Engineering Daily 12 [Set 03] EV - Answer Sheet

Binomial Expansion + Vector

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. A. 3

Explanation: Condition for perpendicularity: $\vec{A} \cdot \vec{B} = 0$

Here, $\vec{A} \cdot \vec{B} = (2)(4) + (m)(-2) + (1)(-2)$

According to the question, $8 - 2m - 2 = 0$

or, $2m = 6$

Therefore, $m = 3$

2. A. $-\frac{2}{3}$

Explanation: Orthogonal projection = $\frac{\vec{P} \cdot \vec{Q}}{|Q|}$

Here, $\vec{P} \cdot \vec{Q} = (2)(1) + (-1)(2) + (1)(-2) = -2$

and, $|\vec{Q}| = \sqrt{1 + 4 + 4} = 3$

Hence, orthogonal projection = $-\frac{2}{3}$

3. B. $5\sqrt{3}$

Explanation: Area of the parallelogram = $\frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

Here, $\vec{d_1} \times \vec{d_2} = \hat{i}(4 - 6) - \hat{j}(12 + 2) + \hat{k}(-9 - 1) = -2\hat{i} - 14\hat{j} - 10\hat{k}$

and, $|\vec{d_1} \times \vec{d_2}| = \sqrt{4 + 196 + 100} = \sqrt{300} = 10\sqrt{3}$

Hence, area = $\frac{1}{2} \times 10\sqrt{3} = 5\sqrt{3}$ square units

4. A. 1

Explanation: Condition for coplanarity: $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

Expanding the determinant, $1(\lambda - 1) - (-1)(2\lambda + \lambda) + 1(-2 - \lambda) = 0$

or, $\lambda - 1 + 3\lambda - 2 - \lambda = 0$

or, $3\lambda = 3$

Therefore, $\lambda = 1$

5. B. $\sqrt{3}$

Explanation: $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$

Here, since they are mutually perpendicular, all dot products are 0, and since they are unit vectors, $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Therefore, $|\vec{a} + \vec{b} + \vec{c}|^2 = 1 + 1 + 1 + 0 = 3$

Hence, $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$

6. A. 3

Explanation: Here, $|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = A^2B^2 \sin^2\theta + A^2B^2 \cos^2\theta = A^2B^2$

According to the question, $A^2B^2 = 144$

or, $16B^2 = 144$

or, $B^2 = 9$

Therefore, $|\vec{B}| = 3$

7. C. 90°

Explanation: According to the question, $|\vec{P} + \vec{Q}|^2 = |\vec{P} - \vec{Q}|^2$

or, $P^2 + Q^2 + 2PQ\cos\theta = P^2 + Q^2 - 2PQ\cos\theta$

or, $4PQ\cos\theta = 0$

or, $\cos\theta = 0$

Therefore, $\theta = 90°$

8. D. $-2\vec{a}$

Explanation: Vector triple product formula: $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

Here, $\hat{i} \times (\hat{i} \times \vec{a}) = (\hat{i} \cdot \vec{a})\hat{i} - (\hat{i} \cdot \hat{i})\vec{a} = a_x\hat{i} - \vec{a}$

Similarly, $\hat{j} \times (\hat{j} \times \vec{a}) = a_y\hat{j} - \vec{a}$ and $\hat{k} \times (\hat{k} \times \vec{a}) = a_z\hat{k} - \vec{a}$

Adding, $(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) - 3\vec{a} = \vec{a} - 3\vec{a}$

Hence, the value of the expression = $-2\vec{a}$

9. A. $\pm\frac{4\hat{i}+3\hat{j}}{5}$

Explanation: Let the required vector be $\vec{v} = x\hat{i} + y\hat{j}$ (since it lies in the xy-plane)

Being perpendicular, $\vec{v} \cdot (3\hat{i} - 4\hat{j} + \hat{k}) = 0$

or, $3x - 4y = 0$

or, $\frac{x}{4} = \frac{y}{3}$

That is, $\vec{v}$ is parallel to $4\hat{i} + 3\hat{j}$

Therefore, the unit vector = $\pm\frac{4\hat{i}+3\hat{j}}{\sqrt{4^2+3^2}} = \pm\frac{4\hat{i}+3\hat{j}}{5}$

10. B. 2

Explanation: Let the perpendicular distance be D

Formula: $D = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

Here, $|2(1) - 3(2) + 6(-1) + 24| = |2 - 6 - 6 + 24| = 14$

and, $\sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{49} = 7$

Hence, $D = \frac{14}{7} = 2$ units

11. B. $\cos^{-1}\left(\frac{1}{3}\right)$

Explanation: Let the side length of the cube be a

Two diagonals: $\vec{d_1} = a\hat{i} + a\hat{j} + a\hat{k}$ and $\vec{d_2} = -a\hat{i} + a\hat{j} + a\hat{k}$

Here, $\vec{d_1} \cdot \vec{d_2} = -a^2 + a^2 + a^2 = a^2$

and, $|\vec{d_1}| = |\vec{d_2}| = \sqrt{3}a$

So, $\cos\theta = \frac{a^2}{3a^2} = \frac{1}{3}$

Hence, $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

12. B. $\frac{2}{\sqrt{5}}$ m

Explanation: Let the distance between the particles be D

Here, $\Delta\vec{r} = \vec{r_2} - \vec{r_1} = (4 - 2t)\hat{i} + (t - 3)\hat{j}$

So, $D^2 = (4 - 2t)^2 + (t - 3)^2 = 5t^2 - 22t + 25$

For minimum distance, $\frac{d(D^2)}{dt} = 10t - 22 = 0$

or, $t = 2.2$ s

Again, $D^2 = 5(2.2)^2 - 22(2.2) + 25 = 24.2 - 48.4 + 25 = 0.8 = \frac{4}{5}$

Hence, $D_{min} = \frac{2}{\sqrt{5}}$ m

13. B. $\frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

Explanation: Let $\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$

From $\vec{A} \cdot \vec{B} = 3$, $x + y + z = 3$

Again, $\vec{A} \times \vec{B} = \hat{i}(z - y) - \hat{j}(z - x) + \hat{k}(y - x) = \hat{j} - \hat{k}$

Equating coefficients, $z - y = 0$ i.e. $y = z$, and $x - z = 1$ i.e. $x = z + 1$

Substituting in $x + y + z = 3$, $(z + 1) + z + z = 3$

or, $z = \frac{2}{3}$, so $y = \frac{2}{3}$ and $x = \frac{5}{3}$

Hence, $\vec{B} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

14. B. -25

Explanation: Let $S = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

Here, $(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

or, $a^2 + b^2 + c^2 + 2S = 0$

or, $9 + 16 + 25 + 2S = 0$

or, $2S = -50$

Therefore, $S = -25$

15. A. 90°

Explanation: To cross the river in the shortest time, the entire velocity of the boat must be used along the width of the river, so the boat must be steered perpendicular to the current, i.e. at 90°.

16. A. $2\sin\frac{\theta}{2}$

Explanation: $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$

Here, $|\vec{a}| = |\vec{b}| = 1$

So, $|\vec{a} - \vec{b}|^2 = 2 - 2\cos\theta = 2(1 - \cos\theta)$

Again, $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

So, $|\vec{a} - \vec{b}|^2 = 4\sin^2\frac{\theta}{2}$

Hence, $|\vec{a} - \vec{b}| = 2\sin\frac{\theta}{2}$

17. B. $2\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$

Explanation: Here, $(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a}) = \vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a}$ [since $\vec{c} \times \vec{c} = \vec{0}$]

When dot-multiplied with $(\vec{a} + \vec{b})$, any term in which a vector repeats gives a scalar triple product equal to zero

So, expression $= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{c} \times \vec{a})$

Again, by cyclic order, $\vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{a} \cdot (\vec{b} \times \vec{c})$

Hence, expression $= 2\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$

18. B. 60°

Explanation: Let the magnitude of each vector be P and the angle between them be θ

According to the question, $|\vec{P} + \vec{Q}| = \sqrt{3}|\vec{P} - \vec{Q}|$

or, $P^2 + P^2 + 2P^2\cos\theta = 3(P^2 + P^2 - 2P^2\cos\theta)$

or, $2P^2(1 + \cos\theta) = 6P^2(1 - \cos\theta)$

or, $1 + \cos\theta = 3 - 3\cos\theta$

or, $4\cos\theta = 2$

or, $\cos\theta = \frac{1}{2}$

Therefore, $\theta = 60°$

19. A. $\frac{\sqrt{35}}{2}$

Explanation: Area of the triangle $= \frac{1}{2}|\vec{A} \times \vec{B}|$

Here, $\vec{A} \times \vec{B} = \hat{i}(1 + 2) - \hat{j}(3 - 4) + \hat{k}(-3 - 2) = 3\hat{i} + \hat{j} - 5\hat{k}$

and, $|\vec{A} \times \vec{B}| = \sqrt{9 + 1 + 25} = \sqrt{35}$

Hence, area $= \frac{\sqrt{35}}{2}$ square units

20. C. 1

Explanation: If two planes are perpendicular, their normal vectors are perpendicular

Here, $\vec{n_1} = 3\hat{i} - 2\hat{j} + k\hat{k}$ and $\vec{n_2} = \hat{i} + 2\hat{j} + \hat{k}$

According to the question, $\vec{n_1} \cdot \vec{n_2} = 0$

or, $3 - 4 + k = 0$

Therefore, $k = 1$

21. C. 90°

Explanation: Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = -a\omega \sin(\omega t)\hat{i} + a\omega \cos(\omega t)\hat{j}$

Here, $\vec{r} \cdot \vec{v} = -a^2\omega \sin \omega t \cos \omega t + a^2\omega \sin \omega t \cos \omega t = 0$

Since the dot product is zero, the two vectors are perpendicular

Hence, the angle = 90° (the particle moves along a circular path)

22. A. $\sin^{-1}\left(\frac{1}{6}\right)$

Explanation: Let the angle between the line and the plane be θ

Direction vector of the line, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and normal vector of the plane, $\vec{n} = \hat{i} - \hat{j} + 2\hat{k}$

Formula: $\sin\theta = \frac{|\vec{b} \cdot \vec{n}|}{|b||\vec{n}|}$

Here, $\vec{b} \cdot \vec{n} = 2 - 1 - 2 = -1$ and $|\vec{b}| = |\vec{n}| = \sqrt{6}$

So, $\sin\theta = \frac{1}{6}$

Hence, $\theta = \sin^{-1}\left(\frac{1}{6}\right)$

23. B. $3x - 2y + 4z = 20$

Explanation: Equation of the plane: $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

Here, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$

So, $3x - 2y + 4z = 3(2) - 2(-1) + 4(3)$

or, $3x - 2y + 4z = 6 + 2 + 12$

Hence, $3x - 2y + 4z = 20$

24. A. $2x - 4y - 5z + 7 = 0$

Explanation: Let the three points be A(1, 1, 1), B(2, −1, 3), C(3, 2, 1)

Here, $\overrightarrow{AB} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 2\hat{i} + \hat{j}$

Normal to the plane, $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = -2\hat{i} + 4\hat{j} + 5\hat{k}$

So, equation of the plane, $-2(x - 1) + 4(y - 1) + 5(z - 1) = 0$

or, $-2x + 4y + 5z = 7$

Hence, $2x - 4y - 5z + 7 = 0$

25. A. 672

Explanation: General term, $T_{r+1} = \binom{9}{r}(2x^2)^{9-r}\left(-\frac{1}{x}\right)^r = \binom{9}{r}2^{9-r}(-1)^r x^{18-3r}$

For the term independent of x, $18 - 3r = 0$

or, $r = 6$

Hence, the required term = $\binom{9}{6}2^3(-1)^6 = 84 \times 8 \times 1 = 672$

26. B. 70

Explanation: Here, the exponent n = 8 is even, so there is one middle term, namely the $\left(\frac{8}{2} + 1\right) = 5$th term

So, $T_5 = \binom{8}{4}\left(\frac{x}{2}\right)^4\left(\frac{2}{x}\right)^4$

$= 70 \times \frac{x^4}{16} \times \frac{16}{x^4}$

Hence, the middle term = 70

27. A. $x - 5y - 3z + 6 = 0$

Explanation: Since the plane is parallel to both $\vec{a}$ and $\vec{b}$, the normal vector is $\vec{n} = \vec{a} \times \vec{b}$

Here, $\vec{n} = \hat{i}(1 - 2) - \hat{j}(-1 - 4) + \hat{k}(1 + 2) = -\hat{i} + 5\hat{j} + 3\hat{k}$

So, equation of the plane, $-1(x - 1) + 5(y - 2) + 3(z + 1) = 0$

or, $-x + 1 + 5y - 10 + 3z + 3 = 0$

or, $-x + 5y + 3z - 6 = 0$

Hence, $x - 5y - 3z + 6 = 0$

28. B. 240

Explanation: General term, $T_{r+1} = \binom{6}{r}(2x)^r = \binom{6}{r}2^r x^r$

For x^4, r = 4

So, coefficient = $\binom{6}{4}2^4 = 15 \times 16 = 240$

29. B. −1

Explanation: To get the sum of coefficients, put the value 1 for each variable

Here, putting x = 1 and y = 1, $(3 - 4)^{15} = (-1)^{15}$

Hence, the sum = −1

30. B. 9

Explanation: From the first condition, $\binom{n}{r-1} = \binom{n}{r+1}$

Since $r - 1 \neq r + 1$, we get $(r - 1) + (r + 1) = n$ i.e. $n = 2r$

From the second condition, $\frac{\binom{n+2}{r+1}}{\binom{n}{r}} = \frac{(n+2)(n+1)}{(r+1)(n-r+1)} = \frac{7}{2}$

Putting $n = 2r$, $\frac{(2r+2)(2r+1)}{(r+1)(r+1)} = \frac{7}{2}$

or, $\frac{2(2r+1)}{r+1} = \frac{7}{2}$

or, $8r + 4 = 7r + 7$

or, r = 3 and n = 6

Hence, n + r = 9

31. C. 66

Explanation: Number of terms in a multinomial expansion = $\binom{n+k-1}{k-1}$, where k is the number of variables

Here, n = 10 and k = 3

So, number of terms = $\binom{12}{2} = \frac{12\times11}{2} = 66$

32. C. 4

Explanation: Here, $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots + (r+1)x^r + \ldots$

For x^3, r = 3

Hence, coefficient = 3 + 1 = 4

33. B. 7

Explanation: The coefficients of the 2nd, 3rd and 4th terms are $\binom{n}{1}$, $\binom{n}{2}$, $\binom{n}{3}$ respectively

According to the question, $2\binom{n}{2} = \binom{n}{1} + \binom{n}{3}$

or, $n(n-1) = n + \frac{n(n-1)(n-2)}{6}$

or, $6(n-1) = 6 + (n-1)(n-2)$ [dividing by n]

or, $6n - 6 = n^2 - 3n + 8$

or, $n^2 - 9n + 14 = 0$

or, $(n-7)(n-2) = 0$

Since $n \geq 3$, n = 7

34. C. $\binom{24}{12}$

Explanation: In the expansion of $(1+x)^n$, when n is even, the coefficient of the middle term is the greatest

Here, n = 24 is even, so the middle term is the $\left(\frac{24}{2}+1\right)$ = 13th term

Hence, the greatest coefficient = $\binom{24}{12}$

35. A. 7 : 8

Explanation: Here, $T_4 = \binom{10}{3}x^7y^3$ and $T_5 = \binom{10}{4}x^6y^4$

According to the question, $\frac{T_4}{T_5} = \frac{1}{2}$

or, $\frac{120x^7y^3}{210x^6y^4} = \frac{1}{2}$

or, $\frac{4x}{7y} = \frac{1}{2}$

or, $\frac{x}{y} = \frac{7}{8}$

Therefore, x : y = 7 : 8

36. C. 9

Explanation: 3rd term, $T_3 = \binom{n}{2}x^2$

According to the question, $\binom{n}{2} = 36$

or, $\frac{n(n-1)}{2} = 36$

or, $n(n-1) = 72 = 9 \times 8$

Therefore, n = 9

37. C. A = B

Explanation: Here, the coefficient of x^2, $A = \binom{4}{2}2^2 \cdot 3^2 = 6 \times 4 \times 9 = 216$

and the coefficient of x^3, $B = \binom{4}{3}2^1 \cdot 3^3 = 4 \times 2 \times 27 = 216$

Hence, A = B

38. B. 12

Explanation: For the middle term to be independent of x, n must be even, and then the middle term = $\binom{n}{\frac{n}{2}}$

According to the question, $\binom{n}{\frac{n}{2}} = 924$

We know, $\binom{12}{6} = \frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1} = 924$

Therefore, n = 12

39. A. $n2^{n-1}$

Explanation: Here, $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$

Differentiating with respect to x, $n(1+x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + \cdots + nC_nx^{n-1}$

Putting x = 1, $n2^{n-1} = C_1 + 2C_2 + \cdots + nC_n$

Hence, $\sum_{r=1}^{n} rC_r = n2^{n-1}$

40. B. 21

Explanation: In the expansion of $(1-x)^{-n}$, the coefficient of $x^r = \binom{n+r-1}{r}$

Here, n = 3 and r = 5

So, coefficient = $\binom{7}{5} = \binom{7}{2} = \frac{7\times6}{2} = 21$

41. C. $|x| < \frac{2}{3}$

Explanation: Here, $(2+3x)^{-1/2} = 2^{-1/2}\left(1+\frac{3x}{2}\right)^{-1/2}$

Condition for the expansion to be valid: $\left|\frac{3x}{2}\right| < 1$

or, $|3x| < 2$

Therefore, $|x| < \frac{2}{3}$

42. A. $\binom{3n}{n}(-1)^n$

Explanation: General term, $T_{r+1} = \binom{3n}{r}x^{3n-r}\left(-x^{-2}\right)^r = \binom{3n}{r}(-1)^r x^{3n-3r}$

Condition for independence of x: 3n − 3r = 0

or, r = n

Hence, the required term = $\binom{3n}{n}(-1)^n$

43. A. -3

Explanation: Here, $(1 + x)^4 = 1 + 4x + 6x^2 + \dots$

and, $(1 - x)^3 = 1 - 3x + 3x^2 - \dots$

Coefficient of x^2 in the product = $(1)(3) + (4)(-3) + (6)(1)$

$= 3 - 12 + 6 = -3$

44. B. 2^{n-1}

Explanation: Putting $x = 1$, $C_0 + C_1 + C_2 + \dots + C_n = 2^n$

Putting $x = -1$, $C_0 - C_1 + C_2 - \dots = 0$

Adding, $2(C_0 + C_2 + C_4 + \dots) = 2^n$

Hence, $C_0 + C_2 + C_4 + \dots = 2^{n-1}$

45. A. $120x^3y^7$

Explanation: Total number of terms = 10 + 1 = 11

4th term from the end = (11 − 4 + 1) = 8th term from the beginning

So, $T_8 = T_{7+1} = \binom{10}{7}x^3y^7$

Hence, the 4th term from the end = $120x^3y^7$

46. B. $a = 2,\ n = 4$

Explanation: From $T_2 = \binom{n}{1}ax = 8x$, $na = 8$ ... (i)

Again, from $T_3 = \binom{n}{2}a^2x^2 = 24x^2$, $\frac{n(n-1)}{2}a^2 = 24$

or, $\frac{n-1}{n} \cdot \frac{(na)^2}{2} = 24$

or, $\frac{n-1}{n} \cdot 32 = 24$ [from (i), $(na)^2 = 64$]

or, $\frac{n-1}{n} = \frac{3}{4}$

or, $n = 4$

Substituting in (i), $a = \frac{8}{4} = 2$

47. B. 3

Explanation: General term, $T_{r+1} = \binom{12}{r}\left(\sqrt{2}\right)^{12-r}\left(\sqrt[3]{3}\right)^r = \binom{12}{r}2^{(12-r)/2}3^{r/3}$

For a rational term, both $\frac{12-r}{2}$ and $\frac{r}{3}$ must be integers

So, r must be a multiple of the L.C.M. of 2 and 3, i.e. 6

Here, since $0 \le r \le 12$, $r = 0, 6, 12$

Hence, the number of rational terms is 3

48. A. 15×2^{14}

Explanation: Differentiating $(1 + x)^{15}$ with respect to x, $15(1 + x)^{14} = C_1 + 2C_2x + \dots + 15C_{15}x^{14}$

Putting $x = 1$, $C_1 + 2C_2 + \dots + 15C_{15} = 15 \times 2^{14}$

Hence, the required value = 15×2^{14}

49. B. 9

Explanation: Here, $\frac{1-x}{(1+x)^2} = (1 - x)(1 + x)^{-2}$

Again, $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \dots$

So, coefficient of $x^4 = (1)(5) + (-1)(-4)$

$= 5 + 4 = 9$

50. A. 2

Explanation: Let $y = -3x$

$(1 + y)^{-1/3} = 1 + \left(-\frac{1}{3}\right)y + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}y^2 + \dots$

Here, the coefficient of $y^2 = \frac{\frac{4}{9}}{2} = \frac{2}{9}$

and, $y^2 = 9x^2$

Hence, the coefficient of $x^2 = \frac{2}{9} \times 9 = 2$