MCQ 50 marks

1. **A.** $-2.253 \times 10^{32}\,\text{J}$

Explanation: Formula: $U = -\frac{3GM^2}{5R}$

Here, $GM^2 = 6.673 \times 10^{-11} \times (6 \times 10^{24})^2 = 2.4023 \times 10^{39}$

So, $3GM^2 = 7.2068 \times 10^{39}$

Again, $5R = 3.2 \times 10^7$

Therefore, $U = -\frac{7.2068 \times 10^{39}}{3.2 \times 10^7} \approx -2.253 \times 10^{32}\,\text{J}$

2. **B.** $347.5\,\text{N}$

Explanation: We know, $g_h = g\left(\frac{R}{R+h}\right)^2$

Here, $g_h = 9.8 \times \left(\frac{6400}{7600}\right)^2 \approx 9.8 \times 0.7092$

So, $g_h \approx 6.95\,\text{ms}^{-2}$

Therefore, $W = mg_h = 50 \times 6.95 \approx 347.5\,\text{N}$

3. **D.** $36\,\text{J}$

Explanation: Before impact, $KE_{\text{before}} = \frac{1}{2}(0.5)(15)^2 = 56.25\,\text{J}$

After impact, $KE_{\text{after}} = \frac{1}{2}(0.5)(9)^2 = 20.25\,\text{J}$

Therefore, the loss $= 56.25 - 20.25 = 36\,\text{J}$

4. **D.** $11.55\,\text{ms}^{-1}$

Explanation: By the work-energy theorem, $Fd = \frac{1}{2}mv^2$

Here, $25 \times 8 = \frac{1}{2}(3)v^2$

So, $200 = 1.5v^2 \implies v^2 = 133.33$

Therefore, $v \approx 11.55\,\text{ms}^{-1}$

5. **C.** $5090\,\text{s}$

Explanation: We know, $T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

Here, $R + h = 6410\,\text{km}$, so $(R+h)^3 \approx 2.634 \times 10^{20}\,\text{m}^3$

Again, $gR^2 \approx 4.014 \times 10^{14}$

So, $T = 2\pi\sqrt{\frac{2.634 \times 10^{20}}{4.014 \times 10^{14}}} = 2\pi\sqrt{6.563 \times 10^5}$

Therefore, $T \approx 2\pi \times 810.1 \approx 5090\,\text{s} \approx 84.8$ minutes

6. **B.** $-3.469 \times 10^{-9}\,\text{J}$

Explanation: We know, $U = -\frac{G}{r}(m_1m_2 + m_1m_3 + m_2m_3)$

Here, $U = -\frac{6.673 \times 10^{-11}}{0.5} \times (6 + 8 + 12)$

So, $U = -1.3346 \times 10^{-10} \times 26$

Therefore, $U \approx -3.469 \times 10^{-9}\,\text{J}$

7. **A.** $2.11\,\text{HP}$

Explanation: Depth of the well, $h = \frac{125}{2} = 62.5\,\text{m}$.

Effective power $= \frac{mgh}{t} = \frac{100 \times 9.8 \times 62.5}{60} \approx 1020.83\,\text{W}$

Actual power of the engine (at 65% efficiency) $= \frac{1020.83}{0.65} \approx 1570.51\,\text{W}$

Therefore, in HP, $P = \frac{1570.51}{746} \approx 2.11\,\text{HP}$

8. **A.** $6.10 \times 10^{-11}\,\text{N}$

Explanation: Here, $F_{AB} = \frac{Gm_Am_B}{AB^2} = \frac{6.673 \times 10^{-11} \times 2 \times 3}{9} \approx 4.449 \times 10^{-11}\,\text{N}$ (directed $A \to B$)

Again, $F_{AC} = \frac{Gm_Am_C}{AC^2} = \frac{6.673 \times 10^{-11} \times 2 \times 5}{16} \approx 4.171 \times 10^{-11}\,\text{N}$ (directed $A \to C$)

Since angle A is a right angle, AB and AC are perpendicular

So, the resultant $F = \sqrt{F_{AB}^2 + F_{AC}^2}$

Therefore, $F \approx 6.10 \times 10^{-11}\,\text{N}$

9. **B.** $20.25\,\text{MJ}$

Explanation: Here, $\Delta PE = mgh = 1500 \times 9.8 \times 500 = 7350000\,\text{J}$

Again, $\Delta KE = \frac{1}{2}(1500)(80)^2 = 4800000\,\text{J}$

Useful energy $= \Delta PE + \Delta KE = 12150000\,\text{J}$

Therefore, input energy $= \frac{12150000}{0.6} = 20250000\,\text{J} \approx 20.25\,\text{MJ}$

10. **A.** $\frac{T}{\sqrt{2}}$

Explanation: We know, $T^2 = \frac{4\pi^2r^3}{GM}$, so $T \propto \frac{1}{\sqrt{G}}$

Doubling G gives, $T = \frac{T_{old}}{\sqrt{2}}$

Therefore, $T \approx 0.707T_{old}$, i.e. the period decreases

11. **A.** $3.756\,\text{ms}^{-1}$

Explanation: Available energy $= 0.9mgh$

We know, $\frac{1}{2}mv^2 = 0.9gh$

Here, $v^2 = 1.8 \times 9.8 \times 0.8 = 14.112$

Therefore, $v \approx 3.756\,\text{ms}^{-1}$

12. **C.** 36%

Explanation: Here, $KE_{\text{before}} \propto h_{\text{drop}} = 5$ and $KE_{\text{after}} \propto h_{\text{rebound}} = 3.2$

Fraction lost $= \frac{h_{\text{drop}} - h_{\text{rebound}}}{h_{\text{drop}}} \times 100\%$

Here, fraction lost $= \frac{1.8}{5} \times 100\%$

Therefore, fraction lost $= 36\%$

13. **D.** $-6.84 \times 10^7\,\text{Jkg}^{-1}$
Explanation: Inside the outer shell ($r < R_1$), the potential is constant, equal to its surface value
Here, $V_1 = -\frac{GM_1}{R_1} \approx -4.171 \times 10^7\,\text{Jkg}^{-1}$
Again, outside the inner shell ($r > R_2$), it behaves like a point mass
Here, $V_2 = -\frac{GM_2}{r} \approx -2.669 \times 10^7\,\text{Jkg}^{-1}$
Therefore, the total potential $= V_1 + V_2 \approx -6.84 \times 10^7\,\text{Jkg}^{-1}$

14. **A.** $10.20\,\text{m}$
Explanation: We know, $\frac{1}{2}kx^2 = mgh$
Here, $\frac{1}{2}(2000)(0.2)^2 = 40\,\text{J}$
Therefore, $h = \frac{40}{0.4 \times 9.8} = \frac{40}{3.92} \approx 10.20\,\text{m}$

15. **D.** $4.65 \times 10^5\,\text{km}$
Explanation: By Kepler's third law (same central body B), $T^2 \propto r^3$
Here, $r_C = r_A \left(\frac{T_C}{T_A}\right)^{2/3} = 3 \times 10^5 \times \left(\frac{135}{70}\right)^{2/3}$
So, $r_C = 3 \times 10^5 \times (1.9286)^{2/3} \approx 3 \times 10^5 \times 1.5494$
Therefore, $r_C \approx 4.65 \times 10^5\,\text{km}$

16. **A.** $300\,\text{J}$
Explanation: At the highest point, the vertical velocity is zero, only the horizontal component remains
Here, $v_x = 40\cos 30^\circ = 34.64\,\text{ms}^{-1}$
Therefore, $KE = \frac{1}{2}(0.5)(34.64)^2 \approx 300\,\text{J}$

17. **B.** $3.43\,\text{ms}^{-1}$
Explanation: Energy released (only the hanging mass does work) $= m_2gd = 4 \times 9.8 \times 1.5 = 58.8\,\text{J}$
We know, $\frac{1}{2}(m_1 + m_2)v^2 = 58.8$
Here, $5v^2 = 58.8$
Therefore, $v \approx 3.43\,\text{ms}^{-1}$

18. **A.** $r = \frac{2R}{3}$
Explanation: The mass enclosed up to r, $M(r) = \int_0^r 4\pi r'^2 \varrho_0 \left(1 - \frac{r'}{R}\right) dr' = 4\pi\varrho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$
Here, $g(r) = \frac{GM(r)}{r^2} = 4\pi G\varrho_0 \left[\frac{r}{3} - \frac{r^2}{4R}\right]$
For a maximum, $\frac{dg}{dr} = 0$
So, $4\pi G\varrho_0 \left[\frac{1}{3} - \frac{r}{2R}\right] = 0$
Therefore, $r = \frac{2R}{3}$

19. **B.** $1 : 3$
Explanation: We know, $h = \frac{KE}{mg}$ (same KE, m), i.e. $h \propto \frac{1}{g}$
Here, $\frac{h_x}{h_y} = \frac{g_y}{g_x}$
Therefore, $\frac{h_x}{h_y} = \frac{1}{3}$, i.e. $h_x : h_y = 1 : 3$

20. **C.** $12.92\,\text{kms}^{-1}$
Explanation: Each star orbits at $r = \frac{D}{2}$ from the center of mass
We know, $\frac{GM^2}{D^2} = \frac{Mv^2}{D/2}$
Here, $v^2 = \frac{GM}{2D} = \frac{6.673 \times 10^{-11} \times 2 \times 10^{30}}{2 \times 4 \times 10^{11}} \approx 1.669 \times 10^8$
Therefore, $v \approx 12.92\,\text{kms}^{-1}$

21. **C.** $h = 3.5R$
Explanation: At the top of the loop, $N + mg = \frac{mv^2}{R}$, with $N = 2mg$
So, $3mg = \frac{mv^2}{R} \implies v^2 = 3gR$
By energy conservation (bottom to top, top is at height 2R), $mgh = mg(2R) + \frac{1}{2}mv^2$
Here, $gh = 2gR + 1.5gR = 3.5gR$
Therefore, $h = 3.5R$

22. **A.** $0.0925\,\text{N}$
Explanation: We know, $\omega = \frac{2\pi}{24 \times 3600} \approx 7.272 \times 10^{-5}\,\text{rads}^{-1}$
Weight loss $= m\omega^2 R\sin^2\theta$ (here, θ = angle from axis $= 30^\circ$)
Here, weight loss $= 10 \times (7.272 \times 10^{-5})^2 \times 7 \times 10^6 \times \sin^2 30^\circ$
So, weight loss $\approx 10 \times 5.288 \times 10^{-9} \times 7 \times 10^6 \times 0.25$
Therefore, weight loss $\approx 0.0925\,\text{N}$

23. **B.** $v < 11.2\,\text{kms}^{-1}$
Explanation: Escape velocity, $v_{esc} = \sqrt{2gR} \approx 11.2\,\text{kms}^{-1}$
If $v \geq v_{esc}$, the path is parabolic/hyperbolic (never returns)
Therefore, if $v < v_{esc}$, the path is a bound ellipse (a circular orbit being a special case) and the object returns

24. **A.** $10.02\,\text{m}$
Explanation: Since all surfaces are frictionless, energy conservation means the block regains the same height ($8\,\text{m}$)
Therefore, distance $= \frac{h}{\sin 53^\circ} = \frac{8}{0.7986} \approx 10.02\,\text{m}$

25. **C.** $6666\,\text{km}$
Explanation: First, check that $v_{esc} = \sqrt{2gR} \approx 11.2\,\text{kms}^{-1} > 8\,\text{kms}^{-1}$, so the rocket does return at a finite height
By energy conservation, $\frac{1}{2}v^2 - gR = -\frac{gR^2}{R+h}$

Here, $\frac{1}{2}(8000)^2 - 9.8 \times 6.4 \times 10^6 = -\frac{4.014 \times 10^{14}}{6.4 \times 10^6 + h}$
Therefore, $h \approx 6.666 \times 10^6\,\text{m} = 6666\,\text{km}$

26. **D.** $3.98\,\text{HP}$
Explanation: Volume $= 3 \times 4 \times 5 = 60\,\text{m}^3$, mass $= 60000\,\text{kg}$
The centre of mass of the water column is at $\frac{h}{2} = 2.5\,\text{m}$
Work, $W = mg\left(\frac{h}{2}\right) = 60000 \times 9.8 \times 2.5 = 1.47 \times 10^6\,\text{J}$
Useful power $= \frac{W}{t} = \frac{1470000}{900} \approx 1633.33\,\text{W}$
Therefore, motor power $= \frac{1633.33}{0.55} \approx 2969.7\,\text{W} \approx 3.98\,\text{HP}$

27. **D.** $257.25\,\text{W}$
Explanation: Vertical speed $= 0.75 \sin 30^\circ = 0.375\,\text{ms}^{-1}$
Time, $t = \frac{6}{0.375} = 16\,\text{s}$
Therefore, $P = \frac{mgh}{t} = \frac{70 \times 9.8 \times 6}{16} = 257.25\,\text{W}$

28. **D.** $5.98 \times 10^{-8}\,\text{Nkg}^{-1}$
Explanation: Since $0.7\,\text{m} < R(2.5\,\text{m})$, the point is inside the sphere
Interior formula: $g(r) = \frac{GMr}{R^3}$
Here, $g(0.7) = \frac{6.673 \times 10^{-11} \times 2 \times 10^4 \times 0.7}{(2.5)^3}$
Therefore, $g(0.7) \approx 5.98 \times 10^{-8}\,\text{Nkg}^{-1}$

29. **B.** $34.29 \times 10^9\,\text{km}$
Explanation: Let the point be at distance x from A
We know, $\frac{G(36M)}{x^2} = \frac{GM}{(D-x)^2} \implies \frac{6}{x} = \frac{1}{D-x}$
So, $6(D - x) = x \implies 6D = 7x$
Therefore, $x = \frac{6D}{7} = \frac{6 \times 40}{7} \times 10^9 \approx 34.29 \times 10^9\,\text{km}$

30. **B.** $383\,\text{ms}^{-1}$
Explanation: The initial orbital radius $r_p = 8000\,\text{km}$ and new apogee radius $r_a = 10000\,\text{km}$
For the initial circular orbit, $v_c = \sqrt{\frac{GM}{r_p}}$ (here $GM = gR^2$)
After the speed increase, the initial point becomes the perigee of the new ellipse, so by vis-viva, $v_p = \sqrt{GM\left(\frac{2}{r_p} - \frac{2}{r_p + r_a}\right)}$
This gives, $v_c \approx 7083.5\,\text{ms}^{-1}$ and $v_p \approx 7466.7\,\text{ms}^{-1}$
Therefore, $\Delta v = v_p - v_c \approx 383\,\text{ms}^{-1}$

31. **D.** $345600\,\text{km}$
Explanation: We know, $\sqrt{\frac{M_E}{M_M}} = \frac{x}{D-x}$
Here, $\sqrt{\frac{5.97 \times 10^{24}}{7.35 \times 10^{22}}} \approx 9.012$
So, $9.012(D - x) = x \implies x = \frac{9.012D}{10.012} \approx 0.9001D$
Therefore, $x \approx 345600\,\text{km}$

32. **A.** $617.6\,\text{kms}^{-1}$
Explanation: We know, $v_{esc} = \sqrt{\frac{2GM}{R}}$
Here, $v_{esc} = \sqrt{\frac{2 \times 6.673 \times 10^{-11} \times 1.989 \times 10^{30}}{6.96 \times 10^8}}$
Therefore, $v_{esc} \approx 617.6\,\text{kms}^{-1}$

33. **C.** $2.042\,\text{kW}$
Explanation: Here, $\dot{m} = \frac{15000}{3600} = 4.167\,\text{kgs}^{-1}$
Useful power $= \dot{m}gh = 4.167 \times 9.8 \times 35 = 1429.17\,\text{W}$
Therefore, input power $= \frac{1429.17}{0.7} \approx 2041.67\,\text{W} = 2.042\,\text{kW}$

34. **A.** $1 : \sqrt{2}$
Explanation: Energy conservation inside a uniform sphere gives, at the centre, $v^2 = \frac{GM}{R}$
Again, escape velocity, $v_{esc}^2 = \frac{2GM}{R}$
Therefore, ratio $\frac{v}{v_{esc}} = \frac{\sqrt{GM/R}}{\sqrt{2GM/R}} = \frac{1}{\sqrt{2}}$

35. **C.** 0
Explanation: By the shell theorem, the field strictly inside a hollow shell ($r < R$) is always zero
Here, $0.2\,\text{m} < 0.5\,\text{m}$, so $g_{0.2} = 0$
Again, $0.8\,\text{m} > 0.5\,\text{m}$ (outside), so $g_{0.8} = \frac{GM}{(0.8)^2} \neq 0$
Therefore, ratio $= 0 : (\text{nonzero}) = 0$

36. **D. Increases by** 36.9%
Explanation: We know, $v_{esc} = \sqrt{\frac{2GM}{R}}$, so $v_{esc} \propto \sqrt{\frac{M}{R}}$
Here, new/old ratio $= \sqrt{\frac{1.5M/0.8R}{M/R}} = \sqrt{\frac{1.5}{0.8}} = \sqrt{1.875} \approx 1.369$
Therefore, the escape velocity increases by about 36.9%

37. **C.** $0.0775\,\text{m}$
Explanation: By energy conservation, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$
Therefore, $x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{(0.3)(16)}{800}} = \sqrt{0.006} \approx 0.0775\,\text{m}$

38. **C.** $2\sqrt{2}\,T_0$

Explanation: Kepler's third law: $T^2 \propto r^3 \Rightarrow T \propto r^{3/2}$
$r_1 = R$, $r_2 = R + h = 2R$
$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{2R}{R}\right)^{3/2} = 2\sqrt{2}$
$T_2 = 2\sqrt{2}\,T_0$

39. C.
Explanation: Inside ($r < R$), only the mass enclosed within radius r matters, $M_{enc} \propto r^3$
So, $g = \frac{GM_{enc}}{r^2} \propto \frac{r^3}{r^2} = r$ (linear and increasing)
Outside ($r > R$), the full mass M acts as a point mass, $g = \frac{GM}{r^2}$ (inverse-square and decreasing)

40. A. $1.41\,ms^{-1}$
Explanation: Useful power $= 7 \times 0.66 \times 746 = 3446.52\,W$
At constant velocity, $P = mgv$
Therefore, $v = \frac{P}{mg} = \frac{3446.52}{250 \times 9.8} \approx 1.41\,ms^{-1}$

41. D. $814.3\,kW$
Explanation: Here, $A = \pi r^2 = 1963.5\,m^2$
$\dot{m} = \varrho A v = 1.2 \times 1963.5 \times 12 = 28274.4\,kgs^{-1}$
Total wind power $= \frac{1}{2}\dot{m}v^2 = \frac{1}{2}(28274.4)(144) \approx 2035757\,W$
Therefore, output $= 0.4 \times 2035757 \approx 814.3\,kW$

42. D. $\frac{1}{2}mg^2(2n-1)$
Explanation: The distance covered in the n-th second, $s_n = \frac{g}{2}(2n-1)$
The potential-energy loss over this distance (equal to the kinetic-energy gain) $= mg \times s_n$
Therefore, the potential-energy loss $= \frac{1}{2}mg^2(2n-1)$

43. C. $53.15\,m$
Explanation: Here, $\Delta KE = \frac{1}{2}mv^2 = 312500\,J$
Friction force, $f = \mu mg = 5880\,N$
Therefore, $d = \frac{\Delta KE}{f} = \frac{312500}{5880} \approx 53.15\,m$

44. B. $0.0786\,HP$
Explanation: Total steps $= \sum_{n=1}^{5}(5n+2) = 5(1+2+3+4+5) + 2 \times 5 = 75 + 10 = 85$
Total height $= 85 \times 0.13 = 11.05\,m$
Work, $W = mgh = 65 \times 9.8 \times 11.05 = 7038.85\,J$
Time, $t = 2$ minutes $= 120\,s$
Power, $P = \frac{W}{t} = 58.657\,W$
Therefore, $P = \frac{58.657}{746} \approx 0.0786\,HP$

45. D. $\frac{1}{8}$
Explanation: We know, $T \propto r^{3/2}$
Therefore, $\frac{T_{new}}{T_{old}} = \left(\frac{1}{4}\right)^{3/2} = \frac{1}{8}$

46. C. 0 **(same point)**
Explanation: Key idea: $E = -\frac{dV}{dr}$
Wherever the field intensity E is zero, $\frac{dV}{dr} = 0$ there too, meaning V is at an extremum (a minimum, since V is more negative on both sides)
Therefore, the two points are always identical, so the separation $= 0$

47. A. $133\,J$
Explanation: Here, $p(1) = 3(1) + 2(1) = 5\,kgms^{-1}$ and $p(3) = 3(9) + 2(3) = 33\,kgms^{-1}$
We know, $KE = \frac{p^2}{2m}$
Here, $KE(1) = \frac{5^2}{2\times4} = 3.125\,J$
And, $KE(3) = \frac{33^2}{2\times4} = 136.125\,J$
By the work-energy theorem, $W = \Delta KE = 136.125 - 3.125$
Therefore, $W = 133\,J$

48. C. $1.229\,MW$
Explanation: Here, $\dot{m} = \frac{200000}{60} = 3333.33\,kgs^{-1}$
Water power $= \dot{m}gh = 3333.33 \times 9.8 \times 45 = 1.47\,MW$
After the turbine, $1.47 \times 0.88 = 1.2936\,MW$
Therefore, after the generator, $1.2936 \times 0.95 \approx 1.229\,MW$

49. A. $856.32\,J$
Explanation: Here, $\Delta KE = \frac{1}{2}(25)(2)^2 = 50\,J$
Work done by gravity, $W_{gravity} = -mgL\sin 15^\circ = -380.44\,J$
Work done by friction, $W_{friction} = -\mu mgL\cos 15^\circ = -425.88\,J$
Therefore, $W_{applied} = \Delta KE - W_{gravity} - W_{friction} = 50 + 380.44 + 425.88 \approx 856.32\,J$

50. C. $20.37\,kJ$
Explanation: Volume, $V = \pi r^2 L = \pi(0.3)^2(7) \approx 1.979\,m^3$
Mass, $m = \varrho V = 600 \times 1.979 \approx 1187.5\,kg$
Rise of the centre of mass, $\Delta h = \left(\frac{L}{2}\right)\sin 30^\circ = 3.5 \times 0.5 = 1.75\,m$
Therefore, work, $W = mg\Delta h = 1187.5 \times 9.8 \times 1.75 \approx 20369\,J \approx 20.37\,kJ$