Engineering Daily 15 [Home Practice] EV

Conics

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. What kind of curve is $y = \sqrt{4 - 2x^2}$?

   (A) Ellipse (B) Hyperbola
   (C) Parabola (D) Circle

2. A point on the parabola $x^2 = 8y$ is joined to $(0, 1)$. Which is the locus of the point that divides this segment in the ratio $1 : 3$?

   (A) $2x^2 - 4y + 5 = 0$ (B) $2x^2 - 4y + 3 = 0$
   (C) $2x - 4y + 5 = 0$ (D) $2x^2 - y + 5 = 0$

3. For $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, one tangent is $y = 2x + 1$. The hypotenuse of a right triangle with legs 8 and 2 equals how many times $2a$?

   (A) $2a$ (B) a (C) $4a$ (D) $8a$

4. If the line $y = 2x + 2$ touches the parabola $y^2 = 4ax$, what is the length of the latus rectum?

   (A) 4 (B) 12 (C) 8 (D) 16

5. P $(6, 3)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal drawn at P meets the x-axis at $(9, 0)$, what is e?

   (A) $\sqrt{\frac{3}{2}}$ (B) $\frac{5}{2}$ (C) $\frac{5}{3}$ (D) 2

6. What is the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4ay$?

   (A) $x + y = a$ (B) $x + y = -a$
   (C) $x + y = 0$ (D) $x - y + a = 0$

7. What are the parametric coordinates of the hyperbola $\frac{x^2}{16} - \frac{y^2}{5} = 1$?

   (A) $5 \sec\theta, 4 \tan\theta$ (B) $2 \sec\theta, 5 \tan\theta$
   (C) $4 \sec\theta, \sqrt{5} \tan\theta$ (D) $4 \sec\theta, 25 \tan\theta$

8. For the conic $4x^2 - 8x - 5y^2 - 20y - 36 = 0$, what is the equation of a latus rectum?

   (A) $x + 4 = 0$ (B) $x + 1 = 0$
   (C) $x - 1 = 0$ (D) $x - 4 = 0$

9. If $r^2 \sin^2\theta = 12r \cos\theta$, what are the coordinates of the focus?

   (A) $(3, 0)$ (B) $(6, 0)$ (C) $(4, 0)$ (D) $(0, 4)$

10. For a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from the centre is 2 units, find its eccentric angle.

    (A) $\pm 30°$ (B) $\pm 45°$ (C) $\pm 60°$ (D) $\pm 75°$

11. If the line $2y = x + 16$ is tangent to the parabola $y^2 = 4ax$, what is the focus of the parabola?

    (A) $(8, 0)$ (B) $(4, 0)$
    (C) $(2, 0)$ (D) $\left(\frac{3}{4}, 0\right)$

12. An ellipse has vertices $(-1, 2)$ and $(9, 2)$, and eccentricity $\frac{4}{5}$. What is its equation?

    (A) $\frac{(x-4)^2}{9} + \frac{(y-2)^2}{25} = 1$ (B) $\frac{(x-4)^2}{16} + \frac{(y-2)^2}{9} = 1$
    (C) $\frac{(x-4)^2}{25} + \frac{(y-2)^2}{9} = 1$ (D) $\frac{(x-4)^2}{4} + \frac{(y-2)^2}{9} = 1$

13. A point moves such that its distance from $(2, 5)$ always equals its distance from the line $2x + 4y - 3 = 0$. What is the name of its locus?

    (A) Ellipse (B) Hyperbola
    (C) A pair of straight lines (D) Parabola

14. For a focal chord of the parabola $y^2 = 4ax$, what is the product of the ordinates of its endpoints?

    (A) $-4a^2$ (B) a (C) $-4a$ (D) a^2

15. What is the equation of the tangent at the vertex of the parabola $(y + 3)^2 = 16(x - 8)$?

    (A) $x - 4 = 0$ (B) $x = 8$ (C) $x + 8 = 0$ (D) $x = -3$

16. A point moves in a plane such that the difference of its distances from $(4, 0)$ and $(-4, 0)$ is always $4\sqrt{2}$. What is the eccentricity of the locus?

    (A) 3 (B) 5 (C) $\sqrt{2}$ (D) $\frac{3}{2}$

17. A point on the hyperbola $x^2 - y^2 = 4$ is joined to $(0, -1)$. Which is the locus of the point that divides this segment in the ratio $1 : 2$?

    (A) $9x^2 - 9y^2 - 12y + 8 = 0$
    (B) $9x^2 - 9y^2 - 10y - 8 = 0$
    (C) $9x^2 - y^2 - 12y + 8 = 0$
    (D) $9x^2 - 9y^2 - 12y - 8 = 0$

18. If the line $y = 3x + c$ touches the parabola $y^2 = 8x$, what is the value of c?

    (A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) $\frac{3}{5}$

19. P (x, y) is a point on an ellipse with foci $F_1(3, 0), F_2(-3, 0)$. For the ellipse $16x^2 + 25y^2 = 400$, what is $PF_1 + PF_2$?

    (A) 32 (B) 10 (C) 16 (D) 8

20. What is the length of the latus rectum of the parabola $y^2 = 64x$?

    (A) 16 (B) 32 (C) 64 (D) 12

21. What is the latus rectum of the ellipse $\frac{x^2}{3\sqrt{3}} + \frac{y^2}{9} = 1$?

    (A) $2\sqrt{3}$ (B) $2\sqrt{5}$ (C) 7 (D) 3

22. A rectangular hyperbola passes through $(3, 5)$. What are the coordinates of its foci?

    (A) $(0, \pm 4\sqrt{2})$ (B) $(\pm 2, 0)$
    (C) $(0, \pm 2)$ (D) $(\pm 3\sqrt{2}, 0)$

23. What is the eccentricity of the hyperbola $\frac{y^2}{2} - x^2 = 1$?

(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\sqrt{\frac{3}{2}}$ (D) $\frac{\sqrt{3}}{2}$

24. A hyperbola has eccentricity $e = \sqrt{5}$ and foci $S(1, 5)$, $S'(1, -1)$. What is the equation of one of its asymptotes?

(A) $x - 2y = 5$ (B) $2x + y = 5$

(C) $x - 2y = 3$ (D) $x + 2y = 5$

25. Find the equation of the ellipse passing through the origin whose foci coincide with the foci of the parabolas $y^2 = 8x$ and $x^2 = 24y$.

(A) $\frac{(x-1)^2}{26} + \frac{(y-3)^2}{16} = 1$

(B) $\frac{(x-1)^2}{16} + \frac{(y-3)^2}{9} = 1$

(C) $15x^2 + 6xy + 8y^2 - 48x - 48y = 0$

(D) $15x^2 + 6xy + 7y^2 - 48x - 48y = 0$

26. What does the equation $\sqrt{5(x-1)^2 + (y-1)^2} = \frac{1}{\sqrt{3}}|-10x - 2y|$ represent?

(A) Parabola (B) Hyperbola

(C) Ellipse (D) A pair of straight lines

27. The equation $3x^2 + by^2 = 2$ represents an ellipse when—

(A) $b \not\equiv 3$ (B) $b > 0$

(C) Both (A) and (B) (D) $b < 0$ and $b \not\equiv 3$

28. If $(x+1)^3 + y^2 + 6y = x^3 + 6$, what are the coordinates of the centre of the conic?

(A) $\left(\frac{1}{2}, -3\right)$ (B) $\left(\frac{1}{2}, 3\right)$

(C) Not a conic (D) $\left(-\frac{1}{2}, -3\right)$

29. Find the equation of the chord of the hyperbola $9x^2 - 16y^2 = 144$ that is bisected at $(4, 2)$.

(A) $9x - 8y = 20$ (B) $9x - 8y = 10$

(C) $8x - 9y = 10$ (D) $9x - 8y = -20$

30. A hyperbola's transverse and conjugate axes lie along the coordinate axes. The distance between its foci is 10 units. If $e = \frac{5}{4}$, what is the equation of the hyperbola?

(A) $\frac{x^2}{25} - \frac{y^2}{9} = 1$ (B) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(C) $\frac{x^2}{25} - \frac{y^2}{12} = 1$ (D) None of these

31. If the line $4y = 4x + 4k$ is tangent to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, what is the value of k?

(A) 1 (B) -4 (C) 5 (D) 3

32. If the curve $(y+3)^3 = 4k(x-5)$ passes through the point $(2, 3)$, what is k?

(A) -16 (B) 18

(C) -18 (D) -15.625

33. The ellipse $px^2 + 9y^2 = 1$ passes through $(\pm 1, 0)$. What is the length of its major axis?

(A) 1 (B) $\frac{2}{3}$ (C) 4 (D) 2

34. What are the coordinates of the vertex of the parabola $4(y-2)^2 = 32(x-3)$?

(A) $(3, 2)$ (B) $(3, 4)$ (C) $(2, 3)$ (D) $(5, 2)$

35. A moving point's distances from the lines $4x - 3y + 11 = 0$ and $4x + 3y + 5 = 0$ have a product of $\frac{144}{25}$. Find the equation of the locus of the point.

(A) $16x^2 - 9y^2 + 64x + 16y + 199 = 0$

(B) $16x^2 - 9y^2 + 64x + 18y + 199 = 0$

(C) $16x^2 - 9y^2 + 64x + 16y + 89 = 0$

(D) $16x^2 - 9y^2 + 64x + 18y + 89 = 0$

36. For a focal chord of the parabola $y^2 = 4ax$ with endpoints (x_1, y_1) and (x_2, y_2), what is $y_1y_2 + x_1x_2$?

(A) $-2a^2$ (B) $-3a$ (C) $-3a^2$ (D) $-4a^2$

37. A hyperbola's eccentricity is the reciprocal of the eccentricity of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through the ellipse's foci, what is its equation?

(A) $3x^2 - y^2 = 3$ (B) $x^2 - 4y^2 = 3$

(C) $x^2 - 3y^2 = 4$ (D) $x^2 - 3y^2 = 3$

38. The Earth's orbit is elliptical with the Sun at one focus. If the semi-major axis is 9.3×10^7 miles and the eccentricity is $\frac{1}{60}$, find the maximum distance between the Sun and the Earth.

(A) 9.455×10^7 miles (B) 9.55×10^7 miles

(C) 9.35×10^7 miles (D) 9.145×10^7 miles

39. An elliptical arch is 4 m wide and 2 m high. What is the height of the arch at a point 1.5 m from either end?

(A) 1.32 m (B) 1.94 m (C) 1.56 m (D) 1.75 m

40. With the axes along the coordinate axes, what is the equation of a hyperbola whose foci are 16 units apart and whose eccentricity is $\sqrt{2}$?

(A) $x^2 - y^2 = 32$ (B) $\frac{x^2}{4} - \frac{y^2}{6} = 1$

(C) $x^2 - y^2 = 42$ (D) $x^2 - y^2 = 36$

41. A parabola has vertex $(-2, 5)$, latus rectum 8, and axis parallel to the y-axis. What is its equation?

(A) $x^2 + 4x + 8y - 36 = 0$ (B) $x^2 + 4x - 8y - 36 = 0$

(C) $x^2 + 4x + 8y - 48 = 0$ (D) $x^2 + 2x + 8y - 36 = 0$

42. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the intersection of $x - 3\sqrt{5}y = 0$ and $\sqrt{5}x - 2y = 13$, and has latus rectum $\frac{4}{3}$ units. What are the coordinates of its foci?

(A) $(\pm 10, 0)$ (B) $(\pm 2\sqrt{13}, 0)$

(C) $(\pm 2\sqrt{3}, 0)$ (D) $(\pm 2\sqrt{10}, 0)$

43. If the line $x - py + 4 = 0$ is tangent to the parabola $y^2 = 8x$, what is the value of p?

Ⓐ ± 3 Ⓑ $\pm\sqrt{2}$ Ⓒ ± 4 Ⓓ $\pm 2\sqrt{3}$

44. A point moves in the xy-plane such that the ratio of its distance from $(0, -3)$ to its distance from the line $3x - 4y + 1 = 0$ is always $\frac{1}{2}$. What type of conic is the locus?

Ⓐ Parabola Ⓑ Hyperbola

Ⓒ Ellipse Ⓓ Circle

45. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is an ellipse. If $a = b$, what is the equation of the directrix?

Ⓐ $x = ae$ Ⓑ $x = \infty$ Ⓒ $x = \frac{a}{e}$ Ⓓ $x = 0$

46. Which line is perpendicular to $x - 3y + 5 = 0$ and tangent to the parabola $y^2 = 12x$?

Ⓐ $3x + y = 1$ Ⓑ $3x + 2y = 1$

Ⓒ $x + 3y + 1 = 0$ Ⓓ $3x + y + 1 = 0$

47. What is the length of the latus rectum of the ellipse $16x^2 + 25y^2 = 400$?

Ⓐ 2.2 Ⓑ 3.8 Ⓒ 6.4 Ⓓ 4.2

48. Which is the equation of a directrix of the ellipse $4x^2 + 5y^2 = 20$?

Ⓐ $y = \pm 5$ Ⓑ $y = \pm\sqrt{5}$ Ⓒ $x = \pm\sqrt{5}$ Ⓓ $x = \pm 5$

49. The area of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ equals the area of a square. What is the perimeter of the square?

Ⓐ 24.56 Ⓑ 20.44 Ⓒ 18.48 Ⓓ 22.78

50. If u is a variable parameter, what is the eccentricity of the conic represented by $x = 3(\cot u + \tan u)$ and $y = 4(\cot u - \tan u)$?

Ⓐ $\frac{4}{3}$ Ⓑ $\frac{5}{3}$ Ⓒ $\frac{5}{2}$ Ⓓ 2