MCQ 50 marks

1. B. -3

Explanation: The condition for perpendicularity is $\vec{P} \cdot \vec{Q} = 0$

Here, $(2)(m) + (m)(-4) + (-3)(2) = 0$

$\Rightarrow 2m - 4m - 6 = 0$

$\Rightarrow -2m = 6$

Therefore, $m = -3$

2. C. 1

Explanation: The formula for perpendicular distance is $d = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

Here, $d = \frac{|1(2)-2(1)+2(-1)+5|}{\sqrt{1^2+(-2)^2+2^2}}$

$= \frac{|2-2-2+5|}{\sqrt{9}} = \frac{3}{3}$

Therefore, $d = 1$ unit

3. B. 3

Explanation: The perpendicular projection is $\text{proj} = \frac{\vec{A}\cdot\vec{B}}{|B|}$

Here, $\vec{A} \cdot \vec{B} = (3)(2) + (4)(2) + (-5)(1) = 6 + 8 - 5 = 9$

And, $|\vec{B}| = \sqrt{2^2 + 2^2 + 1^2} = 3$

Therefore, $\text{proj} = \frac{9}{3} = 3$

4. B. $\frac{7}{2}$

Explanation: Let $A(1, 0, 0), B(0, 2, 0), C(0, 0, 3)$

Here, $\overrightarrow{AB} = -\hat{i} + 2\hat{j}$ and $\overrightarrow{AC} = -\hat{i} + 3\hat{k}$

$\overrightarrow{AB} \times \overrightarrow{AC} = 6\hat{i} + 3\hat{j} + 2\hat{k}$

Area $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{36 + 9 + 4} = \frac{1}{2}\sqrt{49}$

Therefore, area $= \frac{7}{2}$ square units

5. D. $\sin^{-1}\left(\frac{4\sqrt{2}}{15}\right)$

Explanation: The direction vector of the line is $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ and the normal to the plane is $\vec{n} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

By the formula, $\sin\theta = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$

Here, $\vec{b} \cdot \vec{n} = 6 - 4 - 10 = -8$

$|\vec{b}| = 3$ and $|\vec{n}| = \sqrt{50} = 5\sqrt{2}$

Therefore, $\sin\theta = \frac{8}{15\sqrt{2}} = \frac{4\sqrt{2}}{15}$

6. B. 4

Explanation: For coplanar vectors, the scalar triple product is zero

$$\begin{vmatrix} x & -2 & 1 \\ 3 & 1 & -2 \\ 1 & -3 & 3 \end{vmatrix} = 0$$

$\Rightarrow x(3 - 6) + 2(9 + 2) + 1(-9 - 1) = 0$

$\Rightarrow -3x + 22 - 10 = 0$

Therefore, $x = 4$

7. A. $2x - y + 3z = 16$

Explanation: The equation of the parallel plane is $2x - y + 3z = k$

Since it passes through $(1, -2, 4)$, $2(1) - (-2) + 3(4) = k$

$\Rightarrow k = 2 + 2 + 12 = 16$

Therefore, the equation is $2x - y + 3z = 16$

8. C. $90°$

Explanation: Here, $\vec{a} + \vec{b} = -\vec{c}$

Taking the dot product on both sides, $|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = |\vec{c}|^2$

$\Rightarrow 25 + 144 + 120\cos\theta = 169$

$\Rightarrow 120\cos\theta = 0$

Therefore, $\cos\theta = 0 \Rightarrow \theta = 90°$

9. D. $x + y + z = 2$

Explanation: Let the points be A, B, C

Here, $\overrightarrow{AB} = \hat{i} - \hat{j}$ and $\overrightarrow{AC} = \hat{i} - \hat{k}$

The normal vector, $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \hat{i} + \hat{j} + \hat{k}$

The equation, $1(x - 0) + 1(y - 1) + 1(z - 1) = 0$

Therefore, $x + y + z = 2$

10. B. $\frac{4}{\sqrt{5}}$

Explanation: The relative position, $\Delta\vec{r} = \vec{r}_B - \vec{r}_A = (3 - t)\hat{i} + (2t - 2)\hat{j}$

Here, $D^2 = (3 - t)^2 + (2t - 2)^2 = 5t^2 - 14t + 13$

For minimum distance, $\frac{d(D^2)}{dt} = 10t - 14 = 0 \Rightarrow t = 1.4$

$D^2 = 5(1.4)^2 - 14(1.4) + 13 = 3.2$

Therefore, $D_{min} = \sqrt{3.2} = \frac{4}{\sqrt{5}}$

11. B. $8\sqrt{3}$

Explanation: We know, $|\vec{A} \times \vec{B}| = AB \sin\theta$

Here, $AB \sin 30° = 8 \Rightarrow AB = 16$

Now, $\vec{A} \cdot \vec{B} = AB \cos\theta = 16 \cos 30°$

Therefore, $\vec{A} \cdot \vec{B} = 16 \times \frac{\sqrt{3}}{2} = 8\sqrt{3}$

12. C. $x + y - z = 1$

Explanation: The normal to the plane, $\vec{n} = (\hat{i} - \hat{j}) \times (\hat{j} + \hat{k}) = -\hat{i} - \hat{j} + \hat{k}$

The equation, $-1(x - 1) - 1(y - 1) + 1(z - 1) = 0$

$\Rightarrow -x - y + z + 1 = 0$

Therefore, $x + y - z = 1$

13. B. $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Explanation: Let the side length of the cube be a

The body diagonal, $\vec{d_1} = a\hat{i} + a\hat{j} + a\hat{k}$ and the face diagonal, $\vec{d_2} = a\hat{i} + a\hat{j}$

Here, $\vec{d_1} \cdot \vec{d_2} = 2a^2$, $|\vec{d_1}| = \sqrt{3}a$, $|\vec{d_2}| = \sqrt{2}a$

Therefore, $\cos\theta = \frac{2a^2}{\sqrt{3}a \times \sqrt{2}a} = \sqrt{\frac{2}{3}}$

14. C. 2

Explanation: For perpendicular planes, the dot product of their normal vectors is zero

$(p)(3) + (2)(-p) + (-1)(2) = 0$

$\Rightarrow 3p - 2p - 2 = 0$

Therefore, $p = 2$

15. B. $\hat{i} - 2\hat{j}$

Explanation: Let, $\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$

Here, $\vec{A} \cdot \vec{B} = 2x - y = 4$

And, $\vec{A} \times \vec{B} = -z\hat{i} - 2z\hat{j} + (x + 2y)\hat{k} = -3\hat{k}$

Comparing coefficients, $z = 0$ and $x + 2y = -3$

Solving the two equations, $x = 1, y = -2$

Therefore, $\vec{B} = \hat{i} - 2\hat{j}$

16. B. $\pm\frac{4\hat{j}+3\hat{k}}{5}$

Explanation: Let a vector in the yz-plane be of the form $\vec{v} = y\hat{j} + z\hat{k}$

Since it is perpendicular to the given vector, $-3y + 4z = 0 \Rightarrow \frac{y}{4} = \frac{z}{3}$

So the vector is parallel to $4\hat{j} + 3\hat{k}$, and its unit vector is $\pm\frac{4\hat{j}+3\hat{k}}{5}$

17. A. $\frac{7}{6}$

Explanation: Dividing the second equation by 2, $x - 2y + 2z = 7.5$

Distance, $d = \frac{|D_1 - D_2|}{\sqrt{a^2+b^2+c^2}} = \frac{|7.5-4|}{\sqrt{1+4+4}}$

Therefore, $d = \frac{3.5}{3} = \frac{7}{6}$ units

18. A. $\cos^{-1}(-\frac{3}{5})$

Explanation: To reach the exact opposite point, the resultant velocity must be perpendicular to the current

$u + v \cos\theta = 0$

$\Rightarrow 6 + 10 \cos\theta = 0$

Therefore, $\cos\theta = -\frac{3}{5} \Rightarrow \theta = \cos^{-1}\left(-\frac{3}{5}\right)$

19. C. 1

Explanation: Here, $|\vec{u} + \vec{v}|^2 = 1 + 1 + 2\cos\theta = 3 \Rightarrow \cos\theta = \frac{1}{2}$

Now, $|\vec{u} - \vec{v}|^2 = 1 + 1 - 2\left(\frac{1}{2}\right) = 1$

Therefore, $|\vec{u} - \vec{v}| = 1$

20. C. $\vec{0}$

Explanation: The vector triple product formula, $\vec{X} \times (\vec{Y} \times \vec{Z}) = (\vec{X} \cdot \vec{Z})\vec{Y} - (\vec{X} \cdot \vec{Y})\vec{Z}$

Expanding each term using this formula, all the terms cancel out

Therefore, the given expression equals $\vec{0}$

21. C. 0

Explanation: Note that, $(\vec{a} - \vec{b}) + (\vec{b} - \vec{c}) + (\vec{c} - \vec{a}) = \vec{0}$

Since the sum of the three vectors is zero, they are coplanar

And the scalar triple product (box product) of coplanar vectors is always zero

Therefore, the given expression equals 0

22. C. 8

Explanation: Here, $\frac{2n(2n-1)(2n-2)}{6} = 20 \times \frac{n(n-1)}{2}$

$\Rightarrow \frac{4n(2n-1)(n-1)}{6} = 10n(n-1)$

Cancelling $n(n-1)$ from both sides (for $n > 1$), $\frac{2(2n-1)}{3} = 10$

$\Rightarrow 2n - 1 = 15$

Therefore, $n = 8$

23. A. $\cos^{-1}\left(\frac{3}{5}\right)$

Explanation: $|\vec{A} + \vec{B}|^2 = 4|\vec{A} - \vec{B}|^2$

$\Rightarrow 2A^2(1 + \cos\theta) = 4 \times 2A^2(1 - \cos\theta)$

$\Rightarrow 1 + \cos\theta = 4 - 4\cos\theta \Rightarrow \cos\theta = \frac{3}{5}$

24. C. $\sqrt{2+\sqrt{2}}$

Explanation: $|\vec{a}\times\vec{b}| = \vec{a}\cdot\vec{b} \Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = 45°$

Now, $|\vec{a}+\vec{b}|^2 = 1+1+2\cos 45° = 2+\sqrt{2}$

Therefore, $|\vec{a}+\vec{b}| = \sqrt{2+\sqrt{2}}$

25. C. 180°

Explanation: Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = -6\sin(2t)\hat{i} + 6\cos(2t)\hat{j}$

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = -12\cos(2t)\hat{i} - 12\sin(2t)\hat{j} = -4\vec{r}$

Since $\vec{a}$ and $\vec{r}$ are in opposite directions

Therefore, the angle between them is 180°

26. B. $8\sqrt{3}$

Explanation: $\vec{a}\times\vec{b} = \hat{i}(2-(-6)) - \hat{j}(1-9) + \hat{k}(-2-6) = 8\hat{i} + 8\hat{j} - 8\hat{k}$

Area $= |\vec{a}\times\vec{b}| = \sqrt{8^2+8^2+(-8)^2} = \sqrt{3\times 64}$

Therefore, area $= 8\sqrt{3}$ square units

27. D. 0

Explanation: $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$= 1+1+1+2\left(-\frac{3}{2}\right) = 3-3$

Therefore, $|\vec{a}+\vec{b}+\vec{c}| = 0$

28. A. 17010

Explanation: The general term, $T_{r+1} = \binom{10}{r}(3x^3)^{10-r}\left(-\frac{1}{x^2}\right)^r = \binom{10}{r}3^{10-r}(-1)^r x^{30-5r}$

For the term independent of x, $30-5r = 0 \Rightarrow r = 6$

Therefore, the value of the term $= \binom{10}{6}3^4(-1)^6 = 210\times 81 = 17010$

29. A. 924

Explanation: Since the exponent n = 12 is even, there is one middle term, the $\left(\frac{12}{2}+1\right) = 7$ th term

$T_7 = \binom{12}{6}x^6\left(-\frac{1}{x}\right)^6 = \binom{12}{6}\times 1$

Therefore, $T_7 = 924$

30. A. 672

Explanation: The general term, $T_{r+1} = \binom{9}{r}2^{9-r}(-x)^r$

For x^6, putting r = 6

Coefficient $= \binom{9}{6}2^3(-1)^6 = 84\times 8$

Therefore, the coefficient = 672

31. C. 0

Explanation: To find the sum of the coefficients, we set every variable equal to 1

Putting x = y = z = 1, $(2(1)-3(1)+1)^8 = (2-3+1)^8$

Therefore, the sum $= 0^8 = 0$

32. C. 56

Explanation: The number of variables k = 4 and the exponent n = 5

Number of terms $= \binom{n+k-1}{k-1} = \binom{8}{3}$

Therefore, the number of terms $= \frac{8\times 7\times 6}{3\times 2\times 1} = 56$

33. A. 240

Explanation: Writing it as $(1-(-2x))^{-3}$, the coefficient in $(1-y)^{-n}$ is $\binom{n+r-1}{r}$

Here, the coefficient $= \binom{3+4-1}{4}(-2)^4 = \binom{6}{4}\times 16$

Therefore, the coefficient $= 15\times 16 = 240$

34. D. Both B and C

Explanation: Since n = 19 is odd, there are two middle terms, the $\frac{n+1}{2} = 10$th and $\frac{n+3}{2} = 11$ th terms

Their coefficients are $\binom{19}{9}$ and $\binom{19}{10}$ respectively

Since $\binom{19}{9} = \binom{19}{10}$

Therefore, both are the largest coefficients

35. B. 2

Explanation: $T_3 = \binom{8}{2}x^6y^2 = 28x^6y^2$ and $T_4 = \binom{8}{3}x^5y^3 = 56x^5y^3$

By the given condition, $28x^6y^2 = 56x^5y^3$

Therefore, $\frac{x}{y} = 2$

36. B. $3n\,4^{n-1}$

Explanation: We know, $(1+x)^n = \sum_{r=0}^{n}\binom{n}{r}x^r$

Differentiating both sides with respect to x and multiplying by x, $nx(1+x)^{n-1} = \sum_{r=1}^{n} r\binom{n}{r}x^r$

Putting x = 3, $n(3)(1+3)^{n-1}$

Therefore, the sum $= 3n\,4^{n-1}$

37. A. $-20412\,\frac{x^2}{y^5}$

Explanation: The total number of terms = 7 + 1 = 8

The 3rd term from the end is the (8 − 3 + 1) = 6 th term from the start

$T_6 = T_{5+1} = \binom{7}{5}(2x)^2\left(-\frac{3}{y}\right)^5 = 21\times 4x^2\times\left(-\frac{243}{y^5}\right)$

Therefore, $T_6 = -20412\,\frac{x^2}{y^5}$

38. B. 6

Explanation: The 4th term, $T_4 = \binom{n}{3}x^3$

By the given condition, $\binom{n}{3} = 20 \Rightarrow n(n-1)(n-2) = 120$

Since, $6 \times 5 \times 4 = 120$

Therefore, $n = 6$

39. C. 3

Explanation: The coefficient of x^3 is $\binom{7}{3}3^4k^3$ and of x^4 is $\binom{7}{4}3^3k^4$

Since $\binom{7}{3} = \binom{7}{4}$, we get $3^4k^3 = 3^3k^4$

$\Rightarrow 3k^3 = k^4 \Rightarrow k = 0$ or $k = 3$

Therefore, $k = 3$

40. B. $n + 1$

Explanation: In the expansion of $(1-x)^{-m}$, the coefficient of x^n is $\binom{m+n-1}{n}$

Here $m = 2$, so the coefficient = $\binom{n+1}{n}$

Therefore, the coefficient = $n + 1$

41. A. $|x| < \frac{4}{5}$

Explanation: $(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\frac{5x}{4}\right)^{-\frac{1}{2}}$

The condition for convergence is $\left|\frac{5x}{4}\right| < 1$

Therefore, $|x| < \frac{4}{5}$

42. B. 210

Explanation: The general term, $T_{r+1} = \binom{10}{r}x^{\frac{10-r}{3}-\frac{r}{2}}$

For the term independent of x, $\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 5r = 0 \Rightarrow r = 4$

The value of the term = $\binom{10}{4} = \frac{10\times9\times8\times7}{4\times3\times2\times1}$

Therefore, the value of the term = 210

43. B. 4

Explanation: $(1-x^2)^4 = 1 - 4x^2 + 6x^4 - 4x^6 + x^8$

In the product $(1 - 4x^2 + 6x^4 - \cdots)(1-x)$, the x^3 term comes only from $-4x^2$ and $-x$

$(-4x^2) \times (-x) = 4x^3$

Therefore, the coefficient = 4

44. B. 2048

Explanation: In a binomial expansion, the sum of the coefficients of the even or odd power terms is 2^{n-1}

Here $n = 12$

Therefore, the sum = $2^{11} = 2048$

45. C. 6

Explanation: $T_2 = nax = -12x \Rightarrow na = -12$

$T_3 = \binom{n}{2}a^2x^2 = 60x^2 \Rightarrow \frac{n(n-1)}{2}a^2 = 60$

Substituting $a = -\frac{12}{n}$ from $na = -12$, $\frac{144(n-1)}{n} = 120$

$\Rightarrow 144n - 144 = 120n \Rightarrow 24n = 144$

Therefore, $n = 6$

46. C. 6

Explanation: The general term, $T_{r+1} = \binom{20}{r}3^{\frac{20-r}{2}}5^{\frac{r}{4}}$

For a rational term, r must be a multiple of 4 and $(20 - r)$ must be a multiple of 2

$r \in \{0, 4, 8, 12, 16, 20\}$

Therefore, there are 6 rational terms

47. A. $\binom{2n}{n}$

Explanation: $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$ and $(x+1)^n = C_0x^n + C_1x^{n-1} + \cdots + C_n$

Multiplying the two equations, the coefficient of x^n is $C_0^2 + C_1^2 + \cdots + C_n^2$

Also, in the expansion of $(1+x)^{2n}$, the coefficient of x^n is $\binom{2n}{n}$

Therefore, $C_0^2 + C_1^2 + \cdots + C_n^2 = \binom{2n}{n}$

48. C. 16

Explanation: $(1+x)(1-x)^{-3} = (1+x)(1 + 3x + 6x^2 + 10x^3 + \cdots)$

From the product, the coefficient of x^3 is $1 \times 10 + 1 \times 6$

Therefore, the coefficient = 16

49. B. -2

Explanation: The general expansion, $(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \cdots$

Here $n = \frac{1}{2}$, $y = 4x$

The x^2 term = $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}(4x)^2 = -\frac{1}{8} \times 16x^2$

Therefore, the coefficient = -2

50. A. 7, 14

Explanation: The coefficients are $\binom{n}{4}$, $\binom{n}{5}$, $\binom{n}{6}$ respectively

By the given condition, $2\binom{n}{5} = \binom{n}{4} + \binom{n}{6}$

Simplifying this equation gives, $n^2 - 21n + 98 = 0$

$\Rightarrow (n-7)(n-14) = 0$

Therefore, $n = 7$ or $n = 14$