MCQ 50 marks

1. The maximum resultant of two vectors is 28 units and the minimum resultant is 4 units. At what angle should the vectors act so that their resultant is five times the minimum resultant?

(A) 0° (B) 60° (C) 90° (D) 120°

2. Find the value of $(\hat{i} \times \hat{k}) \times (\hat{j} \times \hat{k})$.

(A) 0 (B) $\hat{i}$ (C) $-\hat{k}$ (D) $\hat{k}$

3. What is the angle between the vector $2\hat{i} + 3\hat{j}$ and the y-axis?

(A) $\tan^{-1}(\frac{3}{2})$ (B) $\tan^{-1}(\frac{2}{3})$

(C) $\sin^{-1}(\frac{2}{3})$ (D) $\cos^{-1}(\frac{2}{\sqrt{13}})$

4. Which of the following is a unit vector parallel to $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$?

(A) $\frac{2}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{j}$ (B) $\frac{1}{2}(2\hat{i} + \hat{j} - \hat{k})$

(C) $\frac{1}{\sqrt{6}}(2\hat{i} + \hat{j} - \hat{k})$ (D) $\frac{1}{\sqrt{6}}(2\hat{i} - \hat{j} + \hat{k})$

5. Car A moves at 10 ms^{-1} and Car B follows it at 15 ms^{-1}. Car C approaches A from the opposite direction at 15 ms^{-1}. When AC=AB=1 km, what minimum acceleration is required for B to overtake A before C reaches A?

(A) 0.25 ms^{-2} (B) 1.0 ms^{-2}

(C) 0.50 ms^{-2} (D) 0.10 ms^{-2}

6. The maximum resultant of two vectors is n times the minimum resultant. If the resultant magnitude is half the sum of the magnitudes of the two vectors, which of the following is correct for the angle θ?

(A) $\cos\theta = \frac{n^2+2}{2(1-n^2)}$ (B) $\cos\theta = \frac{n^2+3}{1-n^2}$

(C) $\cos\theta = \frac{n^2-2}{n^2+1}$ (D) $\cos\theta = \frac{2n^2+1}{n^2-1}$

7. The displacement of a particle is given by $\vec{s} = (3t^3)\hat{i} - (4t^2)\hat{j} + (2t)\hat{k}$. Find the scalar projection of acceleration onto the velocity vector at $t = 2$ s.

(A) 36.10 (B) 32.05 (C) 40.15 (D) 28.90

8. A policeman on a motorcycle moving at 10 ms^{-1} chases a jeep 100 m away which is moving away at 9 ms^{-1}. How long will it take for the policeman to catch the thief?

(A) 10 s (B) 50 s (C) 100 s (D) 200 s

9. Given $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$. If a vector $\vec{c}$ satisfies $\vec{a} \times \vec{c} = -\vec{b}$ and $\vec{a} \cdot \vec{c} = 4$, find the value of $|\vec{c}|^2$.

(A) 8.5 (B) 9.5 (C) 10.5 (D) 11.5

10. A motorcyclist travels 500 m straight in a field and then takes a 60° turn. He repeats this process. Find his displacement at the moment he takes the 3rd turn.

(A) 500 m (B) 1000 m

(C) 1500 m (D) 866 m

11. The ratio of the maximum and minimum resultant of two vectors is 5 : 3. What is the ratio of the magnitudes of the vectors?

(A) 4 : 1 (B) 5 : 2 (C) 7 : 3 (D) 10 : 7

12. A stone is thrown upwards at 50 ms^{-1} and another stone is simultaneously thrown downwards from the same spot at 50 ms^{-1}. What is the relative velocity of the first stone w.r.t. the second when the first stone reaches its highest point?

(A) Zero (B) 50 ms^{-1}

(C) 100 ms^{-1} (D) 150 ms^{-1}

13. If a vector makes angles of 30° and 60° with the positive X and Y axes respectively, what angle does it make with the Z axis?

(A) 0° (B) 45° (C) 90° (D) 180°

14. Rain is falling vertically at 10 ms^{-1}. Wind starts blowing North at 10 ms^{-1} and a motorcyclist moves North at 10 ms^{-1}. At what angle will the rain hit him?

(A) 0° (vertically) (B) 45°

(C) 30° (D) 60°

15. If $\phi = 2x^2yz + 2xyz + 2xz^3$, find the value of $\nabla\phi$ (gradient) at $(1, -1, 1)$.

(A) $-4\hat{i} + 4\hat{j} + 2\hat{k}$ (B) $-4\hat{i} - 4\hat{j} + 4\hat{k}$

(C) $-6\hat{i} + 4\hat{j} + 4\hat{k}$ (D) $4\hat{i} + 4\hat{j} + 8\hat{k}$

16. The stream velocity in a river is 5 kmh^{-1} and a swimmer's speed is 3 kmh^{-1}. At what angle with the stream should he swim to minimize drift?

(A) 120° (B) 126.87°

(C) 135° (D) 150°

17. To cross a river along the shortest path, a boatman steers the boat at an angle of 138.59° with the current. If the boat's speed is 4 kmh^{-1}, what is the velocity of the stream?

(A) 2 kmh^{-1} (B) 3 kmh^{-1}

(C) 1 kmh^{-1} (D) 2.5 kmh^{-1}

18. Rain hits a person vertically when he runs at 5 ms^{-1}. If he doubles his speed, the rain hits him at an angle of 30° with the vertical. Find the actual velocity of the rain.

(A) 10 ms^{-1} (B) $5\sqrt{3}$ ms^{-1}

(C) 8.66 ms^{-1} (D) 5 ms^{-1}

19. What is the angle between the vectors $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$?

(A) 79.02° (B) 45.00° (C) 90.00° (D) 120.00°

20. A river is 36 m wide and the current velocity is 3 ms^{-1}. If a boat is steered at 10 ms^{-1} at an angle of 37° with the current, how long will it take to cross the river?

(A) 12 s (B) 3.6 s (C) 5.98 s (D) 8 s

21. Find the volume of a parallelepiped formed by vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = 4\hat{j}$, and $\vec{C} = 5\hat{j} + 3\hat{k}$.

(A) 24 cubic unit (B) 12 cubic unit

(C) 40 cubic unit (D) 0 cubic unit

22. Given the vector field $\vec{v} = 4xy^3z\hat{i} - 3x^2yz\hat{j} - 3xyz\hat{k}$, find the value of div $\vec{v}$ (divergence) at $(1, -1, 1)$.

(A) −4 (B) 0 (C) −1 (D) 10

23. Given $\vec{P} = 3\hat{i} + 4\hat{j}$ and $\vec{Q} = 7\hat{i} + 24\hat{j}$. Find a vector which has the same magnitude as $\vec{Q}$ and is parallel to $\vec{P}$.

(A) $7\hat{i} + 24\hat{j}$ (B) $15\hat{i} + 20\hat{j}$

(C) $3\hat{i} + 4\hat{j}$ (D) $21\hat{i} + 28\hat{j}$

24. A boatman crosses a river in minimum time in 10 minutes, drifting 120 m along the bank. If he crosses along the shortest path, it takes 12.5 minutes. What is the boat's speed in still water?

(A) 15 m min^{-1} (B) 20 m min^{-1}

(C) 25 m min^{-1} (D) 12 m min^{-1}

25. If a vector makes equal angles with the three positive coordinate axes, what is the value of that angle?

(A) 45° (B) 54.74°

(C) 60° (D) 90°

26. If the resultant of two unit vectors $\hat{a}$ and $\hat{b}$ is $\sqrt{3}$, what is the angle between them?

(A) 0° (B) 45° (C) 60° (D) 90°

27. A particle of mass 2 kg has a position vector $\vec{r} = \hat{i} + 2\hat{j} + 2\hat{k}$ and velocity $\vec{v} = 2\hat{i} + \hat{j}$ ms^{-1}. What is its angular momentum relative to the origin?

(A) $-4\hat{i} + 8\hat{j} - 6\hat{k}$ (B) $4\hat{i} + 8\hat{j} + 6\hat{k}$

(C) 10.77 unit (D) (a) and (c) both

28. If $\phi = 2z - x^3y$ is a scalar field, find the value of $\vec{A} \cdot \nabla\phi$ at $(1, -1, 1)$ where $\vec{A} = 2x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$.

(A) 5 (B) 0 (C) −3 (D) 10

29. If $\vec{v} = 4xy^3z\hat{i} - 3x^3yz\hat{j} + 5xyz^2\hat{k}$, find $\nabla \times \vec{v}$ (Curl).

(A) $4xy^2z\hat{i} - 3x^2yz\hat{j} + 5xyz\hat{k}$

(B) $12xy^2z - 9x^2yz + 10xyz$

(C) $(5xz^2 + 3x^3y)\hat{i} + (4xy^3 - 5yz^2)\hat{j} - (9x^2yz + 12xy^2z)\hat{k}$

(D) $(5xz^2 - 3x^3y)\hat{i} + (4xy^3 - 5yz^2)\hat{j} + \hat{k}$

30. In a river with a current of 3 kmh^{-1}, a boatman steers to reach the opposite bank exactly for 1 hour. Then he changes direction to cross in minimum time. If the boat's speed is 4 kmh^{-1}, find the total time taken to cross the 5 km wide river.

(A) 1.500 h (B) 1.588 h
(C) 2.000 h (D) 1.250 h

31. If the velocity of C relative to A is 25 ms^{-1} and the velocity of A relative to B is 5 ms^{-1} in the opposite direction (assume they are on a straight line and the distance AB is 20 m), how long will it take for car C to pass car B?

(A) 1 s (B) 2 s (C) 3 s (D) 4 s

32. If the vectors $3\hat{i} - \hat{j} + 2\hat{k}$ and $x\hat{i} - 2\hat{j} + 4\hat{k}$ are parallel to each other, find the value of x.

(A) -6 (B) 6 (C) 5 (D) -5

33. A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from point (1, 2, 3) to point (5, 4, 1). Find the work done in standard units.

(A) 40 unit (B) 30 unit
(C) 25 unit (D) 15 unit

34. Find the area of triangle ABC with vertices A(1, −2, 3), B(0, 1, 2), and C(2, 1, 4).

(A) 10.5 sq. unit (B) $3\sqrt{2}$ sq. unit
(C) 5.25 sq. unit (D) 15 sq. unit

35. If the vector field $\vec{A} = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$ is irrotational, find the values of a, b, c.

(A) a = 4, b = −3, c = 5 (B) a = 5, b = 3, c = −4
(C) a = −4, b = 5, c = 3 (D) a = 3, b = 4, c = 5

36. Rain velocity is 5 ms^{-1} (vertically down) and wind is 10 ms^{-1} (East to West). If a driver moves East at 20 ms^{-1}, find the magnitude of the relative velocity of rain.

(A) $3\sqrt{25}\ ms^{-1}$ (B) $9\sqrt{25}\ ms^{-1}$
(C) $\sqrt{925}\ ms^{-1}$ (D) $25\ ms^{-1}$

37. Two persons start from points A and B with velocities 10 ms^{-1} and 12 ms^{-1} respectively and meet at a junction O. If AB = 100 m and angle $\angle OAB = 60°$, after how much time will they meet?

(A) 5.51 s (B) 7.51 s
(C) 10.00 s (D) 12.45 s

38. If $\phi = \frac{1}{|\vec{r}|}$ (where $\vec{r}$ is the position vector), find $\nabla\phi$ (gradient).

(A) $\frac{\vec{r}}{r^2}$ (B) $-\frac{\vec{r}}{r^3}$
(C) $\frac{\vec{r}}{r^3}$ (D) 0

39. If $\vec{v} = \vec{\omega} \times \vec{r}$ (where $\vec{\omega}$ is a constant vector), which of the following is consistent with this relation?

(A) $\nabla \times \vec{v} = 2\vec{\omega}$ (B) $\nabla \cdot \vec{v} = \vec{\omega}$
(C) $\nabla\phi = \vec{v}$ (D) curl $\vec{v} = 0$

40. Rain falls vertically at 5 ms^{-1} and wind blows East to West at 2 ms^{-1}. If a car moves East at 3 ms^{-1}, what is the relative velocity of rain?

(A) 5 ms^{-1} (B) $5\sqrt{2}\ ms^{-1}$
(C) 7 ms^{-1} (D) 10 ms^{-1}

41. Find a vector perpendicular to the plane of $\vec{B} = 3\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j} + 2\hat{k}$ having magnitude same as $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$.

(A) $\pm\frac{\sqrt{14}}{\sqrt{122}}(-8\hat{i} - 3\hat{j} + 7\hat{k})$ (B) $4\hat{i} - 2\hat{j} + 3\hat{k}$
(C) 0 (D) $-8\hat{i} - 3\hat{j} + 7\hat{k}$

42. Vector $\vec{c}$ is coplanar with $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{c} \cdot \vec{b} = 24$, find $|\vec{c}|$.

(A) $4\sqrt{21}$ (B) $\sqrt{116}$ (C) 24 (D) 10

43. In triangle ABC, $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$. Find the length of the median through A.

(A) $\sqrt{33}$ (B) $\sqrt{18}$ (C) 5 (D) $\sqrt{72}$

44. Find $\hat{a} \cdot \hat{b} + \hat{b} \cdot \hat{c} + \hat{c} \cdot \hat{a}$ for unit vectors with zero sum.

Ⓐ -1.5 Ⓑ 1 Ⓒ -3 Ⓓ 0

45. The maximum resultant of two vectors is 34 units and the minimum resultant is 6 units. Find the magnitudes of the vectors.

Ⓐ 16, 12 Ⓑ 20, 14

Ⓒ 18, 16 Ⓓ 22, 12

46. The ratio of the maximum and minimum resultant of two vectors is 7 : 3. What is the ratio of the magnitudes of the vectors?

Ⓐ 4 : 1 Ⓑ 5 : 2 Ⓒ 7 : 3 Ⓓ 3 : 1

47. Forces $\vec{F}_1 = 5\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{F}_2 = 2\hat{i} + \hat{j} - 2\hat{k}$ displace a particle from $(2, 1, 4)$ to $(6, 5, 2)$. Find the work done.

Ⓐ 40 unit Ⓑ 50 unit

Ⓒ 60 unit Ⓓ 45 unit

48. Find the area of triangle OBC with vertices $O(0, 0, 0)$, $B(2, 0, 0)$, and $C(0, 3, 0)$.

Ⓐ 2 sq. unit Ⓑ 3 sq. unit

Ⓒ 6 sq. unit Ⓓ 1.5 sq. unit

49. What is the angle between the vectors $\vec{A} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + 2\hat{k}$?

Ⓐ $45.00°$ Ⓑ $63.61°$

Ⓒ $79.02°$ Ⓓ $90.00°$

50. A policeman on a motorcycle moving at 12 ms^{-1} chases a jeep 150 m away which is moving away at 8 ms^{-1}. How long will it take for the policeman to catch the thief?

Ⓐ 25 s Ⓑ 37.5 s

Ⓒ 50 s Ⓓ 18.75 s