}, F = -3, C = 2$
Determinant $\Delta = \begin{vmatrix} 1 & -2 & 3/2 \\ -2 & 4 & -3 \\ 3/2 & -3 & 2 \end{vmatrix} = 0$
Since $\Delta = 0$ and $H^2 = AB$, this represents a pair of parallel straight lines

45. **B.** -4

Explanation: Vertex $(2, 3) \Rightarrow y - 3 = A(x - 2)^2$

Substituting $(0, 7)$: $4 = 4A \Rightarrow A = 1$

$y = x^2 - 4x + 7$

Comparing, $b = -4$

46. **D.** $\pm\sqrt{34}$

Explanation: $a = 5, b = 3, m = 1$

Tangency condition: $c^2 = a^2m^2 + b^2 = 25 + 9 = 34$

$c = \pm\sqrt{34}$

47. **D.** $4\sqrt{7}$

Explanation: $a = 4, b = 3$, centre $(3, 4)$

$e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4} \Rightarrow ae = \sqrt{7}$

Foci: $(3 \pm \sqrt{7}, 4)$

Area $= \frac{1}{2} \times 2\sqrt{7} \times 4 = 4\sqrt{7}$

48. **C.** $y = \pm 2\sqrt{2}$

Explanation: $\frac{x^2}{2} + \frac{y^2}{8} = 1 \Rightarrow a = \sqrt{2}, b = 2\sqrt{2}$

Perpendicular to the y-axis means slope 0; such tangents are $y = \pm b$

$\therefore y = \pm 2\sqrt{2}$

49. **B.** 6

Explanation: $a = 4, b = 3, e = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Focal distance $d = |ey_1 - a|$ (nearer focus)

$d = \frac{5}{4} \times 8 - 4 = 10 - 4 = 6$

50. **C.**

Explanation: