Engineering Daily 13 [Home Practice] EV - Answer Sheet

Structural Properties of Matter + Periodic motion

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. C. $0.0220\,\text{m}$

Explanation: $h = \frac{2T\cos\theta}{r\rho g}$

$h = \frac{2\times4.5\times10^{-2}\times\cos 40^\circ}{0.4\times10^{-3}\times800\times9.8}$

$h \approx 0.0220\,\text{m}$

2. A. 2.25

Explanation: $T \propto \sqrt{m}$

$\frac{m'}{m} = (1.5)^2$

$\frac{m'}{m} = 2.25$

3. D. 8 : 45

Explanation: $\frac{\Delta L_1}{\Delta L_2} = \frac{L_1}{L_2} \times \left(\frac{r_2}{r_1}\right)^2$

$\frac{\Delta L_1}{\Delta L_2} = \frac{2}{5} \times \left(\frac{2}{3}\right)^2 = \frac{2}{5} \times \frac{4}{9}$

$\frac{\Delta L_1}{\Delta L_2} = \frac{8}{45}$

4. B. $4.07 \times 10^{-4}\,\text{J}$

Explanation: 27 droplets merge into a drop of radius $R = n^{1/3}r = 3r$

Heat released $= T \times 4\pi r^2(n - n^{2/3})$

$= 72 \times 10^{-3} \times 4\pi \times (5 \times 10^{-3})^2 \times (27 - 9)$

$\approx 4.07 \times 10^{-4}\,\text{J}$ (the terminal velocity is extra data, not needed here)

5. B. $0.148\,\text{s},\ 3.16\,\text{ms}^{-2}$

Explanation: $\cos\theta = 0.8 \Rightarrow \theta \approx 0.6435\,\text{rad}$

At the mean $\theta = \frac{\pi}{2}$, so $\Delta\theta \approx 0.9273$

With $\omega = 2\pi\,\text{rads}^{-1}$, $\Delta t \approx 0.148\,\text{s}$

Initial $a \approx -3.16\,\text{ms}^{-2}$, final $a = 0$, so the change is $3.16\,\text{ms}^{-2}$

6. C. 3 : 1

Explanation: $\frac{U}{E} = \frac{x^2}{A^2} = \frac{1}{4}$

$\frac{K}{E} = \frac{3}{4}$

$K : U = 3 : 1$

7. B. $0.64,\ -12\,\text{ms}^{-2}$

Explanation: $\frac{K}{E} = 1 - \left(\frac{0.12}{0.20}\right)^2 = 0.64$

$a = -\omega^2 x = -12\,\text{ms}^{-2}$

8. B. $\frac{\pi}{8}\,\text{s}$

Explanation: $v_{max} = \omega A$

$\omega = \frac{80}{5} = 16\,\text{s}^{-1}$

$T = \frac{2\pi}{\omega} = \frac{\pi}{8}\,\text{s}$

9. B. $5.88\,\text{ms}^{-2}$

Explanation: At $t = 0$, velocity is zero, so viscous force is zero

$a_0 = g\left(1 - \frac{\rho_{liquid}}{\rho_{ball}}\right)$

$a_0 = 9.8 \times \left(1 - \frac{1000}{2500}\right) = 9.8 \times 0.6 = 5.88\,\text{ms}^{-2}$

10. D. π

Explanation: Positive extreme to mean is $\frac{\pi}{2}$

Mean to negative extreme is a further $\frac{\pi}{2}$

Total $= \frac{\pi}{2} + \frac{\pi}{2} = \pi$ (half a period)

11. B. 3 times

Explanation: $Re = \frac{\rho v d}{\eta}$, so $Re \propto v$

Tripling the velocity triples Re

12. C. 4.12 : 1

Explanation: For equal-size spheres in the same liquid, $v \propto (\rho_{ball} - \rho_{liquid})$

$\frac{v_1}{v_2} = \frac{8000-1000}{2700-1000} = \frac{7000}{1700}$

$\frac{v_1}{v_2} \approx 4.12$, ratio $= 4.12 : 1$

13. D. $6.60 \times 10^{-7}\,\text{m}$

Explanation: Volume is conserved for $n = 27$ droplets

Each has radius $r = \frac{d}{2} = 1.1 \times 10^{-7}\,\text{m}$

Big drop's radius $R = n^{1/3}r = 3r = 3.3 \times 10^{-7}\,\text{m}$

Diameter $D = 2R \approx 6.60 \times 10^{-7}\,\text{m}$ (surface tension is extra data, not needed for this part)

14. B. $1012.0\,\text{kgm}^{-3}$

Explanation: $\Delta P = \rho g h \approx 1015 \times 9.8 \times 6000 \approx 5.97 \times 10^7\,\text{Pa}$

$\rho_0 = \rho(1 - K\Delta P)$

$\rho_0 \approx 1012.0\,\text{kgm}^{-3}$

15. C. 0.5 J

Explanation: $U = \frac{1}{2}F\Delta L$

$U = \frac{1}{2} \times 500 \times 2 \times 10^{-3}$

$U = 0.5\,\text{J}$

16. D. 0.433 s

Explanation: $\frac{T'}{T} = \sqrt{\frac{m'/m}{k'/k}} = \sqrt{\frac{0.75}{1.44}}$

$\frac{T'}{T} \approx 0.722$

$T' = 0.60 \times 0.722 \approx 0.433\,\text{s}$

17. **B.** 2.5×10^{-4}

Explanation: $\frac{\Delta V}{V} = \frac{\Delta P}{B}$

$\frac{\Delta V}{V} = \frac{4 \times 10^{7}}{1.6 \times 10^{11}}$

$\frac{\Delta V}{V} = 2.5 \times 10^{-4}$

18. **A.** 1.07×10^{-2} m

Explanation: $h = \frac{2T \cos\theta}{r\rho g}$; since $\theta > 90°$, $\cos\theta$ is negative, giving a depression

$|h| = \frac{2 \times 0.465 \times |\cos 140°|}{0.5 \times 10^{-3} \times 13600 \times 9.8}$

$|h| \approx 1.07 \times 10^{-2}$ m

19. **D.** ≈ 1041

Explanation:

New $T' = T\sqrt{1.05}$

Old oscillations $N = \frac{86400}{T} = 43200$

New oscillations $N' = \frac{86400}{T\sqrt{1.05}} = \frac{43200}{\sqrt{1.05}}$

Fewer oscillations $= 43200\left(1 - \frac{1}{\sqrt{1.05}}\right) \approx 1041$

20. **A.** $\frac{\pi}{2}$

Explanation: $v < 0$ requires $\sin\theta > 0$

$a < 0$ requires $\cos\theta > 0$

Both hold for $0 < \theta < \frac{\pi}{2}$, so the interval is $\frac{\pi}{2}$

21. **C.** $\frac{2\pi}{5}$ s, $1\ \text{ms}^{-2}$

Explanation: $y'' + 25y = 0 \Rightarrow \omega^2 = 25 \Rightarrow \omega = 5\ \text{rads}^{-1}$

$T = \frac{2\pi}{5}$ s

With $A = 0.04$ m, $a_{max} = \omega^2 A = 25 \times 0.04 = 1\ \text{ms}^{-2}$

22. **D.** 2.69 s

Explanation: Parallel pair: $k_p = 40 + 60 = 100\ \text{Nm}^{-1}$

Series: $\frac{1}{k_{eq}} = \frac{1}{100} + \frac{1}{120}$, giving $k_{eq} \approx 54.55\ \text{Nm}^{-1}$

$T = 2\pi\sqrt{\frac{10}{54.55}} \approx 2.69$ s

23. **A.** 3, 9

Explanation: $v_{max} = \omega A \Rightarrow$ becomes 3 times

$E = \frac{1}{2}m\omega^2 A^2 \Rightarrow$ becomes 9 times

24. **D.** $\frac{\pi}{3}$

Explanation: $\cos\varphi = \frac{1}{2} \Rightarrow \varphi = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

$v = -A\omega\sin\varphi < 0$ requires $\sin\varphi > 0$

So $\varphi = \frac{\pi}{3}$

25. **C.** $\frac{\sqrt{3}}{2}$

Explanation: At maximum positive velocity the first phase is $-\frac{\pi}{2}$

The second phase is $-\frac{\pi}{2} + \frac{\pi}{3} = -\frac{\pi}{6}$

$x_2 = A\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}A$

26. **A.** 2.462 rad

Explanation: For crossing with opposite velocity directions, $\Delta\varphi = 2\cos^{-1}\left(\frac{x}{A}\right)$

$\Delta\varphi = 2\cos^{-1}\left(\frac{1}{3}\right)$

$\Delta\varphi \approx 2.462$ rad

27. **C.** 1×10^{-2} m

Explanation: $\Delta P = \frac{4T}{r}$

$r = \frac{4T}{\Delta P} = \frac{4 \times 4 \times 10^{-2}}{16}$

$r = 1 \times 10^{-2}$ m

28. **D.** 3.82 mm

Explanation: $A = \pi(1 \times 10^{-3})^2 \approx 3.14 \times 10^{-6}\ \text{m}^2$

$\Delta L_1 = \frac{FL_1}{AY_1} \approx 0.955$ mm

$\Delta L_2 = \frac{FL_2}{AY_2} \approx 2.865$ mm

Total ≈ 3.82 mm

29. **B.** $9.6 \times 10^{7}\ \text{Nm}^{-2}$

Explanation: $\sigma = Y\alpha\Delta T$

$\sigma = 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 40$

$\sigma = 9.6 \times 10^{7}\ \text{Nm}^{-2}$

30. **A.** 0.65 mm

Explanation: $\delta = \frac{FL^3}{3YI}$, $I = \frac{\pi r^4}{4} \approx 1.257 \times 10^{-7}\ \text{m}^4$

$F = 40 \times 9.8 = 392$ N

$\delta \approx 6.5 \times 10^{-4}$ m $= 0.65$ mm

31. **C.** 0.678 s

Explanation: $x = A\sin\omega t$

$\sin\omega t = 0.8 \Rightarrow \omega t = \sin^{-1}(0.8) \approx 0.9273$ rad

$T = \frac{2\pi(0.10)}{0.9273} \approx 0.678$ s

32. **D.** $0.641\ \text{Pa}\cdot\text{s}$

Explanation: $v = \frac{0.6}{12} = 0.05\ \text{ms}^{-1}$

$\eta = \frac{2r^2(\rho_{ball} - \rho_{liquid})g}{9v}$

$\eta \approx 0.641\ \text{Pa}\cdot\text{s}$

33. **A.** $x = 0.10\cos\left(4t + \frac{\pi}{4}\right)$ m

Explanation: $K_{max} = \frac{1}{2}m\omega^2 A^2 = 8 \times 10^{-3}$

$\omega^2 = 16$, $\omega = 4\ \text{rads}^{-1}$

$x = A\cos(\omega t + \varphi) = 0.10\cos\left(4t + \frac{\pi}{4}\right)$ m

34. **B.** 2.04×10^{-9} N

Explanation: $F = 6\pi\eta r v$

$F = 6\pi \times 1.8 \times 10^{-5} \times 2 \times 10^{-4} \times 0.03$

$F \approx 2.04 \times 10^{-9}$ N

35. **A.** 1 : 2

Explanation: Parallel: $k_{eq} = 2k$; Series: $k_{eq} = \frac{k}{2}$

$T \propto \frac{1}{\sqrt{k}}$

$\frac{T_p}{T_s} = \sqrt{\frac{k/2}{2k}} = \frac{1}{2}$, so ratio = 1 : 2

36. **D.** 1 : 6

Explanation: $\frac{\Delta L_1}{\Delta L_2} = \frac{L_1}{L_2} \times \frac{Y_2}{Y_1}$

$\frac{\Delta L_1}{\Delta L_2} = \frac{1}{3} \times \frac{1}{2}$

$\frac{\Delta L_1}{\Delta L_2} = \frac{1}{6}$, ratio = 1 : 6

37. **C.** 13.5 J, 13.5 J

Explanation: $\frac{U}{E} = \frac{x^2}{A^2} = \frac{1}{9}$

$E = 1.5 \times 9 = 13.5$ J

Since K is maximum at $x = 0$, $K_{max} = 13.5$ J

38. **A.** 7.37×10^{-2} J

Explanation: By energy conservation, $K = mgL(\cos 20° - \cos 30°)$

$K = 0.10 \times 10 \times 1 \times (0.9397 - 0.8660)$

$K \approx 7.37 \times 10^{-2}$ J

39. **A.** 0.0735 m

Explanation: $h = \frac{2T\cos\theta}{r\rho g}$

$h = \frac{2 \times 7.2 \times 10^{-2} \times 1}{0.2 \times 10^{-3} \times 1000 \times 9.8}$

$h \approx 0.0735$ m

40. **C.** 1×10^5 Jm^{-3}

Explanation: $u = \frac{(\text{stress})^2}{2Y}$

$u = \frac{(2 \times 10^8)^2}{2 \times 2 \times 10^{11}}$

$u = 1 \times 10^5$ Jm^{-3}

41. **B.** 5 cm, 0.444 s

Explanation: $mg = kx_{eq} \Rightarrow x_{eq} = \frac{0.5 \times 10}{100} = 0.05$ m $= 5$ cm

$T = 2\pi\sqrt{\frac{m}{k}} \approx 0.444$ s

The 2 cm displacement does not affect T

42. **A.** 40 $rads^{-1}$, 400 ms^{-2}

Explanation: $\omega = \frac{v_{max}}{A} = \frac{10}{0.25} = 40$ $rads^{-1}$

$a_{max} = \omega^2 A = 1600 \times 0.25$

$a_{max} = 400$ ms^{-2}

43. **C.** 0.57 J

Explanation: $U = \frac{1}{2} F \Delta L$

$U = \frac{1}{2} \times 760 \times 1.5 \times 10^{-3}$

$U = 0.57$ J (mass and density aren't needed for the energy itself)

44. **B.** $\frac{1}{3}$ s

Explanation: $x = A \sin \omega t$

$\frac{A}{2}$ gives $\sin \omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{6}$

$t = \frac{T}{12} = \frac{1}{3}$ s

45. **D.** 16 : 9

Explanation: $\frac{U}{E} = \left(\frac{0.08}{0.10}\right)^2 = 0.64$

$\frac{K}{E} = 0.36$

$U : K = 0.64 : 0.36 = 16 : 9$

46. **A.** 7.2 ms^{-2}, 0.36

Explanation: $|a| = \omega^2 |x| = 0.60\, a_{max} = 7.2$ ms^{-2}

$\frac{U}{E} = \frac{x^2}{A^2} = 0.36$

47. **A.** 12 Pa

Explanation: A soap bubble has two surfaces

$\Delta P = \frac{4T}{r} = \frac{4 \times 3 \times 10^{-2}}{0.01}$

$\Delta P = 12$ Pa

48. **C.** 0.0181 N

Explanation: The ring has two contact lines (inner and outer)

$F = T \times 2 \times 2\pi r = 4\pi r T$

$F = 4\pi \times 0.02 \times 7.2 \times 10^{-2} \approx 0.0181$ N

49. **D.** 2 : 1

Explanation: At equal strain, $u = \frac{1}{2} Y \times (\text{strain})^2$

$\frac{u_1}{u_2} = \frac{Y_1}{Y_2} = \frac{2 \times 10^{11}}{1 \times 10^{11}}$

$\frac{u_1}{u_2} = 2$, ratio = 2 : 1

50. **C.** 7.5×10^4 Jm^{-3}

Explanation: Stress $= \frac{F}{A} = \frac{600}{2 \times 10^{-6}} = 3 \times 10^8$ Nm^{-2}

Strain $= \frac{\Delta L}{L} = 5 \times 10^{-4}$

$u = \frac{1}{2} \times \text{stress} \times \text{strain} = 7.5 \times 10^4$ Jm^{-3}