MCQ 50 marks

1. B. 7

Explanation: $|3\vec{a} - 2\vec{b}|^2 = 9|\vec{a}|^2 - 12\,\vec{a}\cdot\vec{b} + 4|\vec{b}|^2$

Here, $|\vec{a}| = |\vec{b}| = 1$

and, $\vec{a}\cdot\vec{b} = 1 \times 1 \times \cos 60° = \frac{1}{2}$

So, $|3\vec{a} - 2\vec{b}|^2 = 9 - 12 \times \frac{1}{2} + 4$

Therefore, the value = 7

2. A. $\frac{1}{\sqrt{2}}$

Explanation: The line of intersection is perpendicular to both normals, so its direction vector is $\vec{v} = \vec{n}_1 \times \vec{n}_2$

Here, $\vec{v} = (\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} - \hat{j} + \hat{k}) = \hat{i}(1 + 1) - \hat{j}(1 - 1) + \hat{k}(-1 - 1) = 2\hat{i} - 2\hat{k}$

Unit vector, $\hat{u} = \frac{2\hat{i} - 2\hat{k}}{2\sqrt{2}} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{k})$

Orthogonal projection = $\vec{a}\cdot\hat{u} = \frac{1}{\sqrt{2}}(2 - 1)$

Therefore, the orthogonal projection = $\frac{1}{\sqrt{2}}$

3. B. $-11\hat{i} + 13\hat{j} - 10\hat{k}$

Explanation: Formula: $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

Here, $\vec{a}\cdot\vec{c} = 6 - 1 - 2 = 3$

and, $\vec{b}\cdot\vec{c} = 3 + 2 + 2 = 7$

So, the expression = $3(\hat{i} + 2\hat{j} - \hat{k}) - 7(2\hat{i} - \hat{j} + \hat{k})$

$= (3\hat{i} + 6\hat{j} - 3\hat{k}) - (14\hat{i} - 7\hat{j} + 7\hat{k})$

Therefore, the expression = $-11\hat{i} + 13\hat{j} - 10\hat{k}$

4. B. $\sqrt{21}$

Explanation: Any point on the line is $(2t + 1,\ 3t + 2,\ 4t - 1)$

If it lies on the plane, $(2t + 1) + (3t + 2) + (4t - 1) = 11$

or, $9t + 2 = 11$

or, $t = 1$

So, the point of intersection is $P(3, 5, 3)$

Distance = $\sqrt{(3-1)^2 + (5-1)^2 + (3-2)^2} = \sqrt{4 + 16 + 1}$

Therefore, the distance = $\sqrt{21}$

5. C. 8

Explanation: Let $\vec{v} = \lambda\{\vec{a} \times (\vec{a} \times \vec{b})\} = \lambda\{(\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b}\}$ (since $\vec{v}$ lies in the plane of $\vec{a}, \vec{b}$ and is perpendicular to $\vec{a}$)

Here, $\vec{a}\cdot\vec{b} = 3$ and $|\vec{a}|^2 = 6$

So, $\vec{v} = \lambda\{3(2\hat{i} + \hat{j} - \hat{k}) - 6(\hat{i} + 2\hat{j} + \hat{k})\} = -9\lambda(\hat{j} + \hat{k})$

Let $\vec{v} = k(\hat{j} + \hat{k})$

Given, $\frac{\vec{v}\cdot\vec{c}}{|c|} = \frac{k(-1+2)}{\sqrt{6}} = \frac{2}{\sqrt{6}}$

or, $k = 2$

So, $\vec{v} = 2\hat{j} + 2\hat{k}$

Therefore, $|\vec{v}|^2 = 4 + 4 = 8$

6. B. $6\sqrt{3}$

Explanation: Formula: $|\vec{a} \times \vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

Here, $|\vec{a} \times \vec{b}|^2 + 6^2 = 3^2 \times 4^2$

or, $|\vec{a} \times \vec{b}|^2 = 144 - 36 = 108$

Therefore, $|\vec{a} \times \vec{b}| = \sqrt{108} = 6\sqrt{3}$

7. A. $8\hat{i} + 5\hat{j} - 8\hat{k}$

Explanation: Let the circumcentre be $\vec{p}$, the centroid $\vec{g}$ and the orthocentre $\vec{h}$

By Euler line property, the centroid divides the segment joining the orthocentre and the circumcentre internally in the ratio 2 : 1 (from the orthocentre)

So, $\vec{g} = \frac{\vec{h} + 2\vec{p}}{3}$

or, $\vec{h} = 3\vec{g} - 2\vec{p}$

Here, $\vec{h} = 3(4\hat{i} + \hat{j} - 2\hat{k}) - 2(2\hat{i} - \hat{j} + \hat{k})$

Therefore, $\vec{h} = 8\hat{i} + 5\hat{j} - 8\hat{k}$

8. C. $\frac{\pi}{3}$

Explanation: Direction cosine formula: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Here, $\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{4}$ and $\gamma = \theta$

So, $\frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

or, $\cos^2\theta = \frac{1}{4}$

Since θ is acute, $\cos\theta = \frac{1}{2}$

Therefore, $\theta = \frac{\pi}{3}$

9. B. $\sqrt{2}$

Explanation: The direction vector of the y-axis is $\vec{b}_1 = \hat{j}$ and the position vector of the origin is $\vec{a}_1 = \vec{0}$

The direction vector of the line is $\vec{b}_2 = \hat{i} + \hat{j} + \hat{k}$ and a point on it has position vector $\vec{a}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$

Here, $\vec{b}_1 \times \vec{b}_2 = \hat{i}(1-0) - \hat{j}(0-0) + \hat{k}(0-1) = \hat{i} - \hat{k}$

and, $|\vec{b}_1 \times \vec{b}_2| = \sqrt{2}$

Shortest distance $= \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{|1-3|}{\sqrt{2}}$

Therefore, the distance $= \sqrt{2}$

10. B. $-\frac{1}{9}$

Explanation: Condition for coplanarity: $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

Expanding the determinant, $1(8-1) - \lambda(8+1) + 2(-2-2) = 0$

or, $7 - 9\lambda - 8 = 0$

or, $9\lambda = -1$

Therefore, $\lambda = -\frac{1}{9}$

11. A. $\pm\frac{13\hat{i}-4\hat{j}-5\hat{k}}{\sqrt{210}}$

Explanation: The vector lies in the plane of $\vec{a}$, $\vec{b}$ and is perpendicular to $\vec{c}$, so it is parallel to $\vec{c} \times (\vec{a} \times \vec{b})$

Here, $\vec{a} \times \vec{b} = \hat{i}(8-9) - \hat{j}(-4+6) + \hat{k}(3-4) = -\hat{i} - 2\hat{j} - \hat{k}$

Again, $\vec{c} \times (\vec{a} \times \vec{b}) = \hat{i}(3+10) - \hat{j}(-1+5) + \hat{k}(-2-3) = 13\hat{i} - 4\hat{j} - 5\hat{k}$

and, $|13\hat{i} - 4\hat{j} - 5\hat{k}| = \sqrt{169+16+25} = \sqrt{210}$

Therefore, the unit vector $= \pm\frac{13\hat{i}-4\hat{j}-5\hat{k}}{\sqrt{210}}$

12. C. 27

Explanation: Here, since $\vec{c} = \vec{a} \times \vec{b}$, we have $\vec{c} \perp \vec{a}$

So, $|\vec{a} \times \vec{c}|^2 = |\vec{a}|^2|\vec{c}|^2$

Again, $|\vec{c}| = |\vec{a}||\vec{b}| \sin\theta = 2\sqrt{3}\sin\theta$

So, $|\vec{a} \times \vec{c}|^2 = 3 \times 12 \sin^2\theta = 36\sin^2\theta$

In this interval $\sin\theta$ is maximum at $\theta = \frac{\pi}{3}$

Therefore, the maximum value $= 36 \times \frac{3}{4} = 27$

13. A. $\frac{9}{2}$

Explanation: If the lines intersect, they are coplanar

Here, $\vec{a}_2 - \vec{a}_1 = (2, k+1, -1)$, $\vec{b}_1 = (2,3,4)$ and $\vec{b}_2 = (1,2,1)$

Condition: $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

or, $2(3-8) - (k+1)(2-4) - 1(4-3) = 0$

or, $-10 + 2(k+1) - 1 = 0$

or, $2k = 9$

Therefore, $k = \frac{9}{2}$

14. B. $\frac{22}{3}$

Explanation: Given, $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$

or, $(\vec{r} - \vec{b}) \times \vec{a} = \vec{0}$

So, $\vec{r} = \vec{b} + \lambda\vec{a} = (1+2\lambda)\hat{i} + (\lambda-1)\hat{j} + (2-\lambda)\hat{k}$

Again, from $\vec{r} \cdot \vec{c} = 0$, $(1+2\lambda) + 2(\lambda-1) + (2-\lambda) = 0$

or, $3\lambda + 1 = 0$

or, $\lambda = -\frac{1}{3}$

So, $\vec{r} = \frac{1}{3}\hat{i} - \frac{4}{3}\hat{j} + \frac{7}{3}\hat{k}$

Therefore, $|\vec{r}|^2 = \frac{1+16+49}{9} = \frac{22}{3}$

15. B. $(0,-1,-3)$

Explanation: Formula for image: $\frac{x'-x_1}{a} = \frac{y'-y_1}{b} = \frac{z'-z_1}{c} = -\frac{2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$

Here, the plane is $3x - y + 4z - 2 = 0$ and $(x_1, y_1, z_1) = (3,-2,1)$

Right side $= -\frac{2(9+2+4-2)}{9+1+16} = -\frac{26}{26} = -1$

So, $\frac{x'-3}{3} = -1$ i.e. $x' = 0$

and, $\frac{y'+2}{-1} = -1$ i.e. $y' = -1$

and, $\frac{z'-1}{4} = -1$ i.e. $z' = -3$

Therefore, the image is $(0,-1,-3)$

16. C. $9\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$

Explanation: Here, $(\vec{b} + 2\vec{c}) \times (\vec{c} + 2\vec{a}) = \vec{b} \times \vec{c} + 2(\vec{b} \times \vec{a}) + 4(\vec{c} \times \vec{a})$ [since $\vec{c} \times \vec{c} = \vec{0}$]

When dot-multiplied with $(\vec{a} + 2\vec{b})$, any term in which a vector repeats gives a scalar triple product equal to zero

So, the expression $= \vec{a} \cdot (\vec{b} \times \vec{c}) + 8\{\vec{b} \cdot (\vec{c} \times \vec{a})\}$

Again, by cyclic order, $\vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{a} \cdot (\vec{b} \times \vec{c})$

Therefore, the expression $= 9\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$

17. B. 7

Explanation: Any point on the line is Q(2t + 1, 4t + 3, 3t + 2)

Here, $\overrightarrow{PQ} = (2t-2)\hat{i} + (4t-5)\hat{j} + 3t\hat{k}$

If $\overrightarrow{PQ}$ is parallel to the plane, it is perpendicular to the normal $\vec{n} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

So, $3(2t-2) + 2(4t-5) - 2(3t) = 0$

or, $8t - 16 = 0$

or, $t = 2$

So, $\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Therefore, the distance $= \sqrt{4+9+36} = 7$

[Correction: the given solution uses the line point (2t + 1, 4t + 3, 3t + 2), which matches $\frac{x-1}{2}$; the printed $\frac{x-2}{1}$ in the question was a typo, so the line is taken as $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$.]

18. A. $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

Explanation: Equation of line BC: $\frac{x-4}{1} = \frac{y-7}{2} = \frac{z-1}{-2} = t$

Let the foot of the perpendicular be F (t + 4, 2t + 7, −2t + 1)

Here, $\overrightarrow{AF} = (t+3)\hat{i} + (2t+7)\hat{j} + (-2t-2)\hat{k}$ and the direction vector of BC is $\hat{i} + 2\hat{j} - 2\hat{k}$

Since $\overrightarrow{AF} \perp$ BC, $(t+3) + 2(2t+7) - 2(-2t-2) = 0$

or, $9t + 21 = 0$

or, $t = -\frac{7}{3}$

Therefore, F $= \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

19. C. 8

Explanation: Volume, $V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$

Expanding the determinant, $1(12 + n^2) - 1(6 + n) + n(2n - 4) = 3n^2 - 5n + 6$

Given, $3n^2 - 5n + 6 = 158$

or, $3n^2 - 5n - 152 = 0$

or, $n = \frac{5 \pm 43}{6}$

Since n is a positive integer, n = 8

20. B. $-\frac{29}{2}$

Explanation: Let $S = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

Here, $(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

or, $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2S = 0$

or, $4 + 9 + 16 + 2S = 0$

or, $2S = -29$

Therefore, $S = -\frac{29}{2}$

21. C. 560

Explanation: Area of the triangle, $\Delta = \frac{1}{2}|\vec{u} \times \vec{v}|$

Here, $\vec{u} \times \vec{v} = (3\vec{a} + 2\vec{b}) \times (\vec{a} - 2\vec{b}) = -6(\vec{a} \times \vec{b}) + 2(\vec{b} \times \vec{a}) = -8(\vec{a} \times \vec{b})$

So, $\Delta = 4|\vec{a} \times \vec{b}|$

Again, $\vec{a} \times \vec{b} = \hat{i}(2-1) - \hat{j}(-1-2) + \hat{k}(1+4) = \hat{i} + 3\hat{j} + 5\hat{k}$

and, $|\vec{a} \times \vec{b}| = \sqrt{35}$

So, $\Delta = 4\sqrt{35}$

Therefore, $\Delta^2 = 16 \times 35 = 560$

22. C. 60°

Explanation: Normal vectors: $\vec{n}_1 = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{n}_2 = \hat{i} + \hat{j} + 2\hat{k}$

Formula: $\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

Here, $\vec{n}_1 \cdot \vec{n}_2 = 2 - 1 + 2 = 3$ and $|\vec{n}_1||\vec{n}_2| = \sqrt{6} \times \sqrt{6} = 6$

So, $\cos\theta = \frac{1}{2}$

Therefore, $\theta = 60°$

23. A. −10 J

Explanation: Displacement, $\vec{s} = (3-1)\hat{i} + (1+2)\hat{j} + (-1-3)\hat{k} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

Work done, $W = \vec{F} \cdot \vec{s}$

Here, $W = (4)(2) + (-2)(3) + (3)(-4) = 8 - 6 - 12$

Therefore, $W = -10$ J

24. B. 5x + 4y − z = 8

Explanation: Equation of the required plane: $(2x + y - z - 3) + \lambda(x + 2y + z - 2) = 0$

It passes through (1, 1, 1), so $(2 + 1 - 1 - 3) + \lambda(1 + 2 + 1 - 2) = 0$

or, $-1 + 2\lambda = 0$

or, $\lambda = \frac{1}{2}$

So, $2(2x + y - z - 3) + (x + 2y + z - 2) = 0$

or, $4x + 2y - 2z - 6 + x + 2y + z - 2 = 0$

Therefore, $5x + 4y - z = 8$

25. A. 5376

Explanation: General term, $T_{r+1} = \binom{9}{r}(x^2)^{9-r}\left(-\frac{2}{x}\right)^r = \binom{9}{r}(-2)^r x^{18-3r}$

For the term independent of x, $18 - 3r = 0$

or, $r = 6$

So, the term $= \binom{9}{6}(-2)^6 = 84 \times 64$

Therefore, the value = 5376

26. C. 135°

Explanation: Formula: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Given, $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{\sqrt{2}}|\vec{a}||\vec{b}|\vec{c}$

Since $\vec{b}$ and $\vec{c}$ are not parallel, equating the coefficients of $\vec{c}$, $-\vec{a} \cdot \vec{b} = \frac{1}{\sqrt{2}}|\vec{a}||\vec{b}|$

or, $-\cos\theta = \frac{1}{\sqrt{2}}$

So, $\cos\theta = -\frac{1}{\sqrt{2}}$

Therefore, $\theta = 135°$

27. C. 9

Explanation: Coefficient of the 4th term = $\binom{n}{3}$ and of the 5th term = $\binom{n}{4}$

Given, $\frac{\binom{n}{3}}{\binom{n}{4}} = \frac{2}{3}$

or, $\frac{4!\,(n-4)!}{3!\,(n-3)!} = \frac{2}{3}$

or, $\frac{4}{n-3} = \frac{2}{3}$

or, $2n - 6 = 12$

Therefore, $n = 9$

28. A. 512

Explanation: Let $f(x) = (1 + x + x^2 + x^3)^5 = c_0 + c_1x + c_2x^2 + \dots$

Putting $x = 1$, $f(1) = 4^5 = 1024$

Putting $x = -1$, $f(-1) = (1 - 1 + 1 - 1)^5 = 0$

Adding, $f(1) + f(-1) = 2(c_0 + c_2 + c_4 + \dots)$

or, $1024 = 2(c_0 + c_2 + c_4 + \dots)$

Therefore, the sum of even-power coefficients = 512

29. B. 672

Explanation: In the expansion of $(1 - y)^{-n}$, the coefficient of $y^r = \binom{n+r-1}{r}$

Here, $n = 3$, $y = 2x$ and $r = 5$

So, the coefficient of $x^5 = \binom{7}{5} \times 2^5$

$= 21 \times 32$

Therefore, the coefficient = 672

30. D. Both B and C

Explanation: The given expression = $(1 + x)^n \cdot \frac{(x+1)^n}{x^n} = \frac{(1+x)^{2n}}{x^n}$

To get the coefficient of $\frac{1}{x}$, we need the coefficient of x^{n-1} in $(1 + x)^{2n}$

So, the coefficient = $\binom{2n}{n-1}$

Again, $\binom{2n}{n-1} = \binom{2n}{2n-(n-1)} = \binom{2n}{n+1}$

Therefore, both B and C are correct

31. B. 105

Explanation: In the expansion of $(1 + x)^{10}$, the coefficients of x^4, x^3, x^2 are $\binom{10}{4}$, $\binom{10}{3}$, $\binom{10}{2}$ respectively

So, the coefficient of x^4 in the product = $1 \cdot \binom{10}{4} - 2\binom{10}{3} + 3\binom{10}{2}$

Here, $= 210 - 2(120) + 3(45)$

$= 210 - 240 + 135$

Therefore, the coefficient = 105

32. B. 97

Explanation: General term, $T_{r+1} = \binom{100}{r} 5^{\frac{100-r}{8}} 2^{\frac{r}{6}}$

For a rational term, r must be a multiple of 6 and $(100 - r)$ a multiple of 8

That is, $r = 6k$ and $r \equiv 4 \pmod 8$

For $0 \le r \le 100$, the values satisfying both are $r = 12, 36, 60, 84$, i.e. 4 rational terms

Total number of terms = 101

Therefore, the number of irrational terms = $101 - 4 = 97$

33. B. $\binom{18}{8}$

Explanation: Pascal's identity: $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$

Here, $\binom{15}{5} + \binom{15}{6} = \binom{16}{6}$

Again, $\binom{16}{6} + \binom{16}{7} = \binom{17}{7}$

Again, $\binom{17}{7} + \binom{17}{8} = \binom{18}{8}$

Therefore, the value = $\binom{18}{8}$

34. B. 10

Explanation: 5th term from the beginning, $T_5 = \binom{n}{4} \left(2^{\frac{1}{4}}\right)^{n-4} \left(3^{-\frac{1}{4}}\right)^4$

5th term from the end, $T_5' = \binom{n}{4} \left(3^{-\frac{1}{4}}\right)^{n-4} \left(2^{\frac{1}{4}}\right)^4$

So, $\frac{T_5}{T_5'} = 2^{\frac{n-4}{4}-1} \cdot 3^{\frac{n-4}{4}-1} = 6^{\frac{n-8}{4}}$

Given, $6^{\frac{n-8}{4}} = 6^{\frac{1}{2}}$

or, $\frac{n-8}{4} = \frac{1}{2}$

Therefore, $n = 10$

35. C. 9

Explanation: We know, $3^3 = 27 = 2 \times 13 + 1$

Here, $3^{2024} = 3^2 \cdot (3^3)^{674} = 9(26 + 1)^{674}$

By binomial expansion, $(26 + 1)^{674} = 13k + 1$ (k an integer)

So, $3^{2024} = 9(13k + 1) = 13(9k) + 9$

Therefore, the remainder = 9

36. B. $\frac{3^{10}-1}{2}$

Explanation: Let $f(x) = (1 + 2x)^{10} = c_0 + c_1x + c_2x^2 + \dots$

Putting $x = 1$, $f(1) = 3^{10}$

Putting $x = -1$, $f(-1) = (-1)^{10} = 1$

Subtracting, $f(1) - f(-1) = 2(c_1 + c_3 + c_5 + \dots)$

or, $3^{10} - 1 = 2(c_1 + c_3 + c_5 + \dots)$

Therefore, the sum of odd-power coefficients $= \frac{3^{10}-1}{2}$

37. C. 30

Explanation: Here, $(1 + x + x^2)^5 = \{1 + x(1 + x)\}^5 = \sum_{k=0}^{5} \binom{5}{k} x^k(1 + x)^k$

x^3 comes from $k = 2$ and $k = 3$ (for $k \le 1$ the highest power is 2)

For $k = 2$, the x^3 term of $10x^2(1 + x)^2$ is $10 \times 2 = 20$

For $k = 3$, the x^3 term of $10x^3(1 + x)^3$ is $10 \times 1 = 10$

Therefore, the coefficient $= 20 + 10 = 30$

38. B. 18

Explanation: $(1 + x^2)^4 = 1 + 4x^2 + 6x^4 + 4x^6 + x^8$

and, $(1 + x^3)^3 = 1 + 3x^3 + 3x^6 + x^9$

To get x^{10}, we need x^a from the first and x^b from the second with $a + b = 10$

Here $a \in \{0, 2, 4, 6, 8\}$ and $b \in \{0, 3, 6, 9\}$

The only possible pair is $a = 4$, $b = 6$

So, the coefficient $= 6 \times 3$

Therefore, the coefficient = 18

39. D. $\frac{2^{n+1}-1}{n+1}$

Explanation: We know, $\frac{1}{r+1}\binom{n}{r} = \frac{1}{n+1}\binom{n+1}{r+1}$

So, $\sum_{r=0}^{n} \frac{1}{r+1}\binom{n}{r} = \frac{1}{n+1}\sum_{r=0}^{n}\binom{n+1}{r+1}$

Here, $\sum_{r=0}^{n}\binom{n+1}{r+1} = 2^{n+1} - \binom{n+1}{0} = 2^{n+1} - 1$

Therefore, the value $= \frac{2^{n+1}-1}{n+1}$

40. B. 5

Explanation: Coefficient of $x^r = \binom{14}{r}3^{14-r}2^r$

and of $x^{r+1} = \binom{14}{r+1}3^{13-r}2^{r+1}$

Given, $\binom{14}{r}3^{14-r}2^r = \binom{14}{r+1}3^{13-r}2^{r+1}$

or, $\frac{3}{14-r} = \frac{2}{r+1}$

or, $3r + 3 = 28 - 2r$

or, $5r = 25$

Therefore, $r = 5$

41. B. 11

Explanation: General term, $T_{r+1} = \binom{120}{r}5^{\frac{120-r}{3}}3^{\frac{r}{4}}$

For an integer term, both $\frac{120-r}{3}$ and $\frac{r}{4}$ must be integers

That is, r must be a multiple of both 3 and 4, so a multiple of 12

For $0 \le r \le 120$, $r = 0, 12, 24, \dots, 120$

Therefore, the number of terms $= \frac{120}{12} + 1 = 11$

42. C. $2^{19} + \frac{1}{2}\binom{20}{10}$

Explanation: We know, $\sum_{k=0}^{20}\binom{20}{k} = 2^{20}$

By symmetry, $\sum_{k=0}^{9}\binom{20}{k} = \sum_{k=11}^{20}\binom{20}{k} = S$ (say)

So, $2S + \binom{20}{10} = 2^{20}$

or, $S = 2^{19} - \frac{1}{2}\binom{20}{10}$

Here, $\sum_{k=0}^{10}\binom{20}{k} = S + \binom{20}{10}$

Therefore, the value $= 2^{19} + \frac{1}{2}\binom{20}{10}$

43. B. $-\frac{1920}{x}$

Explanation: Total number of terms $= 10 + 1 = 11$

4th term from the end $= (11 - 4 + 1) = 8$th term from the beginning

So, $T_8 = \binom{10}{7}\left(\frac{x^2}{2}\right)^3\left(-\frac{2}{x}\right)^7$

$= 120 \times \frac{x^6}{8} \times \left(-\frac{128}{x^7}\right)$

Therefore, $T_8 = -\frac{1920}{x}$

44. C. $(n + 1)2^n$

Explanation: The given series $= \sum_{r=0}^{n}(2r + 1)C_r = 2\sum_{r=0}^{n} rC_r + \sum_{r=0}^{n} C_r$

We know, $\sum rC_r = n2^{n-1}$ and $\sum C_r = 2^n$

So, the series $= 2 \cdot n2^{n-1} + 2^n = n2^n + 2^n$

Therefore, the series $= (n + 1)2^n$

45. A. -144

Explanation: Here, $1 - x - x^2 + x^3 = (1 - x) - x^2(1 - x) = (1 - x)(1 - x^2)$

So, the expression $= (1 - x)^6(1 - x^2)^6$

Again, $(1 - x)^6 = 1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$

and, $(1 - x^2)^6 = 1 - 6x^2 + 15x^4 - 20x^6 + \dots$

x^7 arises from $(-6x)(-20x^6) + (-20x^3)(15x^4) + (-6x^5)(-6x^2)$

$= 120 - 300 + 36$

Therefore, the coefficient $= -144$

46. C. T_7

Explanation: Here, $(2 + 3x)^9 = 2^9 \left(1 + \frac{3x}{2}\right)^9$

and for $x = \frac{3}{2}$, $\frac{3x}{2} = \frac{9}{4}$

$\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r} \cdot \frac{9}{4} = \frac{10-r}{r} \cdot \frac{9}{4}$

For $T_{r+1} \geq T_r$, $9(10 - r) \geq 4r$

or, $13r \leq 90$

or, $r \leq 6.92$

So, the terms increase up to $r = 6$ and decrease after T_7

Therefore, the greatest term is T_7

47. C. $ab = 1$

Explanation: General term of the 1st expression: $\binom{11}{r}(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r =$

$\binom{11}{r}a^{11-r}b^{-r}x^{22-3r}$

For x^7, $22 - 3r = 7$ i.e. $r = 5$

So, the coefficient = $\binom{11}{5}a^6b^{-5}$

General term of the 2nd expression:

$\binom{11}{k}(ax)^{11-k}\left(-\frac{1}{bx^2}\right)^k = \binom{11}{k}a^{11-k}(-1)^k b^{-k}x^{11-3k}$

For x^{-7}, $11 - 3k = -7$ i.e. $k = 6$

So, the coefficient = $\binom{11}{6}a^5b^{-6}$

Given, $\binom{11}{5}a^6b^{-5} = \binom{11}{6}a^5b^{-6}$

Since $\binom{11}{5} = \binom{11}{6}$, $a^6b^{-5} = a^5b^{-6}$

Therefore, $ab = 1$

48. D. All of these

Explanation: $9^n - 8n - 1 = (1 + 8)^n - 8n - 1$

Binomial expansion: $(1 + 8)^n = 1 + 8n +$

$\binom{n}{2}8^2 + \binom{n}{3}8^3 + \ldots$

So, the expression = $64\{\binom{n}{2} + 8\binom{n}{3} + \ldots\}$

The expression is a multiple of 64

Therefore, it is divisible by 16, 32 and 64 all

49. D. $\frac{8}{15}$

Explanation: $2^{403} = 2^3 \cdot (2^4)^{100} = 8(15 + 1)^{100}$

By binomial expansion, $(15 + 1)^{100} = 15k + 1$ (k an integer)

So, $2^{403} = 8(15k + 1) = 15(8k) + 8$

Here, $\frac{2^{403}}{15} = 8k + \frac{8}{15}$

Therefore, the fractional part = $\frac{8}{15}$

50. A. $\frac{231}{16}$

Explanation: Here, $n = 12$ is even, so the middle term is the $\left(\frac{12}{2} + 1\right) = 7$th term

So, $T_7 = \binom{12}{6}x^6\left(-\frac{1}{2x}\right)^6$

$= 924 \times x^6 \times \frac{1}{64x^6}$

Therefore, the middle term = $\frac{231}{16}$