MCQ 50 marks

1. **A. Ellipse**
Explanation: $y = \sqrt{4 - 2x^2}$
Squaring, $y^2 = 4 - 2x^2 \Rightarrow 2x^2 + y^2 = 4$
$\frac{x^2}{2} + \frac{y^2}{4} = 1$
This is an ellipse

2. **B.** $2x^2 - 4y + 3 = 0$
Explanation: Let the point on the parabola be $P(x_1, y_1)$, so $x_1^2 = 8y_1$; fixed point $A(0, 1)$
The point (h, k) dividing A, P in ratio $1 : 3$:
$h = \frac{x_1}{4} \Rightarrow x_1 = 4h, \quad k = \frac{y_1+3}{4} \Rightarrow y_1 = 4k - 3$
Substituting, $16h^2 = 8(4k - 3)$
So $2x^2 - 4y + 3 = 0$

3. **C.** $4a$
Explanation: Tangency condition: $c^2 = a^2m^2 - b^2$
Here $b^2 = 16, m = 2, c = 1$: $1 = 4a^2 - 16 \Rightarrow a^2 = \frac{17}{4} \Rightarrow 2a = \sqrt{17}$
Hypotenuse $= \sqrt{8^2 + 2^2} = \sqrt{68} = 2\sqrt{17} = 2 \times 2a$
So the hypotenuse equals $4a$

4. **D.** 16
Explanation: $y = 2x + 2 \Rightarrow m = 2, c = 2$
Tangency condition: $c = \frac{a}{m} \Rightarrow a = mc = 2 \times 2 = 4$
Latus rectum $= 4a = 16$

5. **A.** $\sqrt{\frac{3}{2}}$
Explanation: For a hyperbola, the normal at (x_1, y_1) meets the x-axis at x_1e^2
Given, $x_1 = 6$, intercept $= 9$
$6e^2 = 9 \Rightarrow e^2 = \frac{3}{2} \Rightarrow e = \sqrt{\frac{3}{2}}$

6. **B.** $x + y = -a$
Explanation: A tangent to $y^2 = 4ax$ is $y = mx + \frac{a}{m}$
For it to also touch $x^2 = 4ay$: $x^2 - 4amx - \frac{4a^2}{m} = 0$ has zero discriminant
$(4am)^2 + \frac{16a^2}{m} = 0 \Rightarrow m^3 + 1 = 0 \Rightarrow m = -1$
So $y = -x - a \Rightarrow x + y = -a$

7. **C.** $4\sec\theta, \sqrt{5}\tan\theta$
Explanation: $\frac{x^2}{16} - \frac{y^2}{5} = 1 \Rightarrow a = 4, b = \sqrt{5}$
Parametric coordinates $(a\sec\theta, b\tan\theta) = (4\sec\theta, \sqrt{5}\tan\theta)$

8. **D.** $x - 4 = 0$
Explanation: $4(x - 1)^2 - 5(y + 2)^2 = 36 + 4 - 20 = 20$
$\frac{(x-1)^2}{5} - \frac{(y+2)^2}{4} = 1 \Rightarrow a = \sqrt{5}, b = 2$
$e = \sqrt{1 + \frac{4}{5}} = \frac{3}{\sqrt{5}}$, so $ae = 3$
Latus rectum: $x - 1 = \pm 3 \Rightarrow x = 4$ or $x = -2$
So $x - 4 = 0$

9. **A.** $(3, 0)$
Explanation: $r^2\sin^2\theta = 12r\cos\theta$
$(r\sin\theta)^2 = 12(r\cos\theta) \Rightarrow y^2 = 12x$
$4a = 12 \Rightarrow a = 3$
Focus $(3, 0)$

10. **B.** $\pm 45°$
Explanation: $a = \sqrt{6}, b = \sqrt{2}$; parametric point $(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$
Given, $6\cos^2\theta + 2\sin^2\theta = 4$
$4\cos^2\theta + 2 = 4 \Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$
$\theta = \pm 45°$

11. **B.** $(4, 0)$
Explanation: $2y = x + 16 \Rightarrow y = \frac{1}{2}x + 8 \Rightarrow m = \frac{1}{2}, c = 8$
Tangency condition: $c = \frac{a}{m} \Rightarrow a = mc = \frac{1}{2} \times 8 = 4$
Focus $(4, 0)$

12. **C.** $\frac{(x-4)^2}{25} + \frac{(y-2)^2}{9} = 1$
Explanation: Centre $\left(\frac{-1+9}{2}, 2\right) = (4, 2)$
$2a = |9 - (-1)| = 10 \Rightarrow a = 5$
$e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow \frac{16}{25} = 1 - \frac{b^2}{25} \Rightarrow b^2 = 9$
Equation: $\frac{(x-4)^2}{25} + \frac{(y-2)^2}{9} = 1$

13. **D. Parabola**
Explanation: By definition, $SP = e \cdot PM$
Given, $SP = PM \Rightarrow e = 1$
Since $e = 1$, the locus is a parabola

14. **A.** $-4a^2$

Explanation: Let the endpoints be $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$; for a focal chord, $t_1t_2 = -1$
Product of ordinates $y_1y_2 = (2at_1)(2at_2) = 4a^2t_1t_2 = 4a^2(-1) = -4a^2$

15. **B.** $x = 8$
Explanation: Vertex $(8, -3)$
Since the axis is parallel to the x-axis, the tangent at the vertex is perpendicular to the axis, i.e. a vertical line
Tangent: $x = 8$

16. **C.** $\sqrt{2}$
Explanation: This is a hyperbola, transverse axis $2a = 4\sqrt{2} \Rightarrow a = 2\sqrt{2}$
Distance between foci $2ae = 8 \Rightarrow ae = 4$
$e = \frac{4}{2\sqrt{2}} = \sqrt{2}$

17. **D.** $9x^2 - 9y^2 - 12y - 8 = 0$
Explanation: Let the point on the hyperbola be $P(x_1, y_1)$, so $x_1^2 - y_1^2 = 4$; fixed point $A(0, -1)$
The point (h, k) dividing A, P in ratio $1 : 2$:
$h = \frac{x_1}{3} \Rightarrow x_1 = 3h, \quad k = \frac{y_1 - 2}{3} \Rightarrow y_1 = 3k + 2$
Substituting, $9h^2 - (3k + 2)^2 = 4 \Rightarrow 9h^2 - 9k^2 - 12k - 8 = 0$
So $9x^2 - 9y^2 - 12y - 8 = 0$

18. **A.** $\frac{2}{3}$
Explanation: $y^2 = 8x \Rightarrow 4a = 8 \Rightarrow a = 2$
Tangency condition: $c = \frac{a}{m} = \frac{2}{3}$

19. **B.** 10
Explanation: $16x^2 + 25y^2 = 400 \Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1 \Rightarrow a = 5$
For any point on an ellipse, $PF_1 + PF_2 = 2a = 10$

20. **C.** 64
Explanation: $y^2 = 64x \Rightarrow 4a = 64$
Latus rectum $= 4a = 64$

21. **A.** $2\sqrt{3}$
Explanation: Here $a^2 = 3\sqrt{3}$, $b = 3$ (major axis)
Latus rectum $= \frac{2a^2}{b} = \frac{2 \times 3\sqrt{3}}{3} = 2\sqrt{3}$

22. **A.** $(0, \pm 4\sqrt{2})$
Explanation: Rectangular hyperbola: $y^2 - x^2 = a^2$
Through $(3, 5)$: $a^2 = 5^2 - 3^2 = 16 \Rightarrow a = 4$
For a rectangular hyperbola, $e = \sqrt{2}$
Foci $(0, \pm ae) = (0, \pm 4\sqrt{2})$

23. **C.** $\sqrt{\frac{3}{2}}$
Explanation: $\frac{y^2}{(\sqrt{2})^2} - \frac{x^2}{1^2} = 1 \Rightarrow a^2 = 2, b^2 = 1$
$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$

24. **D.** $x + 2y = 5$
Explanation: Centre $(1, 2)$; $2ae = |5 - (-1)| = 6 \Rightarrow ae = 3$
$e^2 = 1 + \frac{b^2}{a^2} \Rightarrow 5 = 1 + \left(\frac{b}{a}\right)^2 \Rightarrow \frac{b}{a} = 2 \Rightarrow \frac{a}{b} = \frac{1}{2}$
Since the axis is vertical, asymptotes: $y - 2 = \pm\frac{a}{b}(x - 1) = \pm\frac{1}{2}(x - 1)$
Taking the negative sign: $2y - 4 = -(x - 1) \Rightarrow x + 2y = 5$

25. **D.** $15x^2 + 6xy + 7y^2 - 48x - 48y = 0$
Explanation: $y^2 = 8x \Rightarrow$ focus $P(2, 0)$; $x^2 = 24y \Rightarrow$ focus $Q(0, 6)$
Since it passes through the origin, $OP + OQ = 2a \Rightarrow 2 + 6 = 2a \Rightarrow a = 4$
For a point $B(x, y)$, $BP + BQ = 8$; squaring and simplifying,
$15x^2 + 6xy + 7y^2 - 48x - 48y = 0$

26. **B. Hyperbola**
Explanation: Squaring both sides and simplifying gives a general quadratic $Ax^2 + Bxy + Cy^2 + \cdots = 0$ with $B^2 - 4AC > 0$
Since $B^2 - 4AC > 0$, this represents a hyperbola

27. **C. Both (A) and (B)**
Explanation: Equation: $\frac{x^2}{2/3} + \frac{y^2}{2/b} = 1$
For an ellipse, $\frac{2}{b} > 0 \Rightarrow b > 0$
To avoid a circle, $\frac{2}{b} \neq \frac{2}{3} \Rightarrow b \neq 3$
So both $b > 0$ and $b \neq 3$ are required

28. **D.** $\left(-\frac{1}{2}, -3\right)$
Explanation: $(x + 1)^3 = x^3 + 3x^2 + 3x + 1$
$x^3 + 3x^2 + 3x + 1 + y^2 + 6y = x^3 + 6$
$3x^2 + 3x + y^2 + 6y - 5 = 0$
$3\left(x + \frac{1}{2}\right)^2 + (y + 3)^2 = \frac{59}{4}$
Centre $\left(-\frac{1}{2}, -3\right)$

29. **A.** $9x - 8y = 20$

Explanation: Equation of the bisected chord: $T = S_1$
$T = 9x(4) - 16y(2) - 144 = 36x - 32y - 144$
$S_1 = 9(16) - 16(4) - 144 = -64$
$36x - 32y - 144 = -64 \Rightarrow 36x - 32y = 80$
Simplifying, $9x - 8y = 20$

30. **B.** $\frac{x^2}{16} - \frac{y^2}{9} = 1$
Explanation: $2ae = 10 \Rightarrow ae = 5$; with $e = \frac{5}{4}$, $a = \frac{5}{5/4} = 4 \Rightarrow a^2 = 16$
$b^2 = a^2(e^2 - 1) = 16\left(\frac{25}{16} - 1\right) = 9$
Equation: $\frac{x^2}{16} - \frac{y^2}{9} = 1$

31. **C.** 5
Explanation: $4y = 4x + 4k \Rightarrow y = x + k \Rightarrow m = 1, c = k$
Here $a^2 = 16, b^2 = 9$
Tangency condition: $c^2 = a^2m^2 + b^2 \Rightarrow k^2 = 16 + 9 = 25 \Rightarrow k = \pm 5$

32. **C.** -18
Explanation: Through $(2, 3)$: $(3 + 3)^3 = 4k(2 - 5)$
$216 = -12k$
$k = -18$

33. **D.** 2
Explanation: Through $(\pm 1, 0)$: $p(1) + 0 = 1 \Rightarrow p = 1$
Equation: $x^2 + 9y^2 = 1 \Rightarrow \frac{x^2}{1} + \frac{y^2}{1/9} = 1$
Since $1 > \frac{1}{9}$, the major axis is along x, $a = 1$
Major axis length $= 2a = 2$

34. **A.** $(3, 2)$
Explanation: $4(y - 2)^2 = 32(x - 3) \Rightarrow (y - 2)^2 = 8(x - 3)$
Directly from this form, vertex $(3, 2)$

35. **B.** $16x^2 - 9y^2 + 64x + 18y + 199 = 0$
Explanation: Let the point be $P(x, y)$
$\frac{|4x-3y+11|}{5} \times \frac{|4x+3y+5|}{5} = \frac{144}{25}$
$|(4x - 3y + 11)(4x + 3y + 5)| = 144$
Expanding, $16x^2 - 9y^2 + 64x + 18y + 55 = \pm 144$
So $16x^2 - 9y^2 + 64x + 18y + 199 = 0$

36. **C.** $-3a^2$
Explanation: $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$; since it's a focal chord, $t_1t_2 = -1$
$x_1x_2 = a^2t_1^2t_2^2 = a^2(t_1t_2)^2 = a^2$
$y_1y_2 = 4a^2t_1t_2 = -4a^2$
$x_1x_2 + y_1y_2 = a^2 - 4a^2 = -3a^2$

37. **D.** $x^2 - 3y^2 = 3$
Explanation: $x^2 + 4y^2 = 4 \Rightarrow \frac{x^2}{4} + \frac{y^2}{1} = 1 \Rightarrow e_1 = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$
Foci of the ellipse: $(\pm\sqrt{3}, 0)$
Hyperbola's $e_2 = \frac{2}{\sqrt{3}}$; through $(\pm\sqrt{3}, 0)$: $\frac{3}{a^2} = 1 \Rightarrow a^2 = 3$
$e_2^2 = 1 + \frac{b^2}{a^2} \Rightarrow \frac{4}{3} = 1 + \frac{b^2}{3} \Rightarrow b^2 = 1$
Equation: $x^2 - 3y^2 = 3$

38. **A.** 9.455×10^7 **miles**
Explanation: $e = \frac{1}{60}$, $a = 9.3 \times 10^7$
Maximum distance $d_{max} = a(1 + e) = 9.3 \times 10^7\left(1 + \frac{1}{60}\right)$
$= 9.3 \times 10^7 \times \frac{61}{60} = 9.455 \times 10^7$ miles

39. **B.** 1.94 m
Explanation: Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; given $a = 2$ m, $b = 2$ m
$x^2 + y^2 = 4$
At 1.5 m from either end, $x = 2 - 1.5 = 0.5$
$(0.5)^2 + h^2 = 4 \Rightarrow h = \sqrt{3.75} = 1.94$ m

40. **A.** $x^2 - y^2 = 32$
Explanation: $2ae = 16 \Rightarrow ae = 8$; with $e = \sqrt{2}$, $a = \frac{8}{\sqrt{2}} = 4\sqrt{2} \Rightarrow a^2 = 32$
$b^2 = a^2(e^2 - 1) = 32(2 - 1) = 32$
Equation: $\frac{x^2}{32} - \frac{y^2}{32} = 1 \Rightarrow x^2 - y^2 = 32$

41. **A.** $x^2 + 4x + 8y - 36 = 0$
Explanation: $|4a| = 8 \Rightarrow a = \pm 2$; axis parallel to y-axis: $(x + 2)^2 = \pm 8(y - 5)$
Taking the negative sign: $x^2 + 4x + 4 = -8y + 40$
So $x^2 + 4x + 8y - 36 = 0$

42. **D.** $(\pm 2\sqrt{10}, 0)$
Explanation: Intersection of $x - 3\sqrt{5}y = 0$ and $\sqrt{5}x - 2y = 13$ is $(3\sqrt{5}, 1)$
Through this point: $\frac{45}{a^2} - \frac{1}{b^2} = 1$
Latus rectum: $\frac{2b^2}{a} = \frac{4}{3} \Rightarrow b^2 = \frac{2a}{3}$
Solving, $a^2 = 36, b^2 = 4$
$e = \sqrt{1 + \frac{4}{36}} = \frac{\sqrt{10}}{3}$
Foci $(\pm ae, 0) = (\pm 2\sqrt{10}, 0)$

43. **B.** $\pm\sqrt{2}$

Explanation: $x - py + 4 = 0 \Rightarrow y = \frac{1}{p}x + \frac{4}{p} \Rightarrow m = \frac{1}{p}, c = \frac{4}{p}$
$y^2 = 8x \Rightarrow a = 2$
Tangency condition: $c = \frac{a}{m} \Rightarrow \frac{4}{p} = 2p \Rightarrow p^2 = 2$
$p = \pm\sqrt{2}$

44. **C. Ellipse**
Explanation: By definition, the ratio of distances = $e = \frac{1}{2}$
Since $e < 1$, the locus is an ellipse

45. **B.** $x = \infty$
Explanation: $e = \sqrt{1 - \frac{b^2}{a^2}}$; if $a = b$, then $e = 0$
Directrix: $x = \pm\frac{a}{e} = \pm\frac{a}{0} \Rightarrow x = \infty$

46. **D.** $3x + y + 1 = 0$
Explanation: Slope of $x - 3y + 5 = 0$ is $\frac{1}{3}$, so the perpendicular slope $m = -3$
$y^2 = 12x \Rightarrow 4a = 12 \Rightarrow a = 3$
Tangent: $y = mx + \frac{a}{m} = -3x + \frac{3}{-3} = -3x - 1$
So $3x + y + 1 = 0$

47. **C.** 6.4
Explanation: $16x^2 + 25y^2 = 400 \Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1 \Rightarrow a = 5, b^2 = 16$
Latus rectum = $\frac{2b^2}{a} = \frac{32}{5} = 6.4$

48. **D.** $x = \pm 5$
Explanation: $4x^2 + 5y^2 = 20 \Rightarrow \frac{x^2}{5} + \frac{y^2}{4} = 1 \Rightarrow a^2 = 5, b^2 = 4$
$e = \sqrt{1 - \frac{4}{5}} = \frac{1}{\sqrt{5}}$
Directrix: $x = \pm\frac{a}{e} = \pm\left(\sqrt{5} \times \sqrt{5}\right) = \pm 5$

49. **A.** 24.56
Explanation: $a = 3, b = 4$
Area of the ellipse = $\pi ab = \pi \times 3 \times 4 = 37.70$ square units
Side of the square = $\sqrt{37.70} = 6.14$
Perimeter = $4 \times 6.14 = 24.56$

50. **B.** $\frac{5}{3}$
Explanation: $\frac{x}{3} = \cot u + \tan u$, $\frac{y}{4} = \cot u - \tan u$
$\left(\frac{x}{3}\right)^2 - \left(\frac{y}{4}\right)^2 = 4 \cot u \tan u = 4$
$\frac{x^2}{36} - \frac{y^2}{64} = 1 \Rightarrow a^2 = 36, b^2 = 64$
$e = \sqrt{1 + \frac{64}{36}} = \sqrt{\frac{100}{36}} = \frac{5}{3}$