Engineering Daily 15 [Set 03] EV

Conics

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. The line $y - 2x = 1$ intersects the parabola $x^2 = ay$ at two points that are 40 units apart. What is the value of a?

(A) $\frac{1}{2}$ (B) -2 (C) -1 (D) 2

2. PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If O is the centre of the hyperbola and OPQ is an equilateral triangle, which of the following is correct?

(A) $e > \frac{4}{3}$ (B) $e > \frac{1}{3}$ (C) $e < \frac{5}{3}$ (D) $e > \frac{2}{\sqrt{3}}$

3. What is the equation of the directrix of the parabola $y = lx^2 + mx + n$?

(A) $4lx = 4ln - (m^2 + 1)$ (B) $4ly = 4ln + (m^2 + 1)$

(C) $4lx = 4ln + (m^2 + 1)$ (D) $4ly = 4ln - (m^2 + 1)$

4. For all values of t, the point $x = 6 \cdot \frac{1+t^2}{1-t^2}, y = \frac{8t}{1-t^2}$ lies on a hyperbola. What is the eccentricity of that hyperbola?

(A) $\frac{\sqrt{13}}{3}$ (B) $\frac{14}{5}$ (C) 3 (D) $\frac{5}{3}$

5. What is the equation of the line joining the negative end of the latus rectum of $y^2 = 12x$ and the point where the directrix meets the axis?

(A) $x + y + 6 = 0$ (B) $x + y + 3 = 0$

(C) $x + y - 1 = 0$ (D) $x - y + 3 = 0$

6. For the points on the parabola $x^2 = 8y$ whose distance from the focus is 18 units, what is the distance between them?

(A) $8\sqrt{2}$ (B) $8\sqrt{3}$ (C) $16\sqrt{2}$ (D) $\frac{16}{3}$

7. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the intersection point of the lines $7x + 13y = 87$ and $5x - 8y + 7 = 0$, and its latus rectum length is $\frac{32}{25}$ units. Find the value of a.

(A) $\frac{5}{\sqrt{2}}$ (B) $\frac{5}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{5}{2}$

8. An ellipse has a focus at $(1, -1)$ and directrix $x - y - 4 = 0$. If the ellipse passes through $(1, 1)$, find the equation of the ellipse.

(A) $x^2 + y^2 - 3xy + 1 = 0$

(B) $2x^2 + 2y^2 + 3xy + 5 = 0$

(C) $5x^2 + 5y^2 + 9xy - 5 = 0$

(D) $3x^2 + 3y^2 - 2xy - 8 = 0$

9. For the point $\left(-5, \frac{8}{3}\right)$ lying on the hyperbola $\frac{y^2}{9} - \frac{x^2}{4} = 1$, what is the parametric angle?

(A) $53.13°$ (B) $111.8°$

(C) $68.2°$ (D) $126.87°$

10. Find the equation of the ellipse whose foci are $(-1, -1), (1, 1)$ and whose major axis has length $2\sqrt{3}$.

(A) $2x^2 + 2y^2 - 2xy - 6 = 0$

(B) $2x^2 + 2y^2 - 2xy - 3 = 0$

(C) $2x^2 + y^2 - 3xy - 4 = 0$

(D) $2x^2 + 2y^2 - 3xy = 0$

11. The vertices of a triangle are the two foci of the ellipse $16(x - 2)^2 + 25(y - 3)^2 = 400$ and the origin. What is the area of this triangle?

(A) 12 (B) 20 (C) 9 (D) 10

12. For what value of m are the two tangents drawn from the point $(m, 4)$ to the ellipse $4x^2 + 9y^2 = 36$ mutually perpendicular?

(A) ± 3 (B) ± 2 (C) $\pm\sqrt{2}$ (D) $\pm\sqrt{3}$

13. The point $(0, 2)$ lies on the curve $y = 5x^2 + 3x + c$. If the tangent to the curve at that point is parallel to the line $ax + cy + 1 = 0$, what is the value of a?

(A) -1 (B) -6 (C) -4 (D) 4

14. A hyperbola-shaped cooling tower has asymptotes $y = \pm\frac{3}{4}x$. If the hyperbola passes through $(4, 2)$, what is its equation?

(A) $\frac{16x^2}{80} - \frac{y^2}{5} = 1$ (B) $\frac{16x^2}{80} + \frac{y^2}{5} = 1$

(C) $\frac{9x^2}{80} - \frac{y^2}{5} = 1$ (D) $\frac{x^2}{80} - \frac{y^2}{5} = 1$

15. If the angle between two tangents to the parabola $y^2 = 12x$ is α, find the locus of the point of intersection of the tangents.

(A) $(y^2 - 8x)(x + 3)^2 = \tan^2\alpha$

(B) $(y^2 - 12x)(x + 3)^2 = \tan^2\alpha$

(C) $(y^2 - 12x)(x + 2)^2 = \tan^2\alpha$

(D) $(y^2 - 8x)(x + 2)^2 = \tan^2\alpha$

16. An ellipse has centre $\left(\frac{19}{3}, 0\right)$, a focus at $(5, 0)$, and eccentricity $\frac{1}{2}$. What is the area of the ellipse in square units?

(A) $\frac{32}{39}\pi$ (B) $\frac{19}{33}\pi$

(C) $\frac{32}{3\sqrt{3}}\pi$ (D) $\frac{32}{3}\pi$

17. What is the distance between the foci of the parabolas $y^2 = 12z$ and $x^2 = 16z$?

(A) 4 (B) 5 (C) 2 (D) 1

18. If the line $x\cos\frac{\pi}{3} + y\sin\frac{\pi}{3} = p$ is a tangent to the hyperbola $\frac{x^2}{81} - \frac{y^2}{9} = 1$, what is the value of p?

(A) $\pm\frac{9\sqrt{2}}{4}$ (B) $\pm\frac{9}{2}$

(C) $\pm\frac{27}{2}$ (D) $\pm\frac{3\sqrt{6}}{2}$

19. What is the product of the slopes of the two asymptotes of the conic $4x^2 - 9y^2 - 8x + 18y - 41 = 0$?

Ⓐ $-\frac{4}{9}$ Ⓑ -1 Ⓒ $\frac{3}{2}$ Ⓓ $\frac{4}{9}$

20. A hyperbola passes through the points $(6, 4)$ and $(-3, 1)$. Its centre is at the origin and its transverse axis is along the x-axis. Find the equation of the hyperbola.

Ⓐ $\frac{y^2}{4} - \frac{5x^2}{36} = 1$ Ⓑ $\frac{5x^2}{36} - \frac{y^2}{4} = 1$

Ⓒ $\frac{y^2}{36} - \frac{x^2}{4} = 1$ Ⓓ $\frac{x^2}{36} - \frac{y^2}{4} = 1$

21. For the parabola $y^2 = 4ax$, find the length of the chord through the vertex that is inclined at an angle θ to the positive x-axis.

Ⓐ $4a \sec\theta \tan\theta$ Ⓑ $4a \csc\theta$

Ⓒ $4a \cos\theta \cot\theta$ Ⓓ $4a \csc\theta \cot\theta$

22. A hyperbola has the same foci as the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. If the hyperbola's eccentricity is 2, find its equation.

Ⓐ $3x^2 - y^2 = 12$ Ⓑ $x^2 - 3y^2 = 12$

Ⓒ $3x^2 - 3y^2 = 14$ Ⓓ $3x^2 - y^2 = 10$

23. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the intersection point of $x + y = 3$ and $2x - 3y = 1$, and its eccentricity is $\sqrt{3}$. What is the length of its latus rectum?

Ⓐ 27 Ⓑ 26 Ⓒ 7 Ⓓ $2\sqrt{14}$

24. For the parabola given parametrically by $x = 3t^2, y = 6t$, what is its focus?

Ⓐ $(0, 4)$ Ⓑ $(4, 0)$ Ⓒ $(0, 3)$ Ⓓ $(3, 0)$

25. What is the slope of the line joining the intersection points of the parabolas $y^2 = 12x$ and $(x - 9)^2 = 12(y + 9)$?

Ⓐ $\frac{1}{2}$ Ⓑ 1 Ⓒ 2 Ⓓ -1

26. What is the equation of the axis of the parabola $3x^2 + 24x - 4y + 52 = 0$?

Ⓐ $x = -4$ Ⓑ $x = -1$

Ⓒ $x = -3$ Ⓓ $x = 2$

27. A point P on the parabola $y^2 = 8x$ has ordinate 8. What is the focal distance of P ?

Ⓐ $4\sqrt{12}$ Ⓑ 10 Ⓒ 12 Ⓓ 8

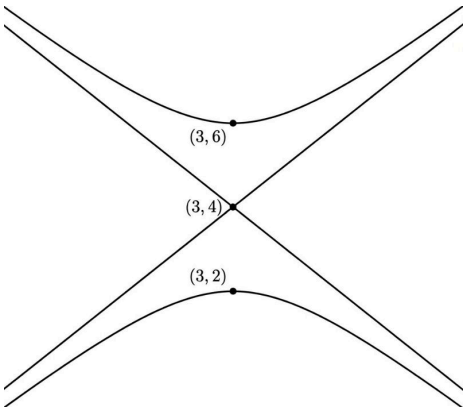
28. Find the locus of the points that divide, in the ratio 1 : 3, the line segment joining the point $(0, -1)$ and points on the parabola $x^2 = 4y$.

Ⓐ $4x^2 - 4y - 1 = 0$ Ⓑ $4x^2 - y - 3 = 0$

Ⓒ $4x^2 - 4y - 3 = 0$ Ⓓ $4x^2 - y - 4 = 0$

29. What is the length of the latus rectum of the parabola $x^2 - 4x - 16y - 12 = 0$?

Ⓐ 12 Ⓑ 6 Ⓒ 16 Ⓓ 10



30. Which is the equation of the hyperbola shown in the figure, given its eccentricity is $\frac{\sqrt{41}}{4}$?

Ⓐ $\frac{(y-4)^2}{2^2} - \frac{(x-3)^2}{3^2} = 1$

Ⓑ $\frac{(y-4)^2}{2^2} - \frac{(x-3)^2}{2.5^2} = 1$

Ⓒ $\frac{(y-4)^2}{2^2} - \frac{(x-3)^2}{4^2} = 1$

Ⓓ $\frac{(y-4)^2}{2.5^2} - \frac{(x-3)^2}{2^2} = 1$

31. For a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from the centre is $\sqrt{3}$ units, what is the eccentric angle of that point?

Ⓐ $\pm 45°$ Ⓑ $\pm 30°$ Ⓒ $\pm 60°$ Ⓓ $\pm 75°$

32. What are the coordinates of the focus of the parabola $(y + \sqrt{3})^2 = 24(x + \sqrt{3})$?

Ⓐ $(-3, 6 - \sqrt{3})$ Ⓑ $(3, 6 - \sqrt{3})$

Ⓒ $(6 - \sqrt{3}, -\sqrt{3})$ Ⓓ $(6 - \sqrt{3}, 3)$

33. An elliptical tunnel has a maximum height of 12 metres and a total width of 20 metres at ground level. What is the height of the tunnel at a horizontal distance of 6 m from the centre?

Ⓐ 10 Ⓑ 8.2 Ⓒ 8.6 Ⓓ 9.6

34. The ellipse $\frac{x^2}{P} + \frac{y^2}{2P} = 1$ passes through the point $(2, 4)$. What is the value of P ?

Ⓐ 12 Ⓑ 10 Ⓒ 8 Ⓓ 14

35. The ellipse $9x^2 + 4y^2 - 18x - 16y = 11$ is symmetric about which lines?

Ⓐ $x = 1, y = 2$ Ⓑ $x = 2, y = 1$

Ⓒ $x = 1, y = 4$ Ⓓ $x = 2, y = 4$

36. Find the distance between a focus of the ellipse $3x^2 + 4y^2 = 1$ and its corresponding directrix.

Ⓐ $\frac{1}{2}$ Ⓑ 2 Ⓒ $\frac{1}{3}$ Ⓓ $\frac{3}{2}$

37. What is the distance between the foci of the hyperbola $9y^2 - 4x^2 = 36$?

Ⓐ $8\sqrt{2}$ Ⓑ 13 Ⓒ $2\sqrt{13}$ Ⓓ $2\sqrt{5}$

38. Find the area of the triangle formed by the foot of the directrix and the two endpoints of the latus rectum of the parabola $(x - 5)^2 = -4(y - 5)$.

Ⓐ 1 Ⓑ 8 Ⓒ 4 Ⓓ 2

39. Which of the following is an equation of one of the asymptotes of the hyperbola $\frac{y^2}{4} - \frac{x^2}{9} = 1$?

(A) $x + 3y = 0$ (B) $3x + 2y = 0$

(C) $3x - 2y = 0$ (D) $2x - 3y = 0$

40. The line $y = 4x + 2$ is tangent to the parabola $y^2 = 32x$. What is the point of tangency?

(A) $(4, 0.5)$ (B) $(4, 2)$

(C) $(0.5, 4)$ (D) $(2, 4)$

41. What is the ratio of the latus rectum of the parabola $y^2 = 8x$ to that of the ellipse $9x^2 + 16y^2 = 144$?

(A) 4 : 3 (B) 16 : 9 (C) 1 : 8 (D) 2 : 9

42. An ellipse has vertices $(0, \pm 5)$ and directrices $y = \pm\frac{25}{3}$. Find the equation of the ellipse.

(A) $9x^2 + 16y^2 = 144$ (B) $16x^2 + 25y^2 = 400$

(C) $25x^2 + 16y^2 = 400$ (D) $25x^2 + 9y^2 = 225$

43. For the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, if (a, b) is the midpoint of a chord drawn from a vertex on the major axis, find the relation between a and b.

(A) $4a^2 + 9b^2 - 12a = 0$ (B) $4a^2 + 9b^2 - 10a = 0$

(C) $9a^2 + 4b^2 - 12a = 0$ (D) $4a^2 + b^2 - 10a = 0$

44. What kind of conic does the equation $x^2 + 3x + 4y^2 - 4xy - 6y + 2 = 0$ represent?

(A) উপবৃত্ত (B) অধিবৃত্ত

(C) সরলরেখা যুগল (D) পরাবৃত্ত

45. The parabola $y = ax^2 + bx + c$ has its vertex at $(2, 3)$ and passes through $(0, 7)$. What is the value of b?

(A) 7 (B) -4 (C) 1 (D) -2

46. If the line $y = x + c$ is tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, what is the value of c?

(A) $\pm\sqrt{26}$ (B) $\pm\sqrt{13}$ (C) ± 17 (D) $\pm\sqrt{34}$

47. The vertices of a triangle are the two foci of the ellipse $\frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} = 1$ and the origin. What is the area of this triangle?

(A) $4\sqrt{5}$ (B) $5\sqrt{7}$ (C) $7\sqrt{5}$ (D) $4\sqrt{7}$

48. For the curve $4x^2 + y^2 = 8$, two tangents are perpendicular to the y-axis. What are the equations of those tangents?

(A) $y = \pm 2\sqrt{3}$ (B) $y = \pm 2\sqrt{5}$

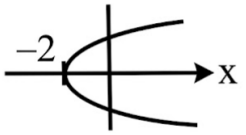
(C) $y = \pm 2\sqrt{2}$ (D) $y = \pm 2$

49. For the hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$, what is the focal distance of the point $(3\sqrt{3}, 8)$?

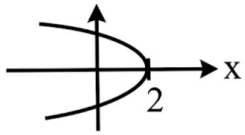
(A) 12 (B) 6 (C) 10 (D) 8

50. Which of the following is the graph of $x = -(y - 2)^2$?

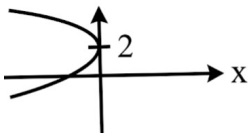
(A)



(B)



(C)



(D)

