Engineering Daily 12 [Set 03] EV

Binomial Expansion + Vector

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. If the vectors $\vec{A} = 2\hat{i} + m\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular to each other, what is the value of m?

Ⓐ 3 Ⓑ 4 Ⓒ −3 Ⓓ −4

2. If $\vec{P} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{Q} = \hat{i} + 2\hat{j} - 2\hat{k}$, what is the orthogonal projection of $\vec{P}$ on $\vec{Q}$?

Ⓐ $-\frac{2}{3}$ Ⓑ $\frac{2}{3}$ Ⓒ $-\frac{1}{3}$ Ⓓ $\frac{1}{3}$

3. If the diagonals of a parallelogram are $\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$, what is its area in square units?

Ⓐ $10\sqrt{3}$ Ⓑ $5\sqrt{3}$ Ⓒ $15\sqrt{2}$ Ⓓ $5\sqrt{6}$

4. If the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar, what is the value of λ?

Ⓐ 1 Ⓑ −1 Ⓒ 2 Ⓓ −2

5. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, what is the value of $|\vec{a} + \vec{b} + \vec{c}|$?

Ⓐ 1 Ⓑ $\sqrt{3}$ Ⓒ 3 Ⓓ 0

6. If $|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = 144$ and $|\vec{A}| = 4$, what is the value of $|\vec{B}|$?

Ⓐ 3 Ⓑ 4 Ⓒ 6 Ⓓ 12

7. For two vectors $\vec{P}$ and $\vec{Q}$, if $|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$, what is the angle between them?

Ⓐ 0° Ⓑ 45° Ⓒ 90° Ⓓ 180°

8. Which of the following is the value of $\hat{i} \times (\hat{i} \times \vec{a}) + \hat{j} \times (\hat{j} \times \vec{a}) + \hat{k} \times (\hat{k} \times \vec{a})$?

Ⓐ $\vec{a}$ Ⓑ $-\vec{a}$ Ⓒ $2\vec{a}$ Ⓓ $-2\vec{a}$

9. Which is the unit vector that is perpendicular to $3\hat{i} - 4\hat{j} + \hat{k}$ and lies in the xy-plane?

Ⓐ $\pm\frac{4\hat{i}+3\hat{j}}{5}$ Ⓑ $\pm\frac{3\hat{i}+4\hat{j}}{5}$ Ⓒ $\pm\frac{3\hat{i}-4\hat{j}}{5}$ Ⓓ $\pm\frac{4\hat{i}-3\hat{j}}{5}$

10. What is the perpendicular distance from the point $(1, 2, -1)$ to the plane $2x - 3y + 6z + 24 = 0$?

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ 4

11. What is the angle between two diagonals of a cube?

Ⓐ $\cos^{-1}\left(\frac{1}{2}\right)$ Ⓑ $\cos^{-1}\left(\frac{1}{3}\right)$
Ⓒ $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ Ⓓ $\cos^{-1}\left(\frac{2}{3}\right)$

12. One particle is at position $\vec{r}_1 = (2t\hat{i} + 3\hat{j})$ m and another particle is at position $\vec{r}_2 = (4\hat{i} + t\hat{j})$ m. What is the minimum distance between them?

Ⓐ $\frac{1}{\sqrt{5}}$ m Ⓑ $\frac{2}{\sqrt{5}}$ m Ⓒ $\frac{4}{\sqrt{5}}$ m Ⓓ $\frac{3}{\sqrt{5}}$ m

13. If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{A} \times \vec{B} = \hat{j} - \hat{k}$, where $\vec{A} \cdot \vec{B} = 3$, what is the vector $\vec{B}$?

Ⓐ $\frac{1}{3}(2\hat{i} + 5\hat{j} + 2\hat{k})$ Ⓑ $\frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$
Ⓒ $\frac{1}{3}(2\hat{i} + 2\hat{j} + 5\hat{k})$ Ⓓ $\frac{1}{3}(\hat{i} + \hat{j} + 7\hat{k})$

14. The magnitudes of three vectors $\vec{a}, \vec{b}, \vec{c}$ are 3, 4, 5 respectively and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. What is the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$?

Ⓐ 25 Ⓑ −25 Ⓒ 50 Ⓓ −50

15. The velocity of a river's current is 4 kmh^{-1}. The velocity of a boat is 5 kmh^{-1}. At what angle to the current must the boat be steered to cross the river in the shortest time?

Ⓐ 90° Ⓑ $\cos^{-1}\left(-\frac{4}{5}\right)$
Ⓒ $\sin^{-1}\left(\frac{4}{5}\right)$ Ⓓ 0°

16. If $\vec{a}$ and $\vec{b}$ are two unit vectors and the angle between them is θ, which of the following is the value of $|\vec{a} - \vec{b}|$?

Ⓐ $2\sin\frac{\theta}{2}$ Ⓑ $2\cos\frac{\theta}{2}$ Ⓒ $\sin\frac{\theta}{2}$ Ⓓ $\cos\frac{\theta}{2}$

17. The value of $(\vec{a} + \vec{b}) \cdot \{(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\}$ is equal to which of the following?

Ⓐ $\vec{a} \cdot (\vec{b} \times \vec{c})$ Ⓑ $2\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$
Ⓒ 0 Ⓓ $3\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$

18. The magnitude of the sum of two vectors is $\sqrt{3}$ times the magnitude of their difference. If the two vectors have equal magnitudes, what is the angle between them?

Ⓐ 30° Ⓑ 60° Ⓒ 90° Ⓓ 120°

19. Two adjacent sides of a triangle are $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$. What is the area of the triangle?

Ⓐ $\frac{\sqrt{35}}{2}$ Ⓑ $\sqrt{35}$ Ⓒ $\frac{\sqrt{38}}{2}$ Ⓓ $\sqrt{38}$

20. If the planes $3x - 2y + kz = 4$ and $x + 2y + z = 5$ are perpendicular to each other, what is the value of k?

Ⓐ 2 Ⓑ −1 Ⓒ 1 Ⓓ −2

21. The position vector of a point is $\vec{r} = a\cos(\omega t)\hat{i} + a\sin(\omega t)\hat{j}$. What is the angle between the vector $\vec{r}$ and the velocity vector $\vec{v}$?

Ⓐ $0°$ Ⓑ $45°$ Ⓒ $90°$ Ⓓ $180°$

22. What is the angle between the straight line $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-1}$ and the plane $x - y + 2z = 5$?

Ⓐ $\sin^{-1}\left(\frac{1}{6}\right)$ Ⓑ $\cos^{-1}\left(\frac{1}{6}\right)$

Ⓒ $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$ Ⓓ $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$

23. Which is the equation of the plane passing through the point $A(2, -1, 3)$ and perpendicular to the vector $\vec{n} = 3\hat{i} - 2\hat{j} + 4\hat{k}$?

Ⓐ $3x - 2y + 4z = 10$ Ⓑ $3x - 2y + 4z = 20$

Ⓒ $2x - y + 3z = 20$ Ⓓ $3x + 2y + 4z = 20$

24. Which is the equation of the plane passing through the points $(1, 1, 1)$, $(2, -1, 3)$ and $(3, 2, 1)$?

Ⓐ $2x - 4y - 5z + 7 = 0$ Ⓑ $2x + 4y - 5z - 7 = 0$

Ⓒ $2x - 4y + 5z - 7 = 0$ Ⓓ $x - 2y - z + 2 = 0$

25. What is the value of the term independent of x in the expansion of $\left(2x^2 - \frac{1}{x}\right)^9$?

Ⓐ 672 Ⓑ -672 Ⓒ 84 Ⓓ -84

26. What is the middle term in the expansion of $\left(\frac{x}{2} + \frac{2}{x}\right)^8$?

Ⓐ 56 Ⓑ 70 Ⓒ 112 Ⓓ 28

27. A plane passes through the point $(1, 2, -1)$ and is parallel to the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Which is the equation of the plane?

Ⓐ $x - 5y - 3z + 6 = 0$ Ⓑ $x + 5y + 3z - 8 = 0$

Ⓒ $x - 5y + 3z + 12 = 0$ Ⓓ $2x - y + 3z + 3 = 0$

28. What is the coefficient of x^4 in the expansion of $(1 + 2x)^6$?

Ⓐ 120 Ⓑ 240 Ⓒ 60 Ⓓ 480

29. What is the sum of the coefficients of all terms in the expansion of $(3x - 4y)^{15}$?

Ⓐ 1 Ⓑ -1 Ⓒ 0 Ⓓ 3^{15}

30. If $\binom{n}{r-1} = \binom{n}{r+1}$ and $\binom{n+2}{r+1} = \frac{7}{2}\binom{n}{r}$, what is the value of $n + r$?

Ⓐ 7 Ⓑ 9 Ⓒ 11 Ⓓ 15

31. How many terms are there in total in the expansion of $(x + y + z)^{10}$?

Ⓐ 11 Ⓑ 55 Ⓒ 66 Ⓓ 286

32. What is the coefficient of x^3 in the expansion of $(1 - x)^{-2}$? (where $|x| < 1$)

Ⓐ 3 Ⓑ -3 Ⓒ 4 Ⓓ -4

33. If the coefficients of the 2nd, 3rd and 4th terms in the expansion of $(1 + x)^n$ are in arithmetic progression (AP), what is the value of n? (where $n \geq 3$)

Ⓐ 5 Ⓑ 7 Ⓒ 9 Ⓓ 14

34. Which is the greatest coefficient in the expansion of $(1 + x)^{24}$?

Ⓐ $\binom{24}{10}$ Ⓑ $\binom{24}{11}$ Ⓒ $\binom{24}{12}$ Ⓓ $\binom{24}{14}$

35. In the expansion of $(x + y)^{10}$, the ratio of the 4th and 5th terms is $1 : 2$. What is the value of $x : y$?

Ⓐ $7 : 8$ Ⓑ $8 : 7$ Ⓒ $3 : 4$ Ⓓ $4 : 3$

36. If the 3rd term in the expansion of $(1 + x)^n$ is $36x^2$, what is the value of n?

Ⓐ 6 Ⓑ 8 Ⓒ 9 Ⓓ 12

37. If the coefficients of x^2 and x^3 in the expansion of $(2 + 3x)^4$ are A and B respectively, what is the relation between A and B?

Ⓐ $A > B$ Ⓑ $A < B$ Ⓒ $A = B$ Ⓓ $A = 2B$

38. If the value of the middle term in the expansion of $\left(x + \frac{1}{x}\right)^n$ is 924, what is the value of n? (where the middle term is independent of x)

Ⓐ 10 Ⓑ 12 Ⓒ 14 Ⓓ 16

39. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$, what is the value of $\sum_{r=1}^{n} rC_r$?

Ⓐ $n2^{n-1}$ Ⓑ $n2^n$

Ⓒ $(n - 1)2^{n-1}$ Ⓓ $(n - 1)2^n$

40. What is the coefficient of x^5 in the expansion of $(1 - x)^{-3}$? (where $|x| < 1$)

Ⓐ 15 Ⓑ 21 Ⓒ -21 Ⓓ 35

41. For which condition on x is the expansion of $(2 + 3x)^{-1/2}$ convergent (valid)?

Ⓐ $|x| < 2$ Ⓑ $|x| < 3$

Ⓒ $|x| < \frac{2}{3}$ Ⓓ $|x| < \frac{3}{2}$

42. Which is the term independent of x in the expansion of $\left(x - \frac{1}{x^2}\right)^{3n}$?

Ⓐ $\binom{3n}{n}(-1)^n$ Ⓑ $\binom{3n}{n}$

Ⓒ $\binom{3n}{2n}(-1)^{2n}$ Ⓓ $\binom{3n}{n-1}$

43. What is the coefficient of x^2 in the expansion of $(1 + x)^4(1 - x)^3$?

Ⓐ -3 Ⓑ 3 Ⓒ -1 Ⓓ 0

44. What is the sum of the coefficients of the even-power terms in the expansion of $(1 + x)^n$?

Ⓐ 2^n Ⓑ 2^{n-1} Ⓒ 2^{n+1} Ⓓ $n2^{n-1}$

45. Which is the 4th term from the end in the expansion of $(x + y)^{10}$?

Ⓐ $120x^3y^7$ Ⓑ $120x^7y^3$

Ⓒ $210x^4y^6$ Ⓓ $210x^6y^4$

46. If the first three terms in the expansion of $(1 + ax)^n$ are $1, 8x, 24x^2$ respectively, what are the values of a and n?

Ⓐ $a = 4, n = 2$ Ⓑ $a = 2, n = 4$

Ⓒ $a = 8, n = 1$ Ⓓ $a = 1, n = 8$

47. How many rational terms are there in the expansion of $\left(\sqrt{2} + \sqrt[3]{3}\right)^{12}$?

Ⓐ 2 Ⓑ 3 Ⓒ 4 Ⓓ 6

48. If $(1 + x)^{15} = C_0 + C_1x + \cdots + C_{15}x^{15}$, what is the value of $C_1 + 2C_2 + 3C_3 + \cdots + 15C_{15}$?

Ⓐ 15×2^{14} Ⓑ 15×2^{15}

Ⓒ 14×2^{15} Ⓓ $2^{15} - 1$

49. What is the coefficient of x^4 in the expansion of $\frac{1-x}{(1+x)^2}$? (where $|x| < 1$)

Ⓐ -9 Ⓑ 9 Ⓒ 5 Ⓓ -5

50. What is the coefficient of x^2 in the expansion of $(1 - 3x)^{-1/3}$?

Ⓐ 2 Ⓑ 6 Ⓒ -2 Ⓓ -6