MCQ 50 marks

1. C. $90°$

Explanation: $P + Q = 28$ and $P - Q = 4$

Adding, $P = 16,\ Q = 12$

Required resultant, $R = 5 \times 4 = 20$

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$\Rightarrow 400 = 256 + 144 + 384\cos\alpha$

$\Rightarrow 384\cos\alpha = 0$

Therefore, $\alpha = 90°$

2. D. $\hat{k}$

Explanation: Here, $\hat{i} \times \hat{k} = -\hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$

So the expression becomes, $(-\hat{j}) \times \hat{i}$

$= -(\hat{j} \times \hat{i})$

$= -(-\hat{k})$

$= \hat{k}$

3. B. $\tan^{-1}(\frac{2}{3})$

Explanation: Here, $A_x = 2,\ A_y = 3$

If θ is the angle with the y-axis,

$\tan\theta = \frac{A_x}{A_y} = \frac{2}{3}$

Therefore, $\theta = \tan^{-1}\left(\frac{2}{3}\right)$

4. C. $\frac{1}{\sqrt{6}}(2\hat{i} + \hat{j} - \hat{k})$

Explanation: $|\vec{A}| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$

The parallel unit vector, $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$

Therefore, $\hat{a} = \frac{1}{\sqrt{6}}(2\hat{i} + \hat{j} - \hat{k})$

5. B. $1.0\ \text{ms}^{-2}$

Explanation: Relative velocity of C and A, $v_{CA} = 15 + 10 = 25\ \text{ms}^{-1}$

So time for C to reach A, $t = \frac{1000}{25} = 40\ \text{s}$

Initial relative velocity of B and A, $u_{BA} = 15 - 10 = 5\ \text{ms}^{-1}$

Now, $1000 = 5(40) + \frac{1}{2}a(40)^2$

$\Rightarrow 1000 = 200 + 800a$

$\Rightarrow a = 1\ \text{ms}^{-2}$

6. A. $\cos\theta = \frac{n^2 + 2}{2(1 - n^2)}$

Explanation: $P + Q = n(P - Q) \Rightarrow P = \frac{n+1}{n-1}Q$

$R = \frac{P + Q}{2}$ and $\cos\theta = \frac{R^2 - P^2 - Q^2}{2PQ}$

By algebraic simplification, $\cos\theta = \frac{n^2 + 2}{2(1 - n^2)}$

7. A. 36.10

Explanation: $\vec{v} = \frac{d\vec{s}}{dt} = 9t^2\hat{i} - 8t\hat{j} + 2\hat{k}$; at $t = 2$, $\vec{v} = 36\hat{i} - 16\hat{j} + 2\hat{k}$

$\vec{a} = \frac{d\vec{v}}{dt} = 18t\hat{i} - 8\hat{j}$; at $t = 2$, $\vec{a} = 36\hat{i} - 8\hat{j}$

$\vec{a} \cdot \vec{v} = 1296 + 128 = 1424$; $|\vec{v}| = \sqrt{1556} \approx 39.446$

Projection $= \frac{1424}{39.446} \approx 36.10$

8. C. 100 s

Explanation: Relative velocity, $v_{rel} = 10 - 9 = 1\ \text{ms}^{-1}$

Relative distance = 100 m

Therefore, $t = \frac{100}{1} = 100\ \text{s}$

9. B. 9.5

Explanation: Let, $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} \cdot \vec{c} = 4 \Rightarrow x - y = 4$ ... (1)

$\vec{a} \times \vec{c} = -z\hat{i} - z\hat{j} + (x + y)\hat{k} = -\vec{b} \Rightarrow z = 1,\ x + y = -1$ ... (2)

Solving (1) and (2), $x = 1.5,\ y = -2.5$

Therefore, $|\vec{c}|^2 = 1.5^2 + (-2.5)^2 + 1^2 = 9.5$

10. B. 1000 m

Explanation: Since each turn is 60°, the path forms a regular hexagon.

$\vec{S} = 500\angle 0° + 500\angle 60° + 500\angle 120°$

x-component $= 500(1 + 0.5 - 0.5) = 500$; y-component $= 500(0 + 0.866 + 0.866) = 866$

Therefore, $|\vec{S}| = \sqrt{500^2 + 866^2} = 1000$ m

11. A. 4 : 1

Explanation: $\frac{P+Q}{P-Q} = \frac{5}{3}$

$\Rightarrow 3P + 3Q = 5P - 5Q$

$\Rightarrow 2P = 8Q$

Therefore, $\frac{P}{Q} = 4$, i.e. the ratio is 4 : 1

12. C. $100\ \text{ms}^{-1}$

Explanation: Time for stone 1 to reach maximum height, $t = \frac{u}{g} = \frac{50}{10} = 5\ \text{s}$

At that moment, $v_1 = 0$

Velocity of stone 2 (downward) at this time, $v_2 = 50 + 10 \times 5 = 100\ \text{ms}^{-1}$

Relative velocity $= |v_1 - v_2| = |0 - 100| = 100\ \text{ms}^{-1}$

13. C. 90°

Explanation: Sum of squares of direction cosines = 1

$\cos^2 30° + \cos^2 60° + \cos^2 \gamma = 1$

$\Rightarrow \frac{3}{4} + \frac{1}{4} + \cos^2 \gamma = 1$

$\Rightarrow \cos^2 \gamma = 0 \Rightarrow \gamma = 90°$

14. A. 0° **(vertically)**

Explanation: Since the wind and rider move at the same speed in the same direction, the wind's effect relative to the rider is zero.

So the rider experiences only the rain's true (vertically downward) velocity.

Therefore, the rain hits at 0°, i.e. vertically.

15. A. $-4\hat{i} + 4\hat{j} + 2\hat{k}$

Explanation: $\frac{\partial \phi}{\partial x} = 4xyz + 2yz + 2z^3$; at $(1, -1, 1)$, $= -4 - 2 + 2 = -4$

$\frac{\partial \phi}{\partial y} = 2x^2z + 2xz$; at $(1, -1, 1)$, $= 2 + 2 = 4$

$\frac{\partial \phi}{\partial z} = 2x^2y + 2xy + 6xz^2$; at $(1, -1, 1)$, $= -2 - 2 + 6 = 2$

Therefore, $\nabla\phi = -4\hat{i} + 4\hat{j} + 2\hat{k}$

16. B. 126.87°

Explanation: Since the stream velocity exceeds the swimmer's speed, the condition to minimize drift is $\cos \alpha = -\frac{v}{u}$

Here, $\cos \alpha = -\frac{3}{5} = -0.6$

Therefore, $\alpha = \cos^{-1}(-0.6) \approx 126.87°$

17. B. $3\ \text{kmh}^{-1}$

Explanation: The condition for the shortest crossing path is $\cos \alpha = -\frac{u}{v}$

$\cos(138.59°) = -\frac{u}{4}$

$\Rightarrow -0.75 = -\frac{u}{4}$

Therefore, $u = 3\ \text{kmh}^{-1}$

18. A. $10\ \text{ms}^{-1}$

Explanation: In the first case, hitting vertically means $v_x = 5\ \text{ms}^{-1}$

In the second case, $\tan 30° = \frac{5}{v_y} \Rightarrow v_y = 5\sqrt{3}\ \text{ms}^{-1}$

Therefore, $v_r = \sqrt{v_x^2 + v_y^2} = \sqrt{25 + 75} = 10\ \text{ms}^{-1}$

19. A. 79.02°

Explanation: $\vec{A} \cdot \vec{B} = 12 - 6 - 2 = 4$

$|\vec{A}| = 3,\ |\vec{B}| = 7$

$\cos \theta = \frac{4}{21} \approx 0.1905$

Therefore, $\theta \approx 79.02°$

20. C. 5.98 s

Explanation: Time to cross the river, $t = \frac{d}{v \sin \alpha}$

$t = \frac{36}{10 \times \sin 37°} = \frac{36}{6.018}$

Therefore, $t \approx 5.98\ \text{s}$

21. A. 24 **cubic unit**

Explanation: Volume, $V = |\vec{A} \cdot (\vec{B} \times \vec{C})|$

$\vec{B} \times \vec{C} = 12\hat{i}$

Therefore, $\vec{A} \cdot (\vec{B} \times \vec{C}) = 2 \times 12 = 24$ cubic units

22. A. -4

Explanation: $\frac{\partial v_x}{\partial x} = 4y^3z$; at $(1, -1, 1)$, $= -4$

$\frac{\partial v_y}{\partial y} = -3x^2z$; at $(1, -1, 1)$, $= -3$

$\frac{\partial v_z}{\partial z} = -3xy$; at $(1, -1, 1)$, $= 3$

Sum $= -4 - 3 + 3 = -4$

23. B. $15\hat{i} + 20\hat{j}$

Explanation: $|\vec{Q}| = \sqrt{49 + 576} = 25$

Unit vector in the direction of $\vec{P} = \frac{3\hat{i} + 4\hat{j}}{5}$

Required vector $= 25 \times \frac{3\hat{i} + 4\hat{j}}{5} = 15\hat{i} + 20\hat{j}$

24. B. $20\ \text{m min}^{-1}$

Explanation: Stream velocity, $u = \frac{120}{10} = 12\ \text{m min}^{-1}$

At minimum time, $d = 10v$

At the shortest path, $\frac{10v}{\sqrt{v^2 - 144}} = 12.5$

Squaring, $100v^2 = 156.25(v^2 - 144)$

$\Rightarrow v^2 = 400 \Rightarrow v = 20\ \text{m min}^{-1}$

25. B. $54.74°$

Explanation: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$; $\alpha = \beta = \gamma$

$3\cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Therefore, $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 54.74°$

26. C. $60°$

Explanation: $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$; here $P = Q = 1,\ R = \sqrt{3}$

$3 = 1 + 1 + 2\cos\alpha \Rightarrow \cos\alpha = 0.5$

Therefore, $\alpha = 60°$

27. A. $-4\hat{i} + 8\hat{j} - 6\hat{k}$

Explanation: Angular momentum, $\vec{L} = \vec{r} \times \vec{p}$

$\vec{p} = 2(2\hat{i} + \hat{j}) = 4\hat{i} + 2\hat{j}$

$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 4 & 2 & 0 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 6\hat{k}$$

Since angular momentum is a vector quantity, only the component-form answer $(-4\hat{i} + 8\hat{j} - 6\hat{k})$ is correct.

28. A. 5

Explanation: $\nabla\phi = -3x^2y\hat{i} - x^3\hat{j} + 2\hat{k}$; at $(1, -1, 1)$, $= 3\hat{i} - \hat{j} + 2\hat{k}$

$\vec{A}$ at this point $= 2\hat{i} + 3\hat{j} + \hat{k}$

Therefore, $\vec{A} \cdot \nabla\phi = (2)(3) + (3)(-1) + (1)(2) = 6 - 3 + 2 = 5$

29. C. $(5xz^2 + 3x^3y)\hat{i} + (4xy^3 - 5yz^2)\hat{j} - (9x^2yz + 12xy^2z)\hat{k}$

Explanation: $\hat{i}$ component: $\partial_y(5xyz^2) - \partial_z(-3x^3yz) = 5xz^2 + 3x^3y$

$\hat{j}$ component: $\partial_z(4xy^3z) - \partial_x(5xyz^2) = 4xy^3 - 5yz^2$

$\hat{k}$ component: $\partial_x(-3x^3yz) - \partial_y(4xy^3z) = -(9x^2yz + 12xy^2z)$

30. B. $1.588\ \text{h}$

Explanation: Angle for the straight crossing, $\cos\alpha = -\frac{3}{4} \Rightarrow \alpha \approx 138.59°$

Distance covered in 1 hour, $y_1 = 4\sin(138.59°) \approx 2.645\ \text{km}$

Remaining distance, $y_2 = 2.355\ \text{km}$; time, $t_2 = \frac{2.355}{4} = 0.588\ \text{h}$

Total time $= 1 + 0.588 = 1.588\ \text{h}$

31. A. $1\ \text{s}$

Explanation: $v_{C/B} = v_{C/A} + v_{A/B} = 25 - 5 = 20\ \text{ms}^{-1}$

Required distance $= 20\ \text{m}$

Therefore, $t = \frac{20}{20} = 1\ \text{s}$

32. B. 6

Explanation: For vectors to be parallel, the ratios of corresponding components are equal.

$\frac{3}{x} = \frac{-1}{-2} = \frac{2}{4} = 0.5$

Therefore, $x = 6$

33. A. 40 **unit**

Explanation: Resultant force, $\vec{F} = 7\hat{i} + 2\hat{j} - 4\hat{k}$

Displacement vector, $\vec{s} = 4\hat{i} + 2\hat{j} - 2\hat{k}$

Therefore, $W = \vec{F} \cdot \vec{s} = 28 + 4 + 8 = 40$ units

34. B. $3\sqrt{2}$ **sq. unit**

Explanation: $\overrightarrow{AB} = -\hat{i} + 3\hat{j} - \hat{k}$, $\overrightarrow{AC} = \hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{AB} \times \overrightarrow{AC} = 6\hat{i} - 6\hat{k}$; $|\overrightarrow{AB} \times \overrightarrow{AC}| = 6\sqrt{2}$

Therefore, area $= \frac{1}{2}(6\sqrt{2}) = 3\sqrt{2}$ sq. units

35. A. $a = 4, b = -3, c = 5$

Explanation: Comparing coefficients under $\nabla \times \vec{A} = 0$:

$\hat{i}$: $c - 5 = 0 \Rightarrow c = 5$

$\hat{j}$: $a - 4 = 0 \Rightarrow a = 4$

$\hat{k}$: $b - (-3) = 0 \Rightarrow b = -3$

36. C. $\sqrt{925}\,ms^{-1}$

Explanation: True velocity of rain plus wind $= -10\hat{i} - 5\hat{k}$

Relative velocity w.r.t. the car $= (-10\hat{i} - 5\hat{k}) - 20\hat{i} = -30\hat{i} - 5\hat{k}$

Therefore, $|\vec{v}_{rc}| = \sqrt{(-30)^2 + (-5)^2} = \sqrt{900 + 25} = \sqrt{925}\,ms^{-1}$

37. B. 7.51 s

Explanation: By the sine rule, $\sin\theta = \frac{10\sin 60°}{12} \approx 0.7217 \Rightarrow \theta \approx 46.19°$

$\angle AOB = 180° - 60° - 46.19° = 73.81°$

Applying the sine rule again, $12t = \frac{100 \times 0.866}{0.9603} \approx 90.18$

Therefore, $t \approx 7.51$ s

38. B. $-\frac{\vec{r}}{r^3}$

Explanation: $\phi = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$

$\nabla\phi = \sum \left(-\frac{1}{2}\right)(x^2 + y^2 + z^2)^{-\frac{3}{2}}(2x)$

Therefore, $\nabla\phi = -\frac{\vec{r}}{r^3}$

39. A. $\nabla \times \vec{v} = 2\vec{\omega}$

Explanation: From a vector identity, $\nabla \times (\vec{\omega} \times \vec{r}) = 2\vec{\omega}$

Since $\vec{v} = \vec{\omega} \times \vec{r}$, $\nabla \times \vec{v} = 2\vec{\omega}$

40. B. $5\sqrt{2}\,ms^{-1}$

Explanation: True velocity of rain with wind $= -2\hat{i} - 5\hat{k}$

Relative velocity w.r.t. the car $= (-2\hat{i} - 5\hat{k}) - 3\hat{i} = -5\hat{i} - 5\hat{k}$

Therefore, $|\vec{v}_{rel}| = \sqrt{25 + 25} = 5\sqrt{2}\,ms^{-1}$

41. A. $\pm\frac{\sqrt{14}}{\sqrt{122}}(-8\hat{i} - 3\hat{j} + 7\hat{k})$

Explanation: Normal vector to the plane, $\vec{n} = \vec{B} \times \vec{C} = -8\hat{i} - 3\hat{j} + 7\hat{k}$; $|\vec{n}| = \sqrt{122}$

$|\vec{A}| = \sqrt{14}$

Therefore, the required vector $= \pm|\vec{A}|\hat{n} = \pm\frac{\sqrt{14}}{\sqrt{122}}(-8\hat{i} - 3\hat{j} + 7\hat{k})$

42. A. $4\sqrt{21}$

Explanation: Coplanarity condition, $2x - y + z = 0$ ... (1)

$\vec{c} \perp \vec{a}$: $2x + 3y - z = 0$ ... (2)

$\vec{c} \cdot \vec{b} = 24$: $y + z = 24$ ... (3)

From (1) and (2), $y = -2x,\ z = 2y$;

substituting into (3), $y = 8,\ z = 16,\ x = -4$

Therefore, $|\vec{c}| = \sqrt{16 + 64 + 256} = \sqrt{336} = 4\sqrt{21}$

43. A. $\sqrt{33}$

Explanation: Median vector, $\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = 4\hat{i} - \hat{j} + 4\hat{k}$

Therefore, $|\overrightarrow{AD}| = \sqrt{16 + 1 + 16} = \sqrt{33}$

44. A. -1.5

Explanation: $(\hat{a} + \hat{b} + \hat{c})^2 = 1 + 1 + 1 + 2(\hat{a} \cdot \hat{b} + \hat{b} \cdot \hat{c} + \hat{c} \cdot \hat{a}) = 0$

$3 + 2X = 0$

Therefore, $X = -\frac{3}{2} = -1.5$

45. B. 20, 14

Explanation: Let the magnitudes of the two vectors be P and Q.

Maximum resultant, $P + Q = 34$

Minimum resultant, $P - Q = 6$

Adding the two equations, $2P = 40 \Rightarrow P = 20$

Subtracting, $2Q = 28 \Rightarrow Q = 14$

Therefore, the magnitudes of the vectors are 20 and 14 units.

46. B. 5 : 2

Explanation: $\frac{P + Q}{P - Q} = \frac{7}{3}$

$\Rightarrow 3P + 3Q = 7P - 7Q$

$\Rightarrow 10Q = 4P$

Therefore, $\frac{P}{Q} = \frac{5}{2}$, i.e. the ratio is 5 : 2

47. B. 50 **unit**

Explanation: Resultant force, $\vec{F} = \vec{F}_1 + \vec{F}_2 = 7\hat{i} + 3\hat{j} - 5\hat{k}$

Displacement vector, $\vec{s} = (6 - 2)\hat{i} + (5 - 1)\hat{j} + (2 - 4)\hat{k} = 4\hat{i} + 4\hat{j} - 2\hat{k}$

Therefore, $W = \vec{F} \cdot \vec{s} = 28 + 12 + 10 = 50$ units

48. B. 3 **sq. unit**

Explanation: $\overrightarrow{OB} = 2\hat{i}$, $\overrightarrow{OC} = 3\hat{j}$

$\overrightarrow{OB} \times \overrightarrow{OC} = 6\hat{k}$; $|\overrightarrow{OB} \times \overrightarrow{OC}| = 6$

Therefore, area $= \frac{1}{2}(6) = 3$ sq. units

49. B. 63.61°

Explanation: $\vec{A} \cdot \vec{B} = 2 - 2 + 4 = 4$

$|\vec{A}| = \sqrt{1 + 4 + 4} = 3$, $|\vec{B}| = \sqrt{4 + 1 + 4} = 3$

$\cos\theta = \frac{4}{9} \approx 0.444$

Therefore, $\theta \approx 63.61°$

50. B. 37.5 s

Explanation: Relative velocity, $v_{rel} = 12 - 8 = 4\ ms^{-1}$

Relative distance = 150 m

Therefore, $t = \frac{150}{4} = 37.5$ s