Engineering Daily 12 [Home Practice] EV

Binomial Expansion + Vector

Total Marks: 50 **Negative Marks:** 0.25 **Time:** 1 Hour

MCQ 50 marks

1. If $\vec{a}$ and $\vec{b}$ are two unit vectors and the angle between them is $60°$, what is the value of $|3\vec{a} - 2\vec{b}|^2$?

(A) 5 (B) 7 (C) 13 (D) 19

2. What is the orthogonal projection of the vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on the line of intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 3$?

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) 1

3. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$, what is the value of $(\vec{a} \times \vec{b}) \times \vec{c}$?

(A) $11\hat{i} - 13\hat{j} + 10\hat{k}$ (B) $-11\hat{i} + 13\hat{j} - 10\hat{k}$
(C) $10\hat{i} - 13\hat{j} + 11\hat{k}$ (D) $-10\hat{i} + 13\hat{j} - 11\hat{k}$

4. The line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{4}$ meets the plane $x + y + z = 11$ at a point. What is the distance of the point $(1, 1, 2)$ from that point?

(A) $\sqrt{19}$ (B) $\sqrt{21}$ (C) 5 (D) $\sqrt{29}$

5. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. The vector $\vec{v}$ lies in the plane of $\vec{a}$ and $\vec{b}$ and is perpendicular to $\vec{a}$. If the orthogonal projection of $\vec{v}$ on $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$ is $\frac{2}{\sqrt{6}}$, what is the value of $|\vec{v}|^2$?

(A) 4 (B) 6 (C) 8 (D) 16

6. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 6$. What is the value of $|\vec{a} \times \vec{b}|$?

(A) $3\sqrt{3}$ (B) $6\sqrt{3}$ (C) 12 (D) 18

7. If the circumcentre of a triangle is $2\hat{i} - \hat{j} + \hat{k}$ and its centroid is $4\hat{i} + \hat{j} - 2\hat{k}$, which is the position vector of its orthocentre?

(A) $8\hat{i} + 5\hat{j} - 8\hat{k}$ (B) $6\hat{i} + 3\hat{j} - 5\hat{k}$
(C) $10\hat{i} + 3\hat{j} - 7\hat{k}$ (D) $8\hat{i} - 5\hat{j} + 8\hat{k}$

8. A unit vector $\hat{u}$ makes an angle $\frac{\pi}{3}$ with the positive x-axis and $\frac{\pi}{4}$ with the positive y-axis. If it makes an acute angle θ with the positive z-axis, what is the value of θ?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

9. What is the shortest distance between the y-axis and the straight line $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$?

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

10. If the vectors $\vec{a} = \hat{i} + \lambda\hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$ are coplanar, what is the value of λ?

(A) $\frac{1}{9}$ (B) $-\frac{1}{9}$ (C) 9 (D) -9

11. Which is the unit vector that is coplanar with $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and perpendicular to $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$?

(A) $\pm\frac{13\hat{i} - 4\hat{j} - 5\hat{k}}{\sqrt{210}}$ (B) $\pm\frac{13\hat{i} + 4\hat{j} - 5\hat{k}}{\sqrt{210}}$
(C) $\pm\frac{5\hat{i} - 4\hat{j} - 13\hat{k}}{\sqrt{210}}$ (D) $\pm\frac{5\hat{i} + 4\hat{j} - 13\hat{k}}{\sqrt{210}}$

12. Let $\vec{a}$ and $\vec{b}$ be two vectors with $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{c} = \vec{a} \times \vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$, what is the maximum value of $|\vec{a} \times \vec{c}|^2$?

(A) 9 (B) 18 (C) 27 (D) 36

13. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, what is the value of k?

(A) $\frac{9}{2}$ (B) $\frac{7}{2}$ (C) $\frac{5}{2}$ (D) $\frac{3}{2}$

14. Three vectors are $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$. If a vector $\vec{r}$ satisfies $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \cdot \vec{c} = 0$, what is the value of $|\vec{r}|^2$?

(A) $\frac{22}{9}$ (B) $\frac{22}{3}$ (C) $\frac{44}{9}$ (D) $\frac{66}{3}$

15. What is the image of the point $(3, -2, 1)$ with respect to the plane $3x - y + 4z = 2$?

(A) $(0, 1, 3)$ (B) $(0, -1, -3)$
(C) $(1, 0, -3)$ (D) $(-3, 1, 0)$

16. The value of $(\vec{a} + 2\vec{b}) \cdot \{(\vec{b} + 2\vec{c}) \times (\vec{c} + 2\vec{a})\}$ is equal to which of the following?

(A) $3\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$ (B) $6\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$
(C) $9\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$ (D) $8\{\vec{a} \cdot (\vec{b} \times \vec{c})\}$

17. What is the distance of the point P $(3, 8, 2)$ from the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$, measured parallel to the plane $3x + 2y - 2z = 5$?

(A) 5 (B) 7 (C) 9 (D) 11

18. What are the coordinates of the foot of the perpendicular drawn from the point A$(1, 0, 3)$ to the straight line through B$(4, 7, 1)$ and C$(3, 5, 3)$?

(A) $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ (B) $\left(\frac{2}{3}, \frac{4}{3}, \frac{8}{3}\right)$
(C) $(5, 7, 17)$ (D) $\left(\frac{4}{3}, \frac{5}{3}, \frac{11}{3}\right)$

19. Three adjacent edges of a parallelepiped are $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$. If its volume is 158 cubic units, what is the value of the positive integer n?

Ⓐ 6 Ⓑ 7 Ⓒ 8 Ⓓ 9

20. For three vectors $\vec{a}, \vec{b}, \vec{c}$, if $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 4$, what is the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$?

Ⓐ $\frac{29}{2}$ Ⓑ $-\frac{29}{2}$ Ⓒ 29 Ⓓ -29

21. What is the square of the area of the triangle whose adjacent sides are $\vec{u} = 3\vec{a} + 2\vec{b}$ and $\vec{v} = \vec{a} - 2\vec{b}$, where $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$?

Ⓐ 140 Ⓑ 280 Ⓒ 560 Ⓓ 1120

22. What is the angle between the planes $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5$?

Ⓐ 30° Ⓑ 45° Ⓒ 60° Ⓓ 90°

23. A force $\vec{F} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ displaces a body from the point $(1, -2, 3)$ to the point $(3, 1, -1)$. What is the work done?

Ⓐ −10 J Ⓑ 10 J Ⓒ −12 J Ⓓ 12 J

24. Which is the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = 3$ and $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 2$ and through the point $(1, 1, 1)$?

Ⓐ $3x + 3y = 6$ Ⓑ $5x + 4y - z = 8$
Ⓒ $4x + 5y - z = 8$ Ⓓ $x - y + z = 1$

25. What is the value of the term independent of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^9$?

Ⓐ 5376 Ⓑ −5376 Ⓒ 2688 Ⓓ −2688

26. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{\sqrt{2}}|\vec{a}||\vec{b}|\vec{c}$. If $\vec{b}$ and $\vec{c}$ are not parallel, what is the angle between $\vec{a}$ and $\vec{b}$?

Ⓐ 45° Ⓑ 90° Ⓒ 135° Ⓓ 150°

27. If the ratio of the coefficients of the 4th and 5th terms in the expansion of $(1 + x)^n$ is 2 : 3, what is the value of n?

Ⓐ 7 Ⓑ 8 Ⓒ 9 Ⓓ 10

28. What is the sum of the coefficients of the even-power terms of x in the expansion of $(1 + x + x^2 + x^3)^5$?

Ⓐ 512 Ⓑ 1024 Ⓒ 256 Ⓓ 0

29. What is the coefficient of x^5 in the expansion of $(1 - 2x)^{-3}$? (where $|x| < \frac{1}{2}$)

Ⓐ 336 Ⓑ 672 Ⓒ −672 Ⓓ 1344

30. What is the coefficient of $\frac{1}{x}$ in the expansion of $(1 + x)^n \left(1 + \frac{1}{x}\right)^n$?

Ⓐ $\binom{2n}{n}$ Ⓑ $\binom{2n}{n-1}$
Ⓒ $\binom{2n}{n+1}$ Ⓓ Both B and C

31. What is the coefficient of x^4 in the expansion of $(1 - 2x + 3x^2)(1 + x)^{10}$?

Ⓐ 90 Ⓑ 105 Ⓒ 120 Ⓓ 210

32. How many irrational terms are there in the expansion of $\left(5^{\frac{1}{8}} + 2^{\frac{1}{6}}\right)^{100}$?

Ⓐ 4 Ⓑ 97 Ⓒ 98 Ⓓ 101

33. Which of the following is the value of $\binom{15}{5} + \binom{15}{6} + \binom{16}{7} + \binom{17}{8}$?

Ⓐ $\binom{18}{7}$ Ⓑ $\binom{18}{8}$ Ⓒ $\binom{17}{7}$ Ⓓ $\binom{19}{8}$

34. In the expansion of $\left(2^{\frac{1}{4}} + \frac{1}{3^{\frac{1}{4}}}\right)^n$, the ratio of the 5th term from the beginning to the 5th term from the end is $\sqrt{6} : 1$. What is the value of n?

Ⓐ 8 Ⓑ 10 Ⓒ 12 Ⓓ 14

35. What is the remainder when 3^{2024} is divided by 13?

Ⓐ 1 Ⓑ 3 Ⓒ 9 Ⓓ 12

36. What is the sum of the coefficients of the odd-power terms of x in the expansion of $(1 + 2x)^{10}$?

Ⓐ $\frac{3^{10}+1}{2}$ Ⓑ $\frac{3^{10}-1}{2}$
Ⓒ $3^{10} - 1$ Ⓓ 3^9

37. What is the coefficient of x^3 in the expansion of $(1 + x + x^2)^5$?

Ⓐ 10 Ⓑ 20 Ⓒ 30 Ⓓ 40

38. What is the coefficient of x^{10} in the expansion of $(1 + x^2)^4(1 + x^3)^3$?

Ⓐ 12 Ⓑ 18 Ⓒ 24 Ⓓ 36

39. What is the value of $\sum_{r=0}^{n} \frac{1}{r+1}\binom{n}{r}$?

Ⓐ $\frac{2^n}{n+1}$ Ⓑ $\frac{2^n-1}{n+1}$
Ⓒ $\frac{2^{n+1}}{n+1}$ Ⓓ $\frac{2^{n+1}-1}{n+1}$

40. If the coefficients of x^r and x^{r+1} in the expansion of $(3 + 2x)^{14}$ are equal, what is the value of r?

Ⓐ 4 Ⓑ 5 Ⓒ 6 Ⓓ 7

41. How many integer terms are there in the expansion of $\left(\sqrt[3]{5}+\sqrt[4]{3}\right)^{120}$?

Ⓐ 10 Ⓑ 11 Ⓒ 12 Ⓓ 30

42. Which of the following is the value of $\sum_{k=0}^{10}\binom{20}{k}$?

Ⓐ 2^{19} Ⓑ 2^{20}

Ⓒ $2^{19}+\frac{1}{2}\binom{20}{10}$ Ⓓ $2^{20}+\frac{1}{2}\binom{20}{10}$

43. Which is the 4th term from the end in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^{10}$?

Ⓐ $-960x$ Ⓑ $-\frac{1920}{x}$

Ⓒ $\frac{1920}{x}$ Ⓓ $-\frac{3840}{x^2}$

44. What is the value of $C_0+3C_1+5C_2+\cdots+(2n+1)C_n$? (where $C_r=\binom{n}{r}$)

Ⓐ $(n+1)2^{n-1}$ Ⓑ $(2n+1)2^n$

Ⓒ $(n+1)2^n$ Ⓓ $n2^{n+1}$

45. What is the coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$?

Ⓐ −144 Ⓑ 144 Ⓒ −120 Ⓓ 120

46. In the expansion of $(2+3x)^9$, which is the numerically greatest term when $x=\frac{3}{2}$?

Ⓐ T_5 Ⓑ T_6 Ⓒ T_7 Ⓓ T_8

47. If the coefficient of x^7 in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$ equals the coefficient of x^{-7} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$, what is the relation between a and b?

Ⓐ $a-b=1$ Ⓑ $a+b=1$ Ⓒ $ab=1$ Ⓓ $\frac{a}{b}=1$

48. By which of the following numbers is 9^n-8n-1 always divisible? (where $n\in\mathbb{N}$)

Ⓐ 16 Ⓑ 32

Ⓒ 64 Ⓓ All of these

49. What is the fractional part of $\frac{2^{403}}{15}$?

Ⓐ $\frac{1}{15}$ Ⓑ $\frac{2}{15}$ Ⓒ $\frac{7}{15}$ Ⓓ $\frac{8}{15}$

50. What is the value of the middle term in the expansion of $\left(x-\frac{1}{2x}\right)^{12}$?

Ⓐ $\frac{231}{16}$ Ⓑ $-\frac{231}{16}$

Ⓒ $\frac{462}{16}$ Ⓓ $-\frac{462}{16}$